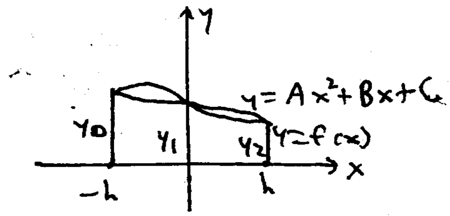
Simpson's Rule

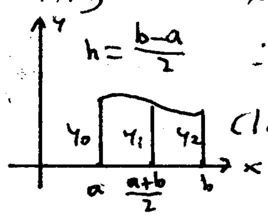


$$y_0 = Ah^2 - Bh + C$$
$$y_1 = C$$
$$y_2 = Ah^2 + Bh + C$$

$$\begin{aligned}
\int_{-h}^{h} f(x)\,dx &\doteq \int_{-h}^{h} (Ax^2+Bx+C)\,dx \\
&= \left[\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx\right]_{-h}^{h} \\
&= \left(\frac{Ah^3}{3} + \frac{Bh^2}{2} + Ch\right) - \left(-\frac{Ah^3}{3} + \frac{Bh^2}{2} - Ch\right) \\
&= \frac{2Ah^3}{3} + 2Ch \\
&= \frac{h}{3}(2Ah^2 + 6C) \\
&= \frac{h}{3}(Ah^2 - Bh + C + 4C + Ah^2 + Bh + C) \\
&= \frac{h}{3}(y_0 + 4y_1 + y_2)
\end{aligned}$$

Shifting sideways does not change area

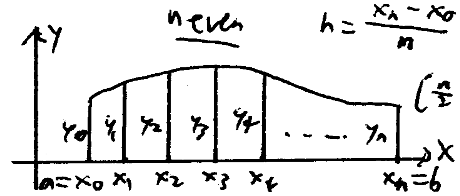
$h = \frac{b-a}{2}$ $\qquad \therefore \int_a^b f(x)\,dx \doteq \frac{h}{3}(y_0 + 4y_1 + y_2)$



(1 application)

More generally,

n even $\qquad h = \frac{x_n - x_0}{n} = \frac{b-a}{n}$



($\frac{n}{2}$ applications)

$$\int_a^b f(x)\,dx \doteq \frac{h}{3}(y_0 + 4y_1 + y_2) + \frac{h}{3}(y_2 + 4y_3 + y_4) + \cdots + \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n)$$

$$\therefore \boxed{\int_a^b f(x)\,dx = \frac{h}{3}\left(y_0 + y_n + 2(y_2 + y_4 + \cdots + y_{n-2}) + 4(y_1 + y_3 + \cdots + y_{n-1})\right) \quad \text{where } h = \frac{b-a}{n}}$$

Exercises

Use Simpson's Rule to estimate

1. $\int_0^4 2^x\,dx$ (1 application)
2. $\int_1^2 \frac{1}{x}\,dx$ (5 applications)
3. $\int_2^5 f(x)\,dx$ given

x	2	2.5	3	3.5	4	4.5	5
f(x)	3.1	3.8	4	3.6	2.8	2.5	2.9