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# Seek distances in two-headed disk systems

M. Manzur Murshed *, M. Kaykobad

*Department of Computer Science & Engineering, Bangladesh University of Engineering & Technology, Dhaka-1000, Bangladesh*

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**Abstract**

We make a correction to the calculation of mean seek time in a recent algorithm of intelligently positioned two-headed disk systems. We also introduce a new two-headed disk system with better mean seek time.

*Keywords:* Operating systems; Performance evaluation; Analysis of algorithms

## 1. Introduction

Two-headed disk systems, as categorized in [3], may have (a) two arms autonomously controlled [2,5], (b) two arms and a single controller [1] and, (c) a single arm with two fixed heads on it [4,5]. In calculating seek distances for intelligently positioned two-headed systems in [5] the authors ignore jockeying time of the dummy head. In this paper we recalculate the seek distance considering the penalty for jockeying. A new two-headed disk system has been introduced in which the disk surface is partitioned into two equal halves and each dedicated head serves the half it is in while the other tries to occupy the center of its half.

## 2. Preliminaries

Consider that a disk is a linear storage medium consisting of $N$ cylinders, each of which is divided into a number of sectors. Let the cylinders be numbered sequentially starting from the innermost cylinder. Let $C_i$ denote the $i$th cylinder and $S_{C_i,C_j}$ denote the seek time when the head is on cylinder $C_i$ and service is done on cylinder $C_j$. Without losing any generality we may assume cylinder numbers on the left of a particular cylinder is less and vice versa. Now, in the disk, cylinders are spaced in such a way that $\forall i,j$, $S_{C_i,C_j} \propto |i-j|$. Let $S_{C_i,C_{i+1}}$ be considered as unit of time and in that case

$$\forall i,\ j \quad S_{C_i,C_j} = |i-j| \text{ units}. \tag{1}$$

Let $L$ denote the latency time plus the data transfer time in a seek expressed in the above-mentioned units. It is assumed that only a few sectors will be read sequentially and therefore, the data transfer time will be a few tenths of the latency time.

In an earlier work by Waters [6] a conventional one-headed disk was examined, and it was estimated that if the head is positioned randomly on top of a cylinder and a random request arrives,

* Corresponding author.


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then the mean seek time is $N/3$.

Manolopoulos and Vakali [5] analyzed a two-headed disk system where each head is attached to an independent arm which can be positioned randomly on any of the $N$ cylinders. It was assumed that when a request arrives, the head with the minimum distance from the requested cylinder will serve it while the other head will remain idle in its position. The mean seek time of this system was calculated to be approximately $5N/24$.

In [4] analysis was made on a two-headed system where both the heads are fixed on a single arm. The optimal distance separating the two heads was shown to be $N/2-1$, and the mean seek time was calculated to be approximately $N/6$.

In [2] another scheduling for two-headed disk systems was introduced where each head is attached to an independent arm which can be positioned randomly on any of the $N$ cylinders. When a request arrives, the closest head moves to serve, while the other head jockeys to take a position anticipating the cylinder number that the next request will hit. Among the two heads, let the serving head be called *active* while the jockeying head be called *dummy*. If no request is under process, and the active head of the last request is on the cylinder $C_a$ then the dummy head will be on the cylinder

$$C_d(C_a)=\begin{cases}\lfloor C_a/3\rfloor, & \text{if } C_a>\lfloor N/2\rfloor,\\ \left\lfloor \dfrac{2N+C_a}{3}\right\rfloor, & \text{otherwise.}\end{cases} \tag{2}$$

Here, $C_d(C_a)$ denotes that $C_d$ is a function of $C_a$, i.e., $C_d$ depends on $C_a$.

Under the assumption that jockeying is performed at no cost the mean seek time for this system is calculated in [5] in the following way. Let $A$ be the cylinder number on which the active head of the previous request is resting, and $X$ be the current requested cylinder number. $A$ is then actually the requested cylinder number of the previous request. In calculating the mean seek value, we may reasonably consider only the half of the disk, and then double the value because of the symmetric nature. Without losing any generality we may assume $N$ even and then, the mean seek time, as calculated in [5], is obtained by considering requests to all possible cylinders to the left and right of the active head and also to the left and right of the dummy head. Hence, the mean seek time is

$$\begin{aligned}\bar{S}=\frac{2}{N^2}\sum_{A=1}^{N/2}\Bigg\{&\sum_{X=1}^{A-1}(A-X)\\ &+\sum_{X=A+1}^{\lfloor(2A+N)/3\rfloor}(X-A)\\ &+\sum_{X=\lfloor(2A+N)/3\rfloor+1}^{\lfloor(2N+A)/3\rfloor-1}\left(\left\lfloor\frac{2N+A}{3}\right\rfloor-X\right)\\ &+\sum_{X=\lfloor(2N+A)/3\rfloor+1}^{N}\left(X-\left\lfloor\frac{2N+A}{3}\right\rfloor\right)\Bigg\}.\end{aligned} \tag{3}$$

If we assume $N$ to be large enough (in practice it is around 1000) so that the argument of every floor function can be considered integer, the mean seek time is then approximated by $5N/36$ in [5].

## 3. Correction in mean seek time of intelligently positioned head system

In the calculation of the mean seek time, stated above, the assumption that jockeying is performed at no cost is not practical. Let, on finishing a request, the active and dummy heads be resting on cylinders $C_a$ and $C_d(C_a)$ respectively. When a new request arrives both the heads start moving. If the previous active head is closer to the requested cylinder $X$, it will move from cylinder $C_a$ to cylinder $X$ while the previous dummy head will move from cylinder $C_d(C_a)$ to cylinder $C_d(X)$. In this case the state of heads is not changed, i.e., the active head remains active and the dummy head remains dummy. But if the previous dummy head is closer to cylinder $X$, the state of the heads will be changed and thus the previous active head (dummy for this request) will move from cylinder $C_a$ to cylinder $C_d(X)$ while the previous dummy head (active for this request) will move from cylinder $C_d(C_a)$ to cylinder $X$. Hence, the jockeying can be ignored only if

$$S_{C_a,X} + L \geqslant S_{C_d(C_a),C_d(X)},$$

if state of heads did not change,

$$S_{C_d(C_a),X} + L \geqslant S_{C_a,C_d(X)},$$

if state of heads changed.

If $L$ is sufficiently large then there is no doubt that the mean seek time is $5N/36$. It is then obvious that as $L$ is decreased the mean seek time will be increased because of the presence of penalty incurred by the incomplete jockeying. To get the maximum possible value of the mean seek time of this system we may rightly assume $L = 0$.

As $2N/3 > N/2 > N/3$, it is obvious from (2) that $C_a \leqslant \lfloor N/2 \rfloor$ implies $C_d(C_a) > \lfloor N/2 \rfloor$ and vice versa. So, if $L$ is not sufficiently large then the condition for ignoring jockeying is violated when the serving head crosses the half way line, i.e., the middle cylinder $C_{\lfloor N/2 \rfloor}$ since dummy head has to traverse too long a distance to be covered even when the latency time is considered.

Let the penalty, incurred when the jockeying head crosses half way cylinder of the disk system, be called the *cross over penalty*. This kind of penalty may be calculated by following the formation of (3).

When the previous active head is serving no cross over penalty exists if the requested cylinder $X \leqslant N/2$. But when $N/2 < X \leqslant \lfloor (2A+N)/3 \rfloor$ cross over penalty will occur if active head movement $X - A$ is less than the dummy head movement $\lfloor (2N+A)/3 \rfloor - \lfloor X/3 \rfloor$, i.e., if $X < N/2 + A$, ignoring the floor functions for large $N$. From now on, whenever possible, we will ignore floor functions without any reference. Now, to the right side the previous active head can serve the request up to $\lfloor (2A+N)/3 \rfloor$ cylinder and possibility of penalty exists only when $\lfloor (2A+N)/3 \rfloor \geqslant N/2$, i.e., $A \geqslant N/4$. It is then obvious that for $N/4 > X \geqslant N/2$ the upper limit of $X$ for cross over penalty includes $\lfloor (2A+N)/3 \rfloor$. Hence, the mean cross over penalty incurred by the previous active head is

$$\bar{P}_a = \frac{2}{N^2} \sum_{A=\lfloor N/4 \rfloor}^{N/2} \sum_{X=N/2}^{\lfloor (2A+N)/3 \rfloor} \left( \left\lfloor \frac{2N+A}{3} \right\rfloor - \left\lfloor \frac{X}{3} \right\rfloor - X + A \right) \simeq \frac{13N}{648}. \tag{4}$$

It may be well mentioned here that throughout the paper simplification of complex expressions are done using *Maple*.

Similarly it can be shown that previous dummy head, while serving on the right side creates no cross over penalty but when the previous dummy head is serving on the left cross over penalty occurs for $\lfloor (2A+N)/3 \rfloor < X < N/2$ and $A \leqslant N/4$. Mean of this penalty is

$$\bar{P}_d = \frac{2}{N^2} \sum_{A=1}^{\lfloor N/4 \rfloor} \sum_{X=\lfloor (2A+N)/3 \rfloor}^{N/2} \left( \left\lfloor \frac{2N+X}{3} \right\rfloor - A - \left\lfloor \frac{2N+A}{3} \right\rfloor + X \right) \simeq \frac{13N}{648}. \tag{5}$$

So, at $L = 0$ overall mean cross over penalty,

$$\bar{P} = \bar{P}_a \bar{P}_d = \frac{26N}{648}. \tag{6}$$

Besides this penalty, significant magnitude of *non cross over penalty* also exists when the previous dummy head serves near its current position. When the previous dummy head is serving on the left non cross over penalty occurs if $\lfloor (2N+A)/3 \rfloor - X < \lfloor X/3 \rfloor - A$, i.e., $X > N/2 + A$ and $\lfloor (2N+A)/3 \rfloor \geqslant N/2 + A$, i.e., $A \leqslant N/4$. Hence, mean non cross over penalty caused by the previous dummy head while serving to the left is

$$\bar{Q}_l = \frac{2}{N^2} \sum_{A=1}^{\lfloor N/4 \rfloor} \sum_{X=N/2+A}^{\lfloor (2N+A)/3 \rfloor} \left( \left\lfloor \frac{X}{3} \right\rfloor - A - \left\lfloor \frac{2N+A}{3} \right\rfloor + X \right) \simeq \frac{N}{324}. \tag{7}$$

Similarly, when the previous dummy head is serving on the right some non cross over penalty is created mean of which is

$$\overline{Q}_r = \frac{2}{N^2} \sum_{A=1}^{\lfloor N/4 \rfloor} \sum_{X=\lfloor (2N+A)/3 \rfloor}^{N-A} \left( \left\lfloor \frac{X}{3} \right\rfloor - A - X + \left\lfloor \frac{2N+A}{3} \right\rfloor \right).$$

$$\simeq \frac{2N}{324}. \tag{8}$$

Hence, at $L = 0$ overall mean non cross over penalty,

$$\overline{Q} = \overline{Q}_l + \overline{Q}_r = \frac{3N}{324}. \tag{9}$$

So, the effective mean seek time of the intelligently positioned head system lies in the range of $\overline{S} + \overline{P} + \overline{Q}$ to $\overline{S}$ i.e., from $61N/324$ to $5N/36$ depending on the latency time of the disk system.

## 4. A new scheduling for two-headed disk system

Suppose there are two heads mounted in two independently controlled arms. Let the total disk surface be divided into two even halves, and each head be assigned to a particular half. Whenever a request arrives the head which is dedicated for the half containing the requested cylinder will serve the request while the non serving head will move towards the center of its half.

Now, if the status of the heads are not changed, i.e., previous active and dummy heads are current active and dummy heads respectively, then the mean seek time will be $(N/2)/3 = N/6$. But if the status of the heads are changed, i.e., previous active and dummy heads are current dummy and active heads respectively, then the mean seek time will be $(N/4)/2 = N/8$. Assuming requests to be uniformly distributed throughout the disk cylinders we may assume that expected value of the probability distribution function of serving the request either by previous active head or by previous dummy head is $1/2$.

Therefore, we may conclude that mean seek time of this scheduling is

$$\overline{S} = \frac{1}{2}\frac{N}{6} + \frac{1}{2}\frac{N}{8} = \frac{7N}{48}. \tag{10}$$

For obvious reasons we can say that no cross over penalty exists for this scheduling. But imagine a situation when the previous dummy head is serving a request for a cylinder in the vicinity of its central position while the previous active head (dummy for this request) tries to move from its extreme position (previous request was for an extreme cylinder) to the center. Surely for small $L$ some penalty exists. If $L$ is sufficiently large then there is no doubt that the mean seek time is $7N/48$. It is then obvious that as $L$ is decreased the mean seek time will be increased. To get the maximum possible value of the mean seek time of this system we may rightly assume $L = 0$.

Let, on finishing a request, the active head be resting on cylinder $A$ while the dummy head resting on cylinder $B$. Using the symmetric nature we can assume $A$ to take any value between 1 to $N/4$, ignoring the floor function. This implies $B$ to be $3N/4$. Now, if any request for cylinder $N/2 < X \leqslant 3N/4$ (for symmetry $X > 3N/4$ is not considered) arrives penalty occurs if $N/4 - A > 3N/4 - X$, i.e., $X > N/2 + A$ and $3N/4 \geqslant N/2 + A$, i.e., $A \leqslant N/4$. It is already pointed out that probability that the previous dummy head will serve the current request is $1/2$. So, the mean penalty is

$$\overline{P} = \frac{1}{2}\frac{1}{N^2/4} 2 \sum_{A=1}^{N/4} 2 \sum_{X=N/2+A}^{3N/4} \left\{ \left( \frac{N}{4} - A \right) - \left( \frac{3N}{4} - X \right) \right\}. \tag{11}$$

which after simplifying using *Maple* comes to

$$\overline{P} \simeq \frac{N}{48}. \tag{12}$$

So, the effective mean seek time of the new system lies in the range of $\overline{S} + \overline{P}$ to $\overline{S}$ i.e., from $N/6$ to $7N/48$ depending on the latency time of the disk system.

This is obviously better than the intelligently positioned head system. When latency time is

Table 1
Simulated and theoretical mean seek distances of two-headed disk systems

| $N$ | Intelligent system | | | | New system | | | |
|---|---|---|---|---|---|---|---|---|
| | Simulated | | Theoretical | | Simulated | | Theoretical | |
| | Avg. | S.D. | $L=0$ | No penalty | Avg. | S.D. | $L=0$ | No penalty |
| 500 | 99 | 0.88 | 95 | 70 | 84 | 0.31 | 84 | 73 |
| 750 | 148 | 0.90 | 142 | 105 | 125 | 0.53 | 126 | 110 |
| 1000 | 198 | 1.36 | 189 | 139 | 167 | 0.65 | 167 | 146 |
| 1250 | 246 | 1.50 | 236 | 174 | 209 | 0.70 | 209 | 183 |
| 1500 | 296 | 1.89 | 283 | 209 | 251 | 0.77 | 250 | 219 |

high the ratio of mean seek time of the new system to the intelligent system is $21/20$, i.e., the mean seek time of the new system is 5% higher. But for lower latency time the situation is reversed in dramatic magnitude. For zero latency time the ratio of mean seek time of the new system to the intelligent system is $54/61$, i.e., the mean seek time of the new system is 11.5% lower.

It may be noted that in the new scheduling if jockeying is not done, irrespective of latency, mean seek time will be $N/6$ which is still better than the effective mean seek time of the intelligently positioned head system with lower latency.

## 5. Experimental results

Both the intelligently positioned head system and the newly introduced disk system have been simulated in C language on a 80486 machine. The range of total number of cylinders, $N$ has been chosen from the knowledge of available disk systems in the market. In the simulation a uniform random number generator is used to generate requested cylinder number. In Table 1, for a particular configuration a run of $20N$ seeks has been completed for 10 times and then the average and standard deviation of the mean seek time are computed. Throughout the simulation the latency time is considered to be zero.

It is easily observed that simulated results follow theoretical findings. Our correction in the calculation of mean seek time of the intelligent system, for low latency time, is clearly reflected in the simulation results. Moreover, the simulation results of the newly introduced system, as expected theoretically, outperformed those of the intelligent system for all $N$.

## 6. Conclusion

Latency time plays a key role in calculating mean seek time of some two-headed disk systems. In a two-headed disk system the simple strategy of dividing the cylinders into equal halves and served by dedicated head has better mean seek time than that of the strategy proposed in [5].

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