Progress in the Determination of a Gravimetric Quasigeoid Model over Sweden

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Introduction

One alternative to the traditional remove-compute-restore procedure that has hitherto been used to compute the Nordic geoid is to use the Least Squares Modification Method (LSMM) with additive corrections. This technique, which has been developed at the Royal Institute of Technology (KTH) in Stockholm, includes least squares kernel modification together with topographic, downward continuation, atmospheric and ellipsoidal corrections.

This poster presents some recent results from an on-going joint project between KTH and the National Land Survey of Sweden (LMV), whose main purpose is to evaluate the KTH approach numerically and to compute a gravimetric quasigeoid model for Sweden. It should be viewed as being conducted under the auspices of the working group for geoid determination of the Nordic Geodetic Commission (NKG). More specifically, the aim of the poster is to evaluate the least squares modification method with additive corrections using 108 high quality GPS/levelling height anomalies covering the major parts of Sweden except for the mountainous areas to the North West. The results are also compared to the Nordic NKG 2004, which is up to now the most accurate gravimetric model available over Sweden.

Determination of the height anomaly using the least squares modification method with additive corrections

In the least squares modification of Stokes' formula (e.g. Sjöberg 1991), Stokes' kernel is modified in such a way that the expected global mean square error is minimised. This technique can be applied with the standard remove-compute-restore estimator (e.g. Ågren 2004), but according to the KTH practice the so-called combined estimator is preferred (Sjöberg 2003). This means that Stokes' formula (truncated to a cap) is applied to the uncorrected surface gravity anomaly, Δg. After that, the height anomaly ζ is computed by adding a number of corrections, i.e.

$$\zeta = \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^M(\psi)\Delta g d\sigma + \frac{R}{2\gamma}\sum_{n=2}^{M}\left(s_n + Q_n^L\right)\Delta g_n^{GGM} + \delta\zeta_{COMB} + \delta\zeta_{DWC} + \delta\zeta_{ATM} + \delta\zeta_{ELL} \quad (1)$$

where σ_0 is the spherical cap, R is the mean Earth radius, γ_0 is mean normal gravity, $S^L(\psi)$ is the modified Stokes' function, s_n are the modification parameter, M is the maximum degree of the Global Geopotential Model (GGM), Q_n^L are the Molodensky truncation coefficients and Δg_n^{GGM} is the Laplace harmonic of degree *n*.

The additive corrections

The four corrections are derived in such a way that the same result should ideally be obtained as when the remove-compute-restore technique is utilised (except for numerical effects). They can be computed as:

- The *combined topographic effect* $\delta\zeta_{COMB}$ vanishes in the height anomaly case (e.g. Sjöberg 2000).
- The *downward continuation effect* $\delta\zeta_{DWC}$ is (Sjöberg 2000; Ågren 2004),

$$\delta\zeta_{DWC}(P) = 3\frac{\zeta_P^0}{r_P} + \frac{R}{2\pi}\sum_{n=2}^{M}\left(s_n + Q_n^M\right)\left[\left(\frac{R}{r_P}\right)^{n+2} - 1\right]\Delta g_n^{GGM}(P) + \frac{R}{4\pi\gamma}\iint_{\sigma_0} S^M(\psi)\left(\left.\frac{\partial \Delta g}{\partial r}\right|_Q \left(H_P - H_Q\right)\right)d\sigma_Q \quad (2)$$

  where P is the computation point, H is the topographic height, $r_P = R + H_P$, ζ_P^0 is an approximate value of the height anomaly and Q is the running point in Stokes' integral.

- The *combined atmospheric effect* $\delta\zeta_{ATM}$ can be approximated to order H by (Sjöberg and Nahavandchi 2000)

$$\delta\zeta_{ATM}(P) \approx \delta N_{ATM}(P) = -\frac{2\pi R\rho_0}{\gamma}\sum_{n=2}^{M}\left(\frac{2}{n-1} - s_n - Q_n^M\right)H_n(P) - \frac{2\pi R\rho_0}{\gamma}\sum_{n=M+1}^{\infty}\left(\frac{2}{n-1} - \frac{n+2}{2n+1}Q_n^M\right)H_n(P) \quad (3)$$

  where ρ_0 is the atmospheric density at sea level and H_n is the Laplace harmonic of degree *n* for the topographic height.

- The *ellipsoidal correction to the modified Stokes' formula* $\delta\zeta_{ELL}$ to order e^2 is (Sjöberg 2004):

$$\delta\zeta_{ELL} \approx \delta N_{ELL}(P) = \frac{R}{2\gamma}\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^* - Q_n^M\right)\left(\frac{a-R}{R}\Delta g_n^{GGM}(P) + \frac{a}{R}(\delta g_e)_n\right) \quad (4)$$

  where $s_n^* = s_n$ if $2 \le n \le M$ and $s_n^* = 0$ otherwise. Furthermore,

$$(\delta g_e)_n = \frac{e^2}{2a}\sum_{m=-n}^{n}\left\{\left[3-(n+2)F_{nm}\right]T_{nm} - (n+1)G_{nm}T_{n-2,m} - (n+7)E_{nm}T_{n+2,m}\right\}Y_{nm}(P) \quad (5)$$

  in which T_{nm} are spherical harmonic coefficients for the disturbing potential. See Sjöberg (2004) for the ellipsoidal coefficients E_{nm}, F_{nm} and G_{nm}.

Some comments on the method

- Analytical continuation is made to point level using the g_1 term in Moritz (1980), but the method differs from Moritz's in that the least squares modification of Stokes' formula is utilised with improved atmospheric and ellipsoidal corrections.
- One problem is that Stokes' quadrature is made on the rough surface gravity anomaly, which results in large *discretisation errors*. However, by taking advantage of the remove-compute-restore philosophy for the gridding of a comparatively dense Δg grid, such errors can be counteracted; cf. below.
- Advantages with the combined estimator are that the "real" importance of the correction terms is apparent and that it is easier t compute the atmospheric and ellipsoidal corrections in this way.

Gravity data and Digital Elevation Model (DEM)

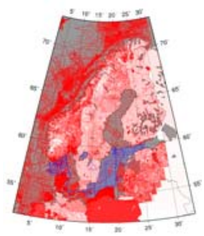


Figure 1: Gravity observations from the NKG database. Blue dots indicate airborne gravity.

- 495 614 gravity observations from the NKC gravity database; see the figure to the left.
- Cleaning of multiple observations at the same location (weighted average).
- Selection of the observation with smallest standard deviation in each compartment of a grid with 2km x 2km resolution using SELECT (GRAVSOFT).
- The Nordic height model SCANDEM 2004 (Bilker 2004) was densified to 500m x 500m resolution using the Swedish photogrammetric DEM. In other areas spline interpolation. It was also extended to the South using SRTM30
- Global 15' x 15' DEM derived from SRTM30 and GTOPO30 (for the atmospheric correction).

Gridding of the surface gravity anomalies

To reduce the discretisation errors, a remove-compute-restore strategy is utilised with a comparatively dense grid (1' x 2'). The following gravity anomaly effects are reduced and restored:

- The long wavelength effect from a GGM with maximum degree *M*. See Table 2 for the GGMs in question.
- The high-frequency part of the topographic effect computed by the RTM method with a smooth reference surface (corresponding to *M)*. The TC program in GRAVSOFT was applied for this task.

The gridding of the reduced gravity anomalies is made in two steps:

- Search for gross errors by cross validation. Each observation is predicted from its neighbours using inverse distance interpolation. In case the obtained difference is larger than 20 mGal, then the observation is rejected (the limit is suitable for Sweden).
- The gridding is made using least squares collocation with individual weights for the reduced gravity anomaly observations. GEOGRID (GRAVSOFT) with 25 km correlation length.

Height anomalies from GPS/levelling

108 GPS/levelling height anomalies in the reference systems SWEREF 99 and RH 2000 are used to evaluate the different models; the locations are illustrated on the maps to the right. The stations are either permanent GPS stations (SWEPOS) or benchmarks of the third precise levelling of Sweden.

- The normal heights in the new RH 2000 system have either been determined in the RH 2000 adjustment (benchmarks) or by utilising high quality levelling connections (SWEPOS stations)
- The coordinates of the SWEPOS stations are very well determined. In fact, they define SWEREF 99. All other GPS heights have been derived using at least 48 hours of observations, a Dorne Margolin T antenna and processing in the Bernese software.

Test of the gravity anomaly weighting and cap radius

- Three different strategies were tested: low, medium and high weighting of the gravity anomalies. In all cases, the error degree variances are assumed to be a combination of white noise and the reciprocal distance model (Moritz 1980).
- We characterise the weightings by the spherical harmonic degree *K*, for which the error degree variance of the GGM is equal to that of the terrestrial gravity anomaly. For lower degrees the GGM is more accurate and vice versa.

Table 1: Evaluation of different weighting schemes and integration radii. Statistics for the 108 GPS/levelling residuals after a 4 parameter fit. GGM02C/EGM 96. Unit: mm.

Δg weighting	K	ψ_0 (deg.)	Min	Max	Mean	StdDev
High	65	3.0	-42	51	0	17
Medium	85	3.0	-38	56	0	16
Low	105	3.0	-45	46	0	22
Medium	85	2.0	-38	43	0	17

Evaluation of a three GGMs originating from GRACE

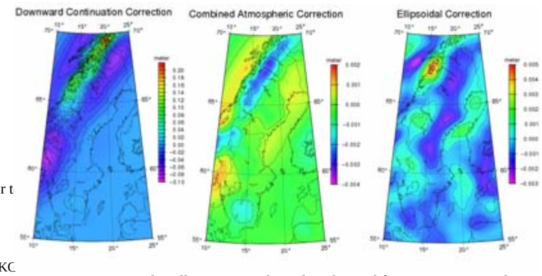
- Two combined GGMs (GGM02C/EGM 96 and EIGEN GL04C) and one satellite-only model (GGM02S) have been tested
- The GGM02C/EGM 96 model is constructed by using GGM02C up to degree 200. Above that, the EGM 96 coefficients are utilised. The model is very similar to the GGM used for NKG 2004.

Table 2: Comparison of using different GGMs. Statistics for the 108 GPS/levelling residuals after a 4 parameter fit. Medium gravity anomaly weighting. Unit: mm.

GGM	M	Min	Max	Mean	StdDev
GGM02C/EGM 96	360	-38	56	0	16
EIGEN GL04C	360	-44	53	0	17
GGM02S	110	-41	52	0	20

- The medium gravity anomaly weighting (K = 85), $\psi_0 = 3°$ and GGM02C/EGM 96 were chosen for further analysis.

Magnitude of the additive corrections



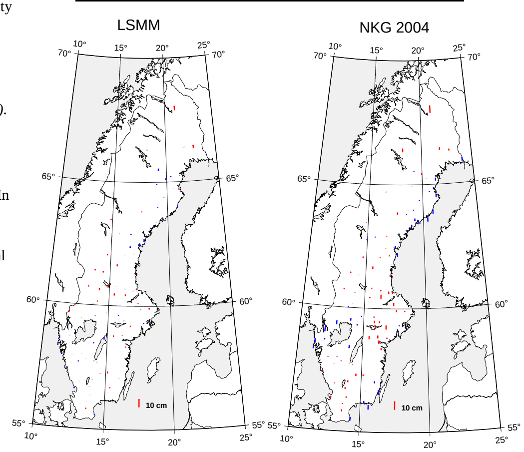
- Notice that all corrections depend on the modification, cap size and maximum degree for the GGM being used.
- The atmospheric and ellipsoidal corrections are small for the LSMM, but are significant for the unmodified Stokes' function (Ellmann 2004)

Comparison with NKG 2004

- The Nordic NKG 2004 was computed by the remove-compute-restore method using the RTM reduction and a Wong and Gore type of modification.
- Approximately the same GGM were used in both cases (GGM02C/S and EGM 96 for the highest degrees to M = 360)

Table 3: Comparison of the LSMM (medium weighting of Δg, GGM02C/EGM96) with NKG 2004. Statistics for the 108 GPS/levelling residuals after a 4 parameter fit. Unit: mm.

Geoid model	Min	Max	Mean	StdDev
LSMM	-38	56	0	16
NKG 2004	-70	88	0	28



Conclusions

- The 16 mm standard error obtained by the least squares modification method with additive corrections is promising. If the standard errors for GPS and levelling are taken as 10 mm and 5 mm, respectively (which is quite optimistic), then the propagated standard error for the fitted gravimetric quasigeoid becomes 11 mm.

  However, it should be noticed that the GPS/levelling observations are located in comparatively flat areas of the country. In the near future, they will be extended also to the mountainous parts to the North West, which will provide a tougher test.
- The results are significantly better for the present method compared to NKG 2004. As can be seen from the above figures, the main improvement is due to the reduction of the long wavelength errors that are present in NKG 2004; cf. for instance the systematic bulges south of the 60° latitude. It is believed that these errors are due to systematic errors in the terrestrial gravity (not shown here).
- The atmospheric and ellipsoidal corrections are very small for the present modification, integration radius and maximum degree *M*.

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