Facts About e

- The number e was first used by William Oughtred and John Napier in 1614:
  Napier, J., *Mirifici Logarithmorum Canonis Descriptio*, 1614.

  But the symbol e was first used by Leonhard Euler in 1727:
  Euler, L., *Meditatio in Experimenta explosione tormentorum nuper instituta*, 1727.

- $\int_1^e \frac{1}{x}\,dx = 1$

- $e = \sum_{n=0}^{\infty} \frac{1}{n!} = \lim_{n\to\infty}(1+\frac{1}{n})^n = 2.7182818284...$

- e is irrational.

- e is transcendental.

- $\frac{d}{dx}e^x = e^x$

- $\int \frac{1}{x}\,dx = \ln x + c$ where $\ln x = \log_e x$.

- If $i = \sqrt{-1}$, then $e^{i\theta} = \cos\theta + i\sin\theta$ and $e^{\pi i} = -1$.

- An equivalence to the Riemann Hypothesis is that for $n > 1$,

  $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}+e^{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}}\ln(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}) >$ sum of divisors of n

  and there exists a \$1,000,000 prize for the proof of this at www.claymath.org.

- $$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{\vdots}}}}$$

- $e = \lim_{n\to\infty} \frac{n}{\sqrt[n]{n!}}$

- Where $p_n :=$ the n-th prime, $e = \lim_{n\to\infty}\left(\prod_{k=1}^{n} p_k\right)^{1/p_n}$