Homework Set 1

$\boxed{11.4}$ $I = 20\,kgm^2\left(\frac{.06852\,slug}{1kg}\right)\left(\frac{3.28\,ft}{1\,m}\right)^2$

$= 14.75\,slugft^2\left(\frac{1\,lbs^2/ft}{1\,slug}\right) = \boxed{14.75\,lbfts^2 = I}$

$\boxed{11.6}$ $a = \frac{gkx}{W}$

Dimensions

$a = \frac{L}{T^2}$ $g = \frac{L}{T^2}$

$k =$ ~~Unitless~~ F/L $x = L$

$W = F$

$$\therefore \frac{L}{T^2} = \frac{\frac{L}{T^2}\left(\frac{\cancel{F}}{\cancel{L}}\right)(\cancel{L})}{\cancel{F}} = \frac{L}{T^2}\ ✓$$

$\boxed{11.10}$ $I = \frac{2}{5}mR^2$

a) Find dimensions in FLT system
b) Find dimensions in MLT system

$F = ma = \frac{ML}{T^2} \therefore M = \frac{FT^2}{L}$

c) $\therefore I = \frac{FT^2}{L}(L^2) = \boxed{FLT^2}$

b) $I = \boxed{\underset{\substack{\nearrow \\ mass}}{M}\ \underset{\substack{\wedge \\ radius^2}}{L^2}}$

$\boxed{11.12}$ $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P_0 \sin\omega t$

What are dimensions of c, k, P_0, ω in FLT

$m\frac{d^2x}{dt^2} = m\ddot{x} = ma = F = \frac{ML}{T^2} \Rightarrow M = \frac{FT^2}{L}$

need all pieces in terms of Forces $ma = \frac{FT^2}{L}\left(\frac{L}{T^2}\right) = F$

$c\frac{dx}{dt} = c\dot{x} = c\frac{L}{T} \Rightarrow c = \frac{M}{T} = FTL^{-1}$

$\boxed{c)\ FTL^{-1}}$

$kx = kL \Rightarrow k = \frac{M}{T^2} = FL^{-1}$

$\boxed{k) = FL^{-1}}$

$P_0 \underbrace{\sin\omega t}_{\text{unitless}} \therefore \boxed{P_0 = F}$

for $\sin \omega t$ unitless $\omega = \frac{1}{T} = \boxed{T^{-1} = \omega}$

$\boxed{11.16}$ $w = 2\text{lb each}$
$R = 8\text{in}$

$F = G\left(\frac{m_1 m_2}{R^2}\right) = 3.44\times10^{-8}\frac{ft^3}{slug\,s^2}\left(\frac{\left(\frac{2lb}{32.2ft/s^2}\right)\left(\frac{2lb}{32.2ft/s^2}\right)}{\left(\frac{16}{12}\right)^2}\right)$

$\boxed{F = 7.46\times10^{-11}\ lb}$

$\boxed{11.18}$ $F = G\frac{m_1 m_2}{R^2}$

$m_{earth} = 5.97 \times 10^{24}$

$m_{moon} = .07348 \times 10^{24}$

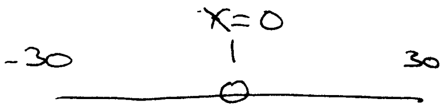
$r_{earth} = 6378$

$r_{moon} = 1738$

$\frac{F_{moon}}{F_{earth}}$ on m_1

$$\frac{G m_1 \frac{m_{moon}}{R_{moon}^2}}{G m_1 \frac{m_{earth}}{R_{earth}^2}} = \frac{\frac{m_{moon}}{R_{moon}^2}}{\frac{m_{earth}}{R_{earth}^2}} = \frac{m_{moon} R_{earth}^2}{M_{earth} R_{moon}^2} =$$

$$= .1656 \simeq \boxed{\frac{1}{6} \checkmark}$$



$\boxed{12.4}$ $x = t^2 - 10t$ in   x measured from center
$v = 2t - 10$

$x = 0 \Rightarrow t = 10\,s$ after 10 seconds bead halfway along wire

$x = 30 = t^2 - 10t$

$30 = t(t-10)$

$t^2 - 10t - 30 = 0$

$$\frac{10 \pm \sqrt{100-4(-30)}}{2} = \frac{10 \pm 14.832}{2}$$

$t = 12.42\,s$

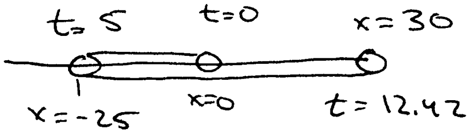
For $v = 0$ at turning point

$0 = 2t - 10$
$t = 5\,s$

$x(5) = 25 - 50 = -25$ in

and at $t=0$ $x(0) = 0$

∴ path of ball



∴ total distance traveled = 25+25+30
= 80 in

$\boxed{12.8}$ $v^2 = 2gr_o\left(\frac{r_o}{r} - 1\right) + v_o^2$

$v = v(t)$
$r = r(t)$

$$\frac{d}{dt}\left[(v(t))^2 = 2gr_o\left(\frac{r_o}{r(t)} - 1\right) + v_o^2\right]$$

$$= 2v(t)\dot{v} = 2gr_o\left(-r_o r(t)^{-2}\dot{r}\right)$$

$\dot{v} = a$
$\dot{r} = v$

$$\cancel{2}\cancel{v}a = \cancel{2}gr_o\left(-r_o r^{-2}\cancel{v}\right)$$

$$a = -g\frac{r_o^2}{r^2} = \boxed{-g\left(\frac{r_o}{r}\right)^2 = a}$$

b) For escape velocity $v \to 0$ as $r \to \infty$

$\therefore$ Plug $v=0$, $r=\infty$ into initial eqn.

$$0 = 2gr_o\left(\frac{r_o}{\infty} - 1\right) + v_o^2$$

$$2gr_o = v_o^2$$

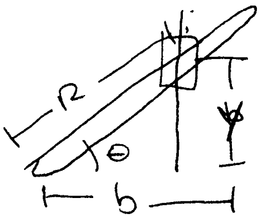
$$\boxed{V_o = \sqrt{2gr_o}}$$

c) $V_o = \sqrt{2\left(32.2\frac{ft}{s^2}\right)(3960\,mile)\left(\frac{5280\,ft}{1\,mile}\right)}$

$$\boxed{V_o = 36700\,ft/s}$$

12.20

$$y = R\sin\theta = \overset{\text{const}}{b}\tan\theta$$



$$\dot{y} = b\sec^2\theta\,\dot{\theta}$$
$$= b\sec\theta\sec\theta\,\dot{\theta}$$

Apply product rule

$$\ddot{y} = b\Big[\sec\theta\sec\theta\,\ddot{\theta} + \dot{\theta}\sec\theta(\sec\theta\tan\theta)\dot{\theta}$$
$$+\dot{\theta}\sec\theta(\sec\theta\tan\theta)\dot{\theta}\Big]$$

$$= b\left[\sec^2\theta\,\ddot{\theta} + 2\dot{\theta}^2\sec^2\theta\tan\theta\right]$$

$$\boxed{\ddot{y} = b\sec^2\theta\left[\ddot{\theta} + 2\dot{\theta}^2\tan\theta\right]}$$

12.22

$$x = R\left[\cos\theta + (9-\sin^2\theta)^{1/2}\right]$$

$$\dot{x} = R\Big[-\sin\theta(\dot{\theta}) + \tfrac{1}{2}(9-\sin^2\theta)^{-1/2}(-2\sin\theta\cos\theta(\dot{\theta}))$$

$$= -R\sin\theta\,\dot{\theta}\left[1 + \frac{\cos\theta}{(9-\sin^2\theta)^{1/2}}\right]$$

$$\dot{\theta} = \omega$$

$$\dot{x} = -R\omega\sin\theta\Big[1 + \frac{\cos\theta}{(9-\sin^2\theta)^{1/2}}$$