2. Electromagnetism fundamentals

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In this book, the whole theory of electromagnetism is derived from a couple of elementary experiments.
From one experiment is derived the expression for electromagnetic force, and from the other, the expression for magnetic field of current carrying wire.
From these two experiments, so-called Lorentz equations are derived.
From Lorentz equations are derived Maxwell equations.

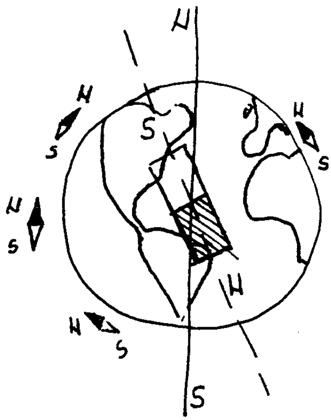

Figure II-1. Planet Earth behaves as a gigantic magnet (magnetic dipole). North pole of the magnet points toward (geographic) South Pole, and vice versa. The axis of the dipole meets the Earth rotation axis at an angle of about 11°.

Historical prelude

By the eighteenth century, it was known that every magnet is characterized by two poles, one that attracts north pole of compass needle and the other the south pole. Being that the same kinds of poles repel each other, and the opposite poles attract each other, the pole of the magnet that attracts the south pole of compass needle is named north, and vice versa. In the same time when he revealed the fundamental law of electrostatics, Charles-Augustin de Coulomb showed that the magnetic force among magnetic poles is inversely proportional to the square of the distance r between them:

(2-1) $$\mathrm{F} = \frac{\mu_0}{4\pi}\cdot\frac{Q_\mathrm{m}\cdot q_\mathrm{m}}{\mathrm{r}^2},$$

where Q_m, and Q_m are named “magnetic masses”. Taking that north poles have positive magnetic mass and south negative, it may conclude, in the same manner as in electrostatic, that corresponding magnetic field (which is inadequately named magnetic induction):

(2-2) $$\mathbf{B} = \frac{\mathbf{F}}{q_\mathrm{m}},$$

and its lines of force “go” from north to south.

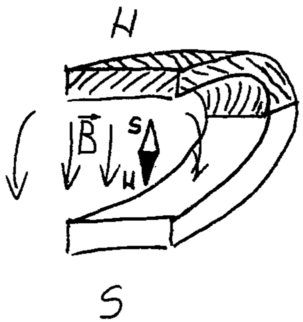

Figure II-2. Magnetic field has (by convention) direction from north to south magnetic pole. Direction of the magnetic field may be determined by the direction of magnetic needle north pole.

Rapid development of electromagnetism begun by the discovery that magnetic needle aligns itself perpendicularly to a current-carrying wire, which Danish physicist Ørsted (Hans Christian) published in the summer of 1820. It was definite experimental evidence of the relationship between electricity and magnetism, and led to many discoveries and experiments. Those experiments we will reduce on two and from them (including the results obtained in electrostatics) we will deduce the whole theory of electromagnetism.

Assuming that magnetism relates to current-carrying wires, the first experiment should be based on study of force between such a wire and an ordinary magnet. In the other experiment it should examine the force between two current-carrying wires.

2.1 Basic experiments of electromagnetism

2.1.1. Electromagnetic force

In the first experiment, it is necessary to have two wide magnetic poles, posed one in front of the other. Between these poles, it should set two fixed parallel metal bars, connected at one side to a constant voltage source E, as it may be seen on Fig. II-3a. On the other side of the bars, one should put a metal conductor rod, which can freely move along the bars. The role of the potentiometer R is to reduce the current of the so obtained circuit. The rod should be tied to a dynamo-meter to measure electromagnetic force, which will be exerted on it.

It is shown, by such experiments, that the electromagnetic force is propor-

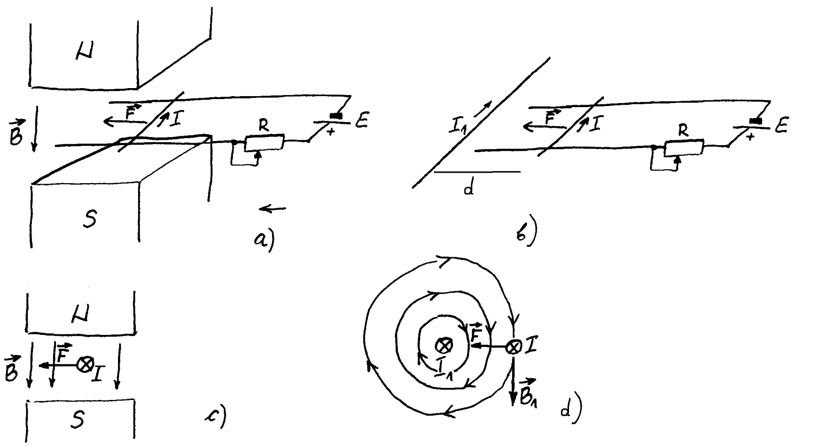

Figure II-3. Illustration of experiments that are used to determine magnetic field of current-carrying wire.

tional to the current I that flows through conductor rod, to the length Δl of the rod and to sine of angle, which rod makes with magnetic lines of forces, as well as to the intensity of magnetic field **B** at the bar. All mentioned relations are involved in a single mathematical expression:

(2-3) $\Delta \mathbf{F} = \mathrm{I} \cdot \Delta \boldsymbol{l} \times \mathbf{B}$,

where × denotes vector products.

If the rod and magnetic lines are perpendicular, this expression is, simply:

(2-4) $\Delta F = I \cdot \Delta l \cdot B$.

2.1.2. Magnetic field of current-carrying wire

In the following experiment, we will use the same accessories as in the previous, except that we will use a long wire with current I_1 in place of ordinary magnet (Fig. II-3b).

By this experiment, it is shown that there is no force between wires if they are mutually perpendicular. If they are parallel, they attract each other if the currents I and I_1 have the same directions, and repel in the case of contrary directions. The intensity of the force is given by the expression:

(2-5) $\Delta F_1 = \mu_0 \frac{I_1 I}{2\pi d} \cdot \Delta l$,

where d denotes the distance between wires and $\mu_0 = 4\pi \cdot 10^{-7}$ N/A^2, is a constant of proportionality. Varying the distance d and current I_1, it is possible to equalize forces $\Delta\mathbf{F}$ and $\Delta\mathbf{F}_1$:

(2-6) $I \cdot \Delta l \cdot B = \mu_0 \frac{I_1 I}{2\pi d} \cdot \Delta l$.

From this expression, we have:

(2-7) $B = \mu_0 \frac{I_1}{2\pi d}$.

This result means that every long straight wire with current I generates in its vicinity a circular magnetic field **B**, the intensity of which on distance r is given by the expression:

(2-8) $B = \mu_0 \frac{I}{2\pi r}$.

The direction of the field is defined by the right screw rule.

2.2. Generalized Lorentz equations

Analyzing expressions (2-3) and (2-8) for electromagnetic force and magnetic field of a current-carrying wire we will obtain deeper expressions, which will make easier understanding the theory of electromagnetism.

2.2.1. Lorentz force

Let's look what happens inside a segment Δl of current-carrying wire, on which is exerted electromagnetic force (2-3):

$$\Delta \mathbf{F} = \mathrm{I} \cdot \Delta \boldsymbol{l} \times \mathbf{B} \;.$$

Assuming that the volume of the segment of the wire is

$$\Delta \mathrm{V} = \mathrm{S} \cdot \Delta l,$$

and that (1-4):

$$\mathrm{I} = J \cdot \mathrm{S},$$

electromagnetic force becomes:

(2-9) $$\Delta \mathbf{F} = \Delta \mathrm{V} \cdot \boldsymbol{J} \times \mathbf{B} \;.$$

If we denote by $\Delta \mathrm{N}$ the number of free electrons that move by velocity $\mathbf{v}_e$ inside the segment $\Delta \mathrm{V}$ of wire, the current density (1-3, Electrostatics) is:

(2-10) $$\boldsymbol{J} = \frac{\Delta \mathrm{N}}{\Delta \mathrm{V}} (-e) \mathbf{v}_{\mathrm{e}} \;,$$

and the electromagnetic force:

(2-11) $$\Delta \mathbf{F} = \Delta \mathrm{N} (-e) \cdot \mathbf{v}_{\mathrm{e}} \times \mathbf{B} \;.$$

It was Dutch physicist Hendrik A. Lorentz who supposed that electromagnetic force originate from particular forces $\mathbf{F}_{el}$, that are exerted on free electrons, which moves by the velocity $\mathbf{v}_e$:

(2-12) $$\mathbf{F}_{\mathrm{el}} = (-e) \cdot \mathbf{v}_{\mathrm{e}} \times \mathbf{B} \;.$$

In the next step of inductive reasoning it is concluded that the same force should be exerted on any charge particle q that moves by the velocity $\mathbf{v}_q$ in static magnetic field $\mathbf{B}$:

(2-13) $$\mathbf{F} = q \cdot \mathbf{v}_{\mathrm{q}} \times \mathbf{B} \;.$$

This expression is verified experimentally and theoretically.

2.2.2. Induced electric field $\mathbf{E}_B$

If a point charge q moves by velocity $\mathbf{v}_q$ in electrostatic field $\mathbf{E}_Q$ and static magnetic field $\mathbf{B}$, total force that is exerted on the charge is:

(2-14) $$\mathbf{F} = q \cdot (\mathbf{E}_{\mathrm{Q}} + \mathbf{v}_{\mathrm{q}} \times \mathbf{B}) \;.$$

Taking that electric field is determined by the expression:

(2-15) $$\mathbf{E} = \frac{\mathbf{F}}{q} \;,$$

we get:

(2-16) $\mathbf{E} = \mathbf{E}_Q + \mathbf{v}_d \times \mathbf{B}$.

Now, we have $\mathbf{v}_d$ denoting the velocity of an arbitrary electric field detector, instead of charge velocity $\mathbf{v}_q$. The first component is electrostatic field (given by expressions 1-10 and 1-15, Electrostatics), while the second component is induced electric field, which we will denote by $\mathbf{E}_B$:

(2-17) $\mathbf{E}_B = \mathbf{v}_d \times \mathbf{B}$.

This expression is obtained for a particle moving by the velocity $\mathbf{v}_d = \mathbf{v}_q$ (the particle has a roll of detector) in static magnetic field **B**. The particle would detect the same electric field if it stayed in magnetic field **B**, which lines of forces move by the opposite velocity $\mathbf{v}_B = -\mathbf{v}_q$:

(2-18) $\mathbf{E}_B = -\mathbf{v}_B \times \mathbf{B}$.

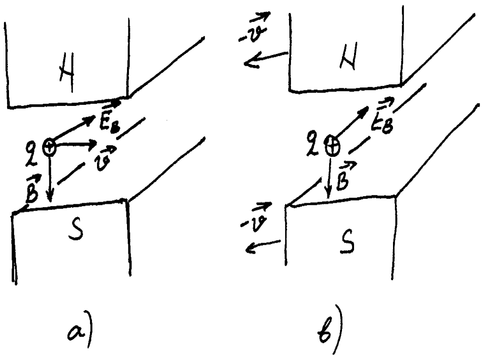

Figure II-4. Moving a) of charge *q* by the velocity in static magnetic field makes the same induced electric field as the moving b) of magnetic field by the velocity around static charge *q*.

In case of particle (or arbitrary detector) moving by velocity $\mathbf{v}_d = \mathbf{v}_q$ in magnetic field **B**, the lines of which move by the velocity $\mathbf{v}_B$, induced electric field is given by the expression:

(2-19) $\mathbf{E}_B = (\mathbf{v}_d - \mathbf{v}_B) \times \mathbf{B}$.

2.2.3. Induced magnetic field B

By similar reasoning, we will show that magnetic field **B** originates from moving of electric field lines.

Let's look upon expression for magnetic field on distance r from a long wire through which flows current I:

(2-20) $B = \mu_0 \frac{I}{2\pi r}$.

Assuming that according to expressions (1-6, Electrostatics) and (2-10) the current I is:

(2-21) $\mathrm{I} = \frac{\Delta \mathrm{N}}{\Delta \mathrm{V}}(-e)\mathrm{v_e S} = \frac{\Delta \mathrm{N}}{\Delta l}(-e)\mathrm{v_e}$,

this expression becomes:

(2-22) $\mathrm{B} = \mu_0 \frac{\Delta \mathrm{N}(-e)}{2\pi \mathrm{r} \cdot \Delta l}\mathrm{v_e}$.

In the next step, we will integrate the Gauss equation:

(*Electrostatics*, 1-22) $\mathrm{div}\, \mathbf{E}_\mathrm{Q}(\mathbf{r}) = \frac{\rho}{\varepsilon_0}$

over a cylinder *Δ*C, with height *Δl*, radius of base r and the axis of which coincides with the wire:

(2-23) $\int_{\Delta \mathrm{V}} \mathrm{div}\, \mathbf{E}_\mathrm{Q}\, \mathrm{dV} = \int_{\Delta \mathrm{V}} \frac{\rho}{\varepsilon_0} \mathrm{dV}$.

After integration it becomes:

(2-24) $\mathrm{E_Q} \cdot 2\mathrm{r}\pi \Delta l = \frac{\Delta \mathrm{N}(-e)}{\varepsilon_0}$,

or:

(2-25) $\mathrm{E_Q} = \frac{\Delta \mathrm{N}(-e)}{2\varepsilon_0 \mathrm{r} \pi \Delta l}$.

So, the expression for magnetic field (2-22) becomes:

(2-26) $\mathrm{B} = \varepsilon \mu_0 \mathrm{E_Q v_e}$.

Assuming that:

(2-27) $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.99792458 \cdot 10^8 \mathrm{m/s}$,

coincide with the speed of light and the speed of free electrons $\mathbf{v}_\mathrm{e}$ inside the wire correspond to side velocity of its lines $\mathbf{v}_\mathrm{E}$ of electrostatic field, we get:

(2-28) $\mathrm{B} = \frac{1}{c^2} \mathrm{E_Q v_E}$.

Assuming that electrostatic field of electrons has direction toward the wire, that the direction of magnetic field is determined by right screw rule relating to current in the wire, and that electrons (and electrostatic field lines) have the direction of moving opposite to the direction of current, the expression for magnetic field gets a vector form:

(2-29) $\mathbf{B} = \frac{1}{c^2} \mathbf{v}_\mathrm{E} \times \mathbf{E}_\mathrm{Q}$.

By the analogue reasoning as in the case of induced electric field, we conclude that the same magnetic field is obtained in case of "static" electrostatic field $\mathbf{E}_\mathrm{Q}$ and detector moving by the opposite velocity $\mathbf{v}_\mathrm{d} = -\mathbf{v}_\mathrm{E}$:

(2-30) $\mathbf{B} = -\frac{1}{c^2} \mathbf{v}_\mathrm{d} \times \mathbf{E}_\mathrm{Q}$.

In case of moving electrostatic field by the velocity $\mathbf{v}_E$ and detector by the velocity $\mathbf{v}_d$, the expression for magnetic field becomes:

(2-31) $$\mathbf{B} = \frac{1}{c^2}(\mathbf{v}_E - \mathbf{v}_d)\times\mathbf{E}_Q \, .$$

Generalization of the expression for magnetic field

Being that every electric field can be divided to electrostatic and induced components:

(2-32) $$\mathbf{E} = \mathbf{E}_Q + \mathbf{E}_B$$

it arises question, whether magnetic field originates only from electrostatic field $\mathbf{E}_B$ (as we get analyzing magnetic field of current-carrying wire), or from total electric field $\mathbf{E}$.

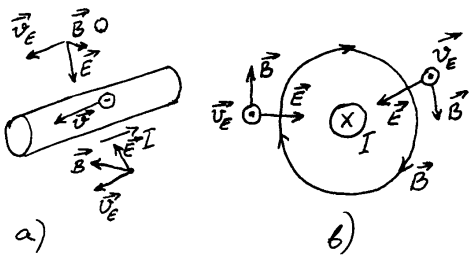

Figure II-5. By the reasoning, similar to the case of Lorentz force, it is shown that magnetic field around a wire originate from the moving electrostatic field .

It seems that it originates from the total field. Direct confirmation of this hypothesis is the so named wave equation obtained from Maxwell's equations. So, the expression for total magnetic field becomes:

(2-33) $$\mathbf{B} = \frac{1}{c^2}(\mathbf{v}_E - \mathbf{v}_d)\times\mathbf{E} \, ,$$

while the magnetic field that originates only from electrostatic field $\mathbf{E}_Q$ (2-31) we will name static magnetic field and denote by $\mathbf{B}_Q$:

(2-34) $$\mathbf{B}_Q = \frac{1}{c^2}(\mathbf{v}_E - \mathbf{v}_d)\times\mathbf{E}_Q \, .$$

Maybe a same notation should be taken for a bar magnet field. It should emphasize that influence of induced electric field becomes significant only at high frequencies of electric current in conducting wires.

Equations:

(2-19) $$\mathbf{E}_B = (\mathbf{v}_d - \mathbf{v}_B)\times\mathbf{B} \, ,$$

(2-33) $$\mathbf{B} = \frac{1}{c^2}(\mathbf{v}_E - \mathbf{v}_d)\times\mathbf{E} \, ,$$

as well as (2-31, i.e. 2-34), as a special case of (2-33) we will name the first and the second **Lorentz equation**, respectively, being that they remind to expression for Lorentz force. Similar expressions I found in “heretical” book from late Thomas G. Barnes [2], where he cited the book [5] from Leigh Page and Norman I. Adams, Jr. (this last I didn’t succeed to get, waiting on interlibrary loan for more then one year). From strange reason, they neglected the velocity of detector **v**.

It is clear that **B** and $\mathbf{E}_Q$ on right side are necessarily vector fields, while the left sides ($\mathbf{E}_B$ and $\mathbf{B}_Q$) are vector fields in 6-dimensional μ-space.

Further, it is obvious that these equations are invariant to Galilean transformations, supposing that vector fields **B** and $\mathbf{E}_Q$ are invariant. Actually, they are invariant even to non-inertial frame transformations. Being that they show a kind of interaction between detector and corresponding (magnetic or electric) field, question is whether they support action at distance or not.

2.3. Solving electromagnetic fields

In this section we will solve some problems that arise in the electromagnetic theory and practice.

2.3.1. Origin of the electromagnetic force

From the expression for electromagnetic force between current carrying wires:

(2-5) $$\Delta F_1 = \mu_0 \frac{I_1 I}{2\pi d} \cdot \Delta l ,$$

as well as from the expression for corresponding magnetic field:

(2-8) $$B = \mu_0 \frac{I}{2\pi r} ,$$

it is generally accepted opinion that the electromagnetic force and the magnetic field originate from parallel electron moving.

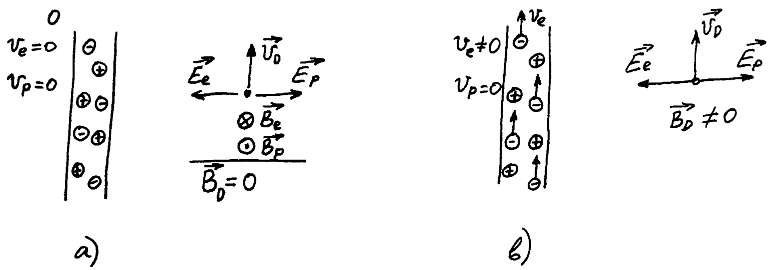

Figure II-6. (a) Detector moving near a currentless wire does not detect magnetic field, because sum of the fields $\mathbf{B}_p$ and $\mathbf{B}_e$ (originating from protons and electrons) is equal to zero. (b) When free electrons move along the wire, the detector detects magnetic field $\mathbf{B}_d$.

Let's start with a wire without current in it. Furthermore, let a detector (of magnetic field) moves by the velocity $\mathbf{v}_d$ on distance $\mathbf{r}$ from the wire. By this moving, the detector crosses the lines of electrostatic field $\mathbf{E}_{p0}$ and $\mathbf{E}_{e0}$, which originate from (fixed) protons and electrons (whose mean velocity is zero), respectively, creating corresponding magnetic fields:

(2-35) $$\mathbf{B}_{e0} = \frac{1}{c^2}(0 - \mathbf{v}_d) \times \mathbf{E}_{e0} , \text{ and}$$

$$\mathbf{B}_{p0} = \frac{1}{c^2}(0 - \mathbf{v}_d) \times \mathbf{E}_{p0} .$$

Being that:

(2-36) $\mathbf{E}_{p0} = -\mathbf{E}_{e0}$,

total magnetic field that detector detects is equal to zero:

(2-37) $\mathbf{B}_0 = \mathbf{B}_{e0} + \mathbf{B}_{p0} = 0$.

On the other hand, when there is a current I along the wire, two groups of electrons and protons may be distinguished. In the first group are those electrons that do not move along the wire and the same number of protons, which we will name compensated protons. As we have just seen, their magnetic field $\mathbf{B}_0$ is equal zero. In the other group are free electrons that move along the wire by the mean velocity $\mathbf{v}_{ef}$, and the rest of (motionless) protons, that are the same in number. The corresponding magnetic fields are:

(2-38) $$\mathbf{B}_{ef} = \frac{1}{c^2}(\mathbf{v}_{ef} - \mathbf{v}_d) \times \mathbf{E}_{ef} \text{ , and}$$

$$\mathbf{B}_{p(f)} = \frac{1}{c^2}(0 - \mathbf{v}_d) \times \mathbf{E}_{p(f)} \text{ ,}$$

where ef in index denotes free electrons, and p(f) denotes just the corresponding number of protons (which are, obviously, not free). Assuming that $\mathbf{E}_{p(f)} = -\mathbf{E}_{ef}$, the total magnetic field is:

(2-39) $$\mathbf{B} = \mathbf{B}_{ef} + \mathbf{B}_{p(f)} = \frac{1}{c^2}\mathbf{v}_{ef} \times \mathbf{E}_{ef} \text{ .}$$

This expression agreeing with the expression (2-28) explains why it seems that magnetic field of current-carrying wire originates only from free electrons. It is self-evident that this field does not depend on the speed of its detector.

We are now able to explain little bit deeply the electromagnetic force by which a long wire with current I_1 acts on a segment Δl of another parallel wire with current I. We will notice that according to just obtained result, both protons and electrons detect the same magnetic field:

(2-40) $$\mathbf{B} = \frac{1}{c^2}\mathbf{v}_{ef1} \times \mathbf{E}_{ef1} \text{ ,}$$

where $\mathbf{v}_{ef1}$ denotes mean velocity of free electrons in the first wire.

If we denote by $\mathbf{v}_B$ the displacement velocity of the magnetic field, induced electric fields that detect protons and (tied and free) electrons are given by the expressions:

(2-41) $$\mathbf{E}_{Bp} = (0 - \mathbf{v}_B) \times \mathbf{B} \text{ ,}$$

$$\mathbf{E}_{Be0} = (0 - \mathbf{v}_B) \times \mathbf{B}$$

$$\mathbf{E}_{Bef} = (\mathbf{v}_{ef} - \mathbf{v}_B) \times \mathbf{B} \text{ ,}$$

where $\mathbf{E}_{Be0}$ denotes electric field that is detected by electrons tied to its nuclei, while $\mathbf{E}_{Bef}$ denotes the field that is detected by free electrons.

Let's denote by ΔN_p the number of protons in segment Δl of second wire, and by ΔN_{e0} and ΔN_{ef} the number of tied and free electrons, respectively. Obviously:

(2-42) $$\Delta N_p = \Delta N_{e0} + \Delta N_{ef} \,.$$

The force exerted on the segment is:

(2-43) $$\Delta \mathbf{F} = \Delta N_p \cdot e \cdot \mathbf{E}_{Bp} + \Delta N_{e0} \cdot (-e) \cdot \mathbf{E}_{Be0} + \Delta N_{ef} \cdot (-e) \cdot \mathbf{E}_{Bef} \,,$$

where $e = 1.60217733 \cdot 10^{-19}$ C denotes the charge of proton. Combining with previous two expressions, we obtain:

(2-44) $$\Delta \mathbf{F} = \Delta N_{ef} \cdot e \cdot (\mathbf{E}_{Bp} - \mathbf{E}_{Bef}) \,,$$

and:

(2-45) $$\Delta \mathbf{F} = \Delta N_{ef} \cdot e \cdot \mathbf{v}_{ef} \times \mathbf{B} \,.$$

Taking in mind, that the number of free electrons inside a segment of wire Δl corresponds to the number of free electrons that pass the starting cross-section S of the wire during the time necessary for a free electron to pass from the beginning to the end of the segment ($\Delta N_{ef,\Delta l} = \Delta N_{ef,S,\Delta t}$):

(2-46) $$\Delta N_{ef} \cdot e \cdot \mathbf{v}_{ef} = \Delta Q_{\Delta l} \cdot \frac{\Delta \boldsymbol{l}}{\Delta t} = \frac{\Delta Q_{S,\Delta t}}{\Delta t} \cdot \Delta \boldsymbol{l} = I \cdot \Delta \boldsymbol{l} \,,$$

we get the expression:

(2-47) $$\Delta \mathbf{F} = I \cdot \Delta \boldsymbol{l} \times \mathbf{B} \,,$$

that agrees with the expression (2-3) for electromagnetic force.

2.3.2. The law of Biot-Savart

The law of Biot-Savart is the name for the expression for the magnetic field $\Delta\mathbf{B}$ in a detector (whose position is defined by vector $\mathbf{r}_d$), and that originates from current segment $\Delta\boldsymbol{l}'$ (whose position is determined by vector $\Delta\mathbf{r}'$) along of which flows (constant) current I (Fig. II-7). That field consists from the field $\Delta\mathbf{B}_{ef}$ that originates from free electrons, and the field $\Delta\mathbf{B}_{p(f)}$ that originates from uncompensated protons of that segment (2-38):

(2-48) $$\Delta \mathbf{B}_{ef} = \frac{1}{c^2} (\mathbf{v}_{ef} - \mathbf{v}_d) \times \Delta \mathbf{E}_{ef} \text{ , and}$$

$$\Delta \mathbf{B}_{p(f)} = \frac{1}{c^2} (0 - \mathbf{v}_d) \times \Delta \mathbf{E}_{p(f)} \,.$$

Being that the current segment can be treated as point charge, the corresponding electrostatic fields are:

(2-49) $$\Delta \mathbf{E}_{ef}(\mathbf{r}_d) = -\Delta \mathbf{E}_{p(f)}(\mathbf{r}_d) = -\frac{\Delta Q}{4 \pi \varepsilon_0} \cdot \frac{\mathbf{r}_d - \mathbf{r}'}{|\mathbf{r}_d - \mathbf{r}'|^3} \,,$$

where $\mathbf{r}'$-$\mathbf{r}_d$ denotes vector that links the current segment with the detector.

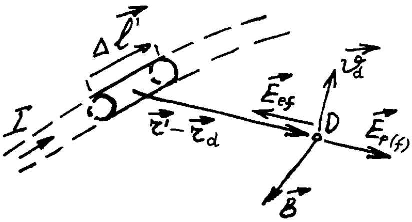

Figure II-7. Illustration for derivation of the expression of Biot-Savart.

From the relation:

(2-50) $$\Delta Q = \mathrm{I}\cdot\Delta\,\mathrm{t} = \frac{\mathrm{I}\cdot\Delta l'}{\mathrm{v_{ef}}}\ ,$$

the above expression becomes:

(2-51) $$\Delta\,\mathbf{E}_{\mathrm{ef}} = -\Delta\,\mathbf{E}_{\mathrm{p(f)}} = \frac{-\mathrm{I}\cdot\Delta\,l'}{4\,\pi\,\varepsilon_0\,\mathrm{v_{ef}}}\cdot\frac{\mathbf{r}_{\mathrm{d}}-\mathbf{r}'}{\left|\mathbf{r}_{\mathrm{d}}-\mathbf{r}'\right|^3}\ .$$

The total magnetic field Δ**B** becomes:

(2-52) $$\Delta\,\mathbf{B} = \Delta\,\mathbf{B}_{\mathrm{ef}}+\Delta\,\mathbf{B}_{\mathrm{p(f)}} = \frac{-\mathbf{v}_{\mathrm{ef}}}{c^2}\times\frac{\mathrm{I}\cdot\Delta\,l'}{4\,\pi\,\varepsilon_0\,\mathrm{v_{ef}}}\cdot\frac{\mathbf{r}_{\mathrm{d}}-\mathbf{r}'}{\left|\mathbf{r}_{\mathrm{d}}-\mathbf{r}'\right|^3}\ .$$

Taking in mind that current segment Δ***l***' has the opposite direction from the velocity of free electrons $\mathbf{v}_{\mathrm{ef}}$, we obtain the final expression for the law of Biot-Savart:

(2-53) $$\Delta\,\mathbf{B}(\mathbf{r}_{\mathrm{d}}) = \frac{\mu_0}{4\pi}\cdot\frac{\mathrm{I}\cdot\Delta\,\boldsymbol{l}'\times(\mathbf{r}_{\mathrm{d}}-\mathbf{r}')}{\left|\mathbf{r}_{\mathrm{d}}-\mathbf{r}'\right|^3}\ ,$$

where we used the expression:

(2-54) $$c = \frac{1}{\sqrt{\varepsilon_0\,\mu_0}} = 2.99792458\cdot10^8\,\mathrm{m/s}\ .$$

The obtained expression may be written in the form:

(2-55) $$\Delta\,\mathbf{B}(\mathbf{r}_{\mathrm{d}}) = \frac{\mu_0}{4\pi}\cdot\frac{\boldsymbol{J}(\mathbf{r}')\times(\mathbf{r}_{\mathrm{d}}-\mathbf{r}')}{\left|\mathbf{r}_{\mathrm{d}}-\mathbf{r}'\right|^3}\,\Delta\,\mathrm{V}'\ ,$$

where we used the relation (4) for current density:

(2-56) $$\boldsymbol{J}\cdot\Delta\,\mathrm{V}' = \frac{\mathrm{I}}{\mathrm{S}'}\cdot\mathrm{S}'\cdot\Delta\,\boldsymbol{l}' = \mathrm{I}\cdot\Delta\,\boldsymbol{l}'\ .$$

2.3.3. The law of Laplace

We are able now to obtain the expression for the electromagnetic force between two current elements $\boldsymbol{l}_1$ and, $\boldsymbol{l}_2$ with currents I_1 and I_2, at places determined by vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.

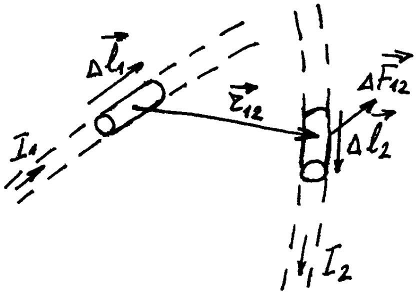

Figure II-8. A pictorial representation of the force between current-carrying elements.

The magnetic field that originates from the first element at the place of the second element is, according to (2-53):

(2-56) $$\Delta \mathbf{B}_1(\mathbf{r}_2) = \frac{\mu_0}{4\pi} \cdot \frac{I_1 \cdot \Delta \boldsymbol{l}_1 \times \mathbf{r}_{12}}{r_{12}^3} .$$

Applying the expression for electromagnetic force that acts on the second current element:

(2-57) $$\Delta \mathbf{F}_1(\mathbf{r}_2) = I_2 \cdot \Delta \boldsymbol{l}_2 \times \mathbf{B}_1 ,$$

we have finally:

(2-58) $$\Delta \mathbf{F}_1(\mathbf{r}_2) = \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2 \cdot \Delta \boldsymbol{l}_2 \times (\Delta \boldsymbol{l}_1 \times \mathbf{r}_{12})}{r_{12}^3} .$$

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