VIVEK VERMA

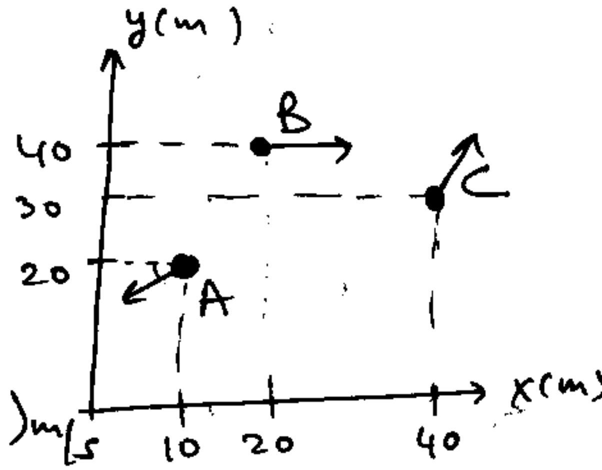
HOMEWORK 10. ~~E Set~~ E Mch 12

15.74. $\overline{30^\circ} \equiv 210^\circ$

$\vec{V_A} = 24(\cos 210\hat{i} + \sin 210\hat{j})$

$\vec{V_B} = 16\hat{i}$ m/s

$\vec{V_c} = 20(\cos 60^\circ\hat{i} + \sin 60^\circ\hat{j})$ m/s



(a) G:

$G(x, y) = ?$

$$G_x = \frac{m_a A_x + m_B B_x + m_c C_x}{m_a + m_B + m_c}$$

$$= \frac{8*10 + 6*20 + 12*40}{8+6+12}$$

$= 26.2$ m

similarly $G_y = 29.2$m

$\therefore \boxed{G(x,y) = (26.2\text{m}, 29.2\text{m})}$

(b) linear momentum:

$= m_A \bar{V_A} + m_B \bar{V_B} + m_c \bar{V_c}$

$= 8*24(\cos 210\hat{i} + \sin 210\hat{j})$
$+ 6*16\hat{i} + 12*(\cos 60\hat{i} + \sin 60\hat{j})$

$$\therefore \boxed{P = 49.7\,\hat{i} + 111.9\,\hat{j} \text{ N·s}}$$

(c) Angular momentum about origin.

$\bar{h}_o = ?$ $\qquad h = \vec{r} \times (m\vec{v})$

for A:

$$(\bar{h}_o)_A = (10\hat{i} + 20\hat{j}) \times (8 * 24 * (\cos 210\hat{i} + \sin 210\hat{j})) \quad \text{--- (a)}$$

for B:

$$(\bar{h}_o)_B = (20\hat{i} + 40\hat{j}) \times (6 * 16\,\hat{i}) \quad \text{--- (b)}$$

for C:

$$(\bar{h}_o)_C = (40\hat{i} + 30\hat{j}) \times (12 * 20\,(\cos 60\hat{i} + \sin 60\hat{j})) \quad \text{--- (c)}$$

adding (a), (b) and (c)

we get

$$\boxed{\bar{h}_o = 3240\,\hat{k} \text{ N·m·s}}$$

(d) angular momentum about G.

$$\bar{h}_G = \Sigma (\bar{r} - \bar{r}_G) \times (m\bar{v})$$

for A:

$$\bar{r}_A = 10\hat{i} + 20\hat{j} \ m$$

$$\bar{r}_G = 26.2\hat{i} + 29.2\hat{j} \ m$$

$$\bar{r}_{A/G} = \bar{r}_A - \bar{r}_G = (10-26.2)\hat{i} + (20-29.2)\hat{j} \ m$$

$$= -16.2\hat{i} - 9.2\hat{j}$$

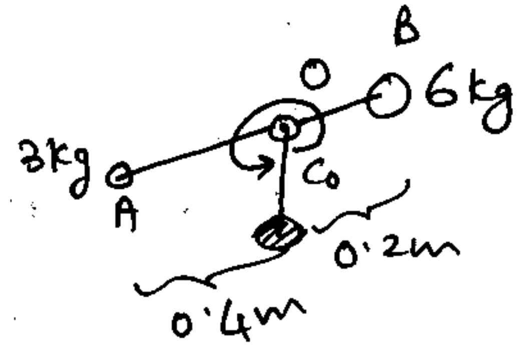
$$\therefore (\bar{h}_G)_A = (-16.2\hat{i} - 9.2\hat{j}) \times (8*24*(\cos 210\hat{i} + \sin 210\hat{j})) \quad \text{---(a)}$$

Similarly for B and C also.

$$\bar{h}_G = (\bar{h}_G)_A + (\bar{h}_G)_B + (\bar{h}_G)_C$$

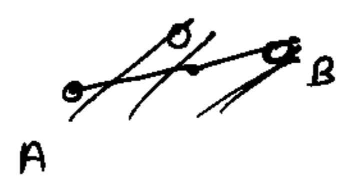
$$= 1768\hat{k} \ N \cdot m \cdot s.$$

76. Initially the system was rotating clockwise about 'O'.

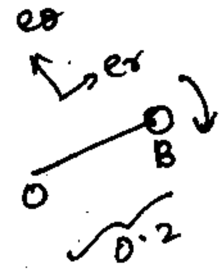


$\dot{\theta}_0 = 120$ rad/s $C_0 = 8$ N·m

B:



Initial angular momentum of B:



$$0.2\,\hat{e}_{r} \times (6\,kg * 0.2 * 120\,(-\hat{e}_\theta))$$

$$= -28.8\,\hat{k}\ N.m.s$$

Initial angular momentum of A:

$$(-0.4\,\hat{e}_r) \times (3 * 0.4 * 120\,\hat{e}_\theta)$$

$$= -57.6\,\hat{k}\ N.m.s$$

$\therefore$ total initial angular momentum

$$= -86.4\,\hat{k}\ N.m.s.$$

Couple: $8\,\hat{k}$ N.m.

~~Using Impulse~~

Using angular Impulse momentum theorem:

Since couple is constant:

$$8\,\hat{k} \cdot \Delta t\ N.m = 0 - (-86.4\,\hat{k}\ N.m.s)$$

$$\therefore \boxed{\Delta t = 10.8\,s}$$

(b). $\alpha = \dfrac{d\omega}{dt} \Rightarrow \omega = \alpha t + c_1$

Initial condition:

$t = 0 \quad \omega = 120,$

final condition $t = 10.8$ $\omega = 0$

Initially:

$\therefore$ $120 = \alpha(0) + C_1 \Rightarrow C_1 = 120\,rad/s$

final condition:

$$0 = \alpha(10.8) + 120 \Rightarrow \boxed{\alpha = 11.11\,rad/s^2}$$

also: $\omega \dfrac{d\omega}{d\theta} = \alpha$

$\therefore$ $\dfrac{\omega^2}{2} = \alpha\theta + C_2$

Initial: $\theta = 0$ $\omega = 120\,rad/s$

$\therefore$ $\dfrac{(120)^2}{2} = 11.11 * 0 + C_2 \Rightarrow C_2 = 7200\left(\dfrac{rad}{s}\right)^2$

final $\theta = ?$ $\omega = 0$

$\therefore$ $0 = 11.11 * \theta + 7200$

$\therefore$ $\theta = \dfrac{-7200}{11.11} = -648.1$ radians.

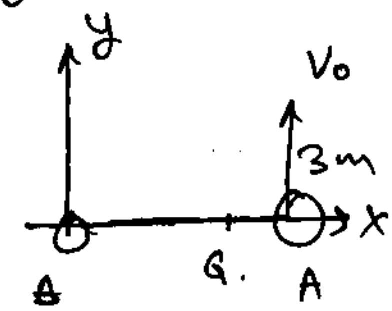
$-ve$ sign means clockwise.

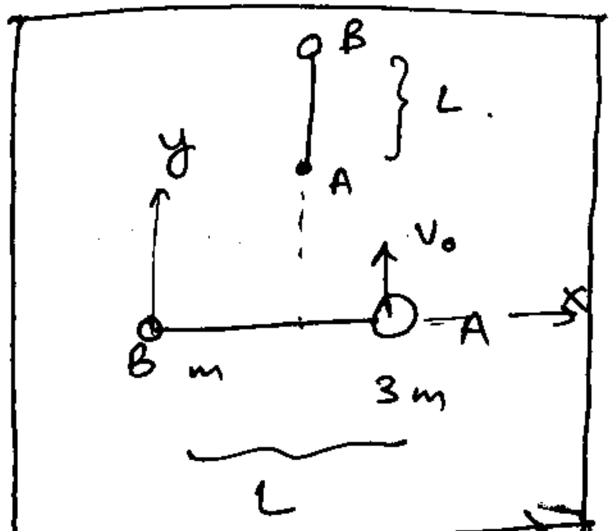
2π (revolution) $= \theta$

$\therefore$ $n = \dfrac{\theta}{2\pi} = 103.1$ revolutions.

5·80. Position ①

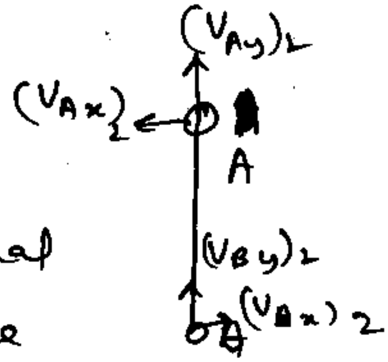
Initially:





$G = \left(\dfrac{3m \cdot L + m(0)}{3m+m} \hat{i} + 0 \hat{j} \right)$ ← Horizontal plane.

$\therefore G = \left(\dfrac{3L}{4}, 0 \right)$.



Finally:

Since there is no external force in 'x' direction, so center of mass along 'x' direction won't change.

$(P_x)_1 = (P_x)_2$:

$$0 = m(V_{Bx})_2 - 3m(V_{Ax})_2$$

$$\therefore (V_{Bx})_2 = 3(V_{Ax})_2$$

$(P_y)_1 = (P_y)_2$: $\quad 3mV_0 = 3m(V_{Ay})_2 + m(V_{By})_2$

$$\therefore (V_{Ay})_2 = (V_{By})_2 = \frac{3V_0}{4}.$$

Since the rod is rigid:

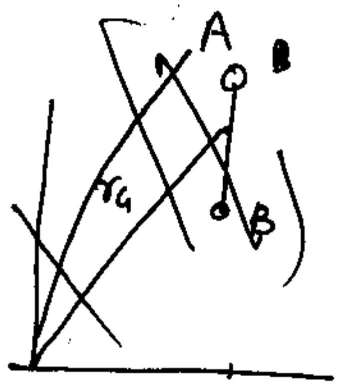
$$(v_{Ay})_2 = (v_{By})_2 .$$

$(h_G)_1$: $$(\bar{h}_G)_1 = (\bar{r}_B - \bar{r}_G) \times (m_B \bar{v}_{B_1}) + (\bar{r}_A - \bar{r}_G) \times (m_A \bar{v}_{A_1})$$

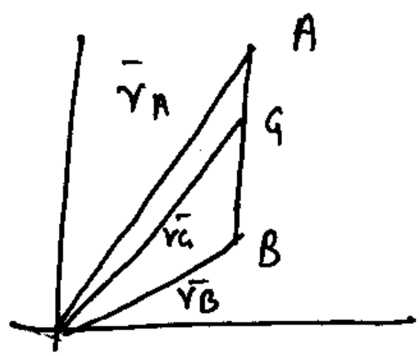
$$= \underset{(v_{B_1}=0)}{0} + \left(\frac{L}{4}\right)\hat{i} \times (3m\, v_0\, \hat{j})$$

$$= \frac{3 m v_0 L}{4} \hat{k}$$

when rod covers $\theta = 90°$



$$\bar{r}_{B/G} = \bar{r}_B - \bar{r}_G$$
$$= \frac{3L}{4}\hat{j}$$
$$\bar{r}_{A/G} = \frac{L}{4}\hat{j}$$



$$(\bar{h}_G)_2 = \cancel{3}\,(\bar{r}_{A/G}) \times (m_A \bar{v}_A) + (\bar{r}_{B/G}) \times (m_B \bar{v}_B)$$

$$= \left(\frac{L}{4}\hat{j}\right)\left(3m \cdot (v_{Ax_2}\hat{i} + v_{Ay_2}\hat{j})\right) +$$
$$\left(\frac{3L}{4}\hat{j}\right)\left(\cancel{3m}\, m \cdot (v_{Bx_2}\hat{i} + v_{By_2}\hat{j})\right)$$

No external force $\Rightarrow$ $(\bar{h}_G)_1 = (\bar{h}_G)_2$

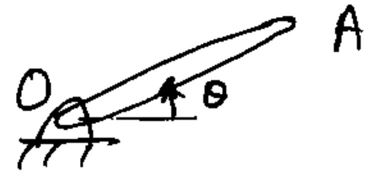
$\therefore$ equating we get:

$$v_0 = (v_{Ax})_2 + (v_{Bx})_2 = (v_{Ax})_2 + 3(v_{Ax})_2$$

$$\therefore (v_{Ax})_2 = \frac{v_0}{4}, \quad (v_{Bx})_2 = \frac{3v_0}{4}$$

$$\therefore \boxed{\begin{aligned} (v_A)_2 &= \frac{v_0}{4}(-\hat{i} + 3\hat{j}) \\ (v_B)_2 &= \frac{3v_0}{4}(\hat{i} + \hat{j}) \end{aligned}}$$

16.4. $\omega = 3t^2 - kt$ rad/s



$t = 0$, $\theta = 8$ rad.

$t = 4s$, $\theta = 16$ rad

(a) $\omega = \frac{d\theta}{dt} \Rightarrow \int \omega\, dt = \int d\theta$

$\therefore \theta = t^3 - \frac{kt^2}{2} + C_1$

$t = 0$, $\theta = 8$ rad clockwise i.e. $\theta = -8$ rad.

$\therefore -8 = 0 - 0 + C_1 \Rightarrow C_1 = -8$ rad

$t = 4$, $\theta = -16$ rad.

$\therefore -16 = 4^3 - \frac{k(4^2)}{2} + (-8)$

$\therefore \boxed{k = 9/s^2}$

(b) $\theta = t^3 - \frac{9t^2}{2} - 8$ rad ; $\omega = 3t^2 - 9$ rad/s

Let t_1 be the times when $\omega = 0$:

$3t_1^2 - 9t_1 = 0 \Rightarrow t_1 = 3$ sec.

when $t = t_1 = 3$ sec $\quad \theta = 3^3 - \frac{9(3)^2}{2} - 8$

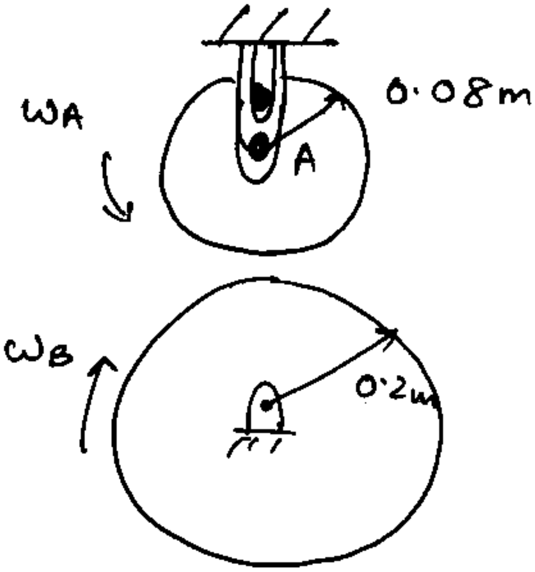
$= -21.5$ rad

$\therefore$ Total angle turned through from $t = 0$

to $t = 4$ s is

$(21.5 - 8) + (21.50 - 16) = 19$ radians.

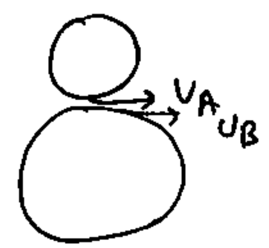
16.10. $(\omega_B)_1 = -18$ rad/s



$(\omega_B)_2 = -12$ rad/s

when both the wheel stop slipping then relative velocity $= 0$

i.e. their ~~radial~~ instantaneous linear velocity at contact $=$ same.



$\boxed{V_A = V_B}$.

A:

$\alpha_A = $ constant.

$\omega_A = \int \alpha_A \, dt$

$\Rightarrow \omega_A = \alpha_A t + c_1$ rad/s

$\therefore \theta = \int \omega_A dt = \dfrac{\alpha_A t^2}{2} + c_1 t + c_2$ rad.

$t = 0, \ \omega_A = 0 \quad \Rightarrow c_1 = 0$

$t = 0, \ \theta_A = 0$ (arbitary choice) $\therefore c_2 = 0$

$\therefore \omega_A = \alpha_A t$ rad/s

$\theta_A = \dfrac{\alpha_A t^2}{2}$ rad.

when $t = 6\,s$, $\omega_B = 12\,rad/s$. Since slipping stops @ that ~~pos~~ time, the point of contact on wheels have the same velocity, i.e., $R_A\omega_A = R_B\omega_B$.

(a) $t = 6\,s$. $0.08\omega_A = 0.20 * 12$

$$\Rightarrow \omega_A = 30\,rad/s$$

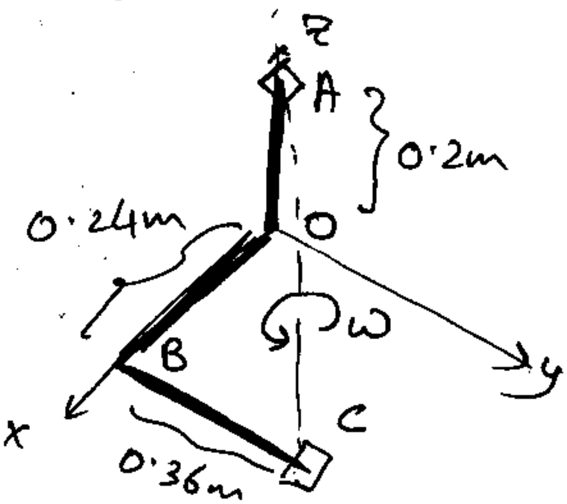
$$\therefore \alpha_A = \frac{\omega_A}{t} = 30/6 = 5.00\ rad/s^2.$$

(b) $t = 6$ seconds. $\theta = \frac{1}{2}\alpha_A t^2$

$$\therefore \theta_A = \frac{1}{2}(5.00)(6)^2 = 90\ radians$$

$$\therefore \text{no. of revolutions} = \frac{90}{2\pi} = 14.32\ rev.$$

16.14.



$\omega = 4\ rad/s$

$\alpha = 9\ rad/s^2$

$V_B = \bar{\omega} \times \bar{r}_{B/C}$

$\bar{r}_{B/C} = -0.36\hat{j}\ m$

$\bar{\omega} = 4\bar{\lambda}_{AC}\ rad/s \quad \bar{\alpha} = 9\lambda_{AC}\ rad/s^2$

$$\bar{\lambda}_{AC} = \frac{0.24\hat{i} + 0.36\hat{j} - 0.2\hat{k}}{0.4767}$$

$$= 0.5035\hat{i} + 0.7552\hat{j} - 0.4196\hat{k}$$

$$\bar{v}_B = 4\begin{vmatrix} i & j & k \\ 0.5035 & 0.7552 & -0.4196 \\ 0 & -0.36 & 0 \end{vmatrix} \cancel{=0.6042}$$

$$\boxed{\bar{v}_B = -0.6042\hat{i} - 0.7250\hat{k} \text{ m/s}}$$

$$\bar{a}_B = \bar{\alpha} \times \bar{r}_{B/C} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{B/C})$$

$$= \bar{\alpha} \times \bar{r}_{B/C} + \bar{\omega} \times \bar{v}_B$$

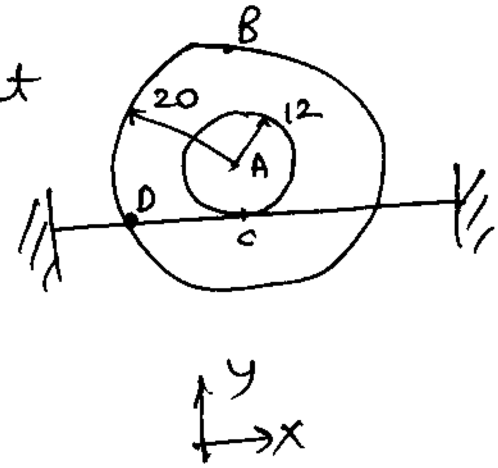
$$\therefore \bar{a}_B = 9\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5035 & 0.7552 & -0.4196 \\ 0 & -0.36 & 0 \end{vmatrix} +$$

$$4\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5035 & 0.7552 & -0.4196 \\ -0.6042 & 0 & -0.7250 \end{vmatrix}$$

$$= (-1.36\hat{i} - 1.631\hat{k}) + (-2.19\hat{i} + 2.474\hat{j} + 1.825\hat{k})$$

$$= -3.55\hat{i} + 2.474\hat{j} + 0.19\hat{k} \text{ m/s}^2.$$

16.18 Let C be the point of contact on the inner hub:



$\bar{\omega} = -3\hat{k}$ rad/s

$\bar{V}_B = \bar{V}_A + \bar{V}_{B/A} = \bar{\omega} \times \bar{r}_{A/C} + \bar{\omega} \times \bar{r}_{B/A}$

$\cancel{= (-3\hat{k} \times 12j}$

$= (-3\hat{k} \times 12\hat{j}) + (-3\hat{k} \times 20\hat{j}) = 36\hat{i} + 60\hat{i}$

$\therefore \bar{V}_B = 96\hat{i}$ in/s

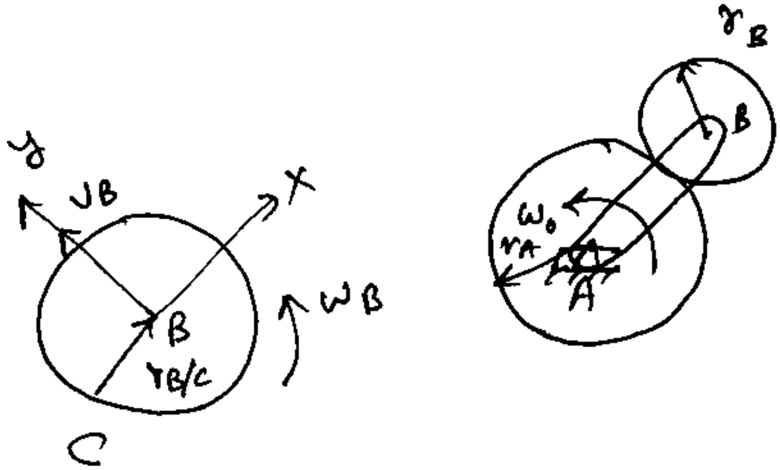
$V_D = V_A + V_{D/A} = 36\hat{i} + \bar{\omega} \times \bar{r}_{D/A}$

$= 36\hat{i} + (-3\hat{k} \times (-16\hat{i} - 12\hat{j}))$

$= 36\hat{i} + (48\hat{j} - 36\hat{i})$

$= 48\hat{j}$ in/s.

16.20.



C is the point on B that is in contact with wheel A. For no slip $\bar{v}_c = 0$

$$\bar{\omega}_0 = \omega_0 \hat{k} \quad , \bar{\omega}_B = \omega_B \hat{k}$$

$$\bar{v}_B = \bar{v}_c + \bar{v}_{B/c} = \bar{0} + \bar{\omega}_B \times \bar{r}_{B/C} = \bar{v}_B$$

$$\therefore \omega_0 \hat{k} \times (\bar{r}_A + \bar{r}_B) \hat{i} = \omega_B \hat{k} \times r_B \hat{i}$$

$$\therefore (r_A + r_B) \omega \hat{j} = r_B \omega_B \hat{j}$$

$$\therefore \boxed{\omega_B = \left(\frac{r_A + r_B}{r_B}\right) \omega_0}$$