$\boxed{15.140}$ Given: Three masses are suspended from a cable and pulley system.

Neglect the masses of the pulleys.

Find: Acceleration of each mass in terms of g.

Solution:

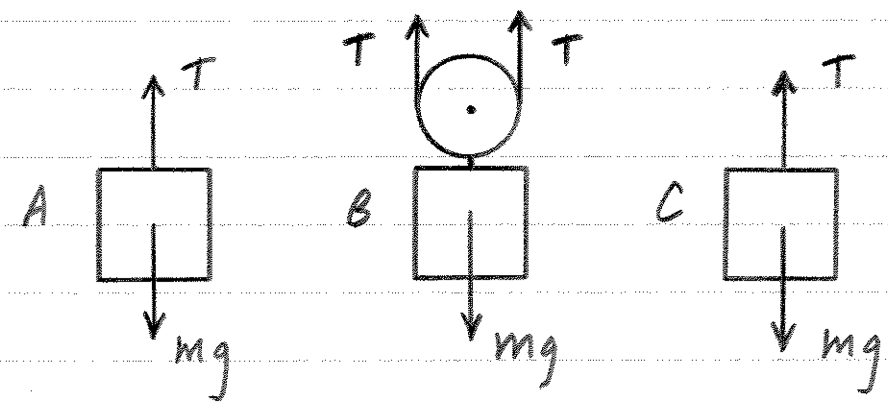
4 Unknowns - T, a_A, a_B, a_C

need 4 eqns.

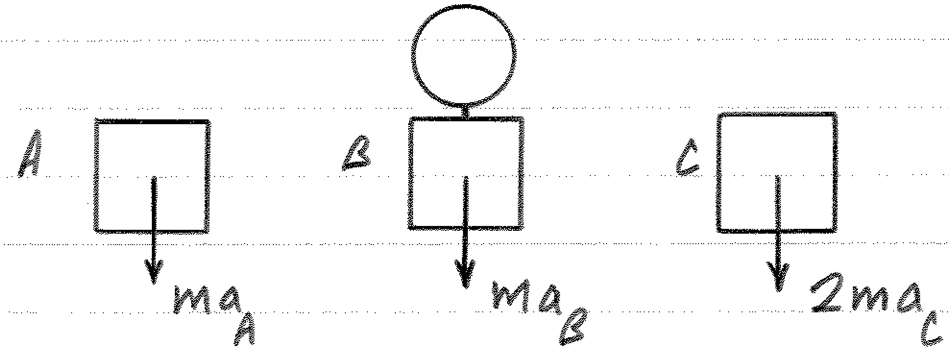
Displacements are pos. down

Length of the rope $= x_A + 2x_B + x_C$

$\longrightarrow a_A + 2a_B + a_C = 0 \quad (1)$



FBDs



MADs

Note: Accelerations are assumed pos. down to stay consistent w/ displacements

$$+\uparrow \Sigma F_A = ma_A : \quad T - mg = -ma_A \quad (2)$$

$$+\uparrow \Sigma F_B = ma_B : \quad 2T - mg = -ma_B \quad (3)$$

$$+\uparrow \Sigma F_C = ma_C : \quad T - 2mg = -ma_C \quad (4)$$

Solving eqns 1-4.

$\longrightarrow$ $T = 0.7273\,mg$, $a_A = 0.2727g$, $a_B = -0.4546g$, $a_C = 0.6364g$

$$a_A = 0.273g \downarrow$$
$$a_B = 0.455g \uparrow$$
$$a_C = 0.636g \downarrow$$

$\boxed{15.146}$ Given: $W_A = 30\text{ lb}$, $V_A = 2000\text{ ft/s}$

$W_B = 800\text{ lb}$, $k = 30(10^3)\text{ lb/ft}$

Find: Max deformation (δ) of the spring after the shell has been fired

Solution:

Define Positions

1 - Just before the shell is fired

2 - Just after the shell is fired

3 - Position of maximum compression (deformation)

- The mechanical energy of the barrel is conserved from 2-3, but the recoil velocity of the barrel in position 2 (V_{B2}) must first be determined to find the max deformation (δ_3).
- The mometum of the system is conserved from 1-2. The spring does not deform ($\delta_2 = 0$) during this instant so $\Sigma F_x = 0$

$$\xrightarrow{} \; (L_{1-2})_x = (P_x)_2 - (P_x)_1 = 0$$

$$\xrightarrow{+} \; (P_x)_1 = (P_x)_2 \; ; \; V_{A_1} = V_{B_1} = 0 \rightarrow (P_x)_1 = 0$$

$$\Rightarrow \; 0 = m_A V_{A2} - m_B V_{B2}$$

$$0 = \frac{30}{g}(2000) - \frac{800}{g} V_{B2}$$

$$\therefore V_{B2} = 75.0\text{ ft/s}$$

$$\cancel{V_2}^{\,0\ (\delta_2 = 0)} + T_2 = V_3 + \cancel{T_3}^{\,(v_{B_3} = 0)} \quad \text{(BARREL)}$$

$$\Rightarrow 0 + \tfrac{1}{2} m_B v_{B_2}^2 = \tfrac{1}{2} k \delta_3^2 + 0$$

$$\Rightarrow \frac{1}{2}\left(\frac{800}{32.2}\right)(75.0)^2 = \frac{1}{2}\left(30\left(10^3\right)\right)\delta_3^2$$

$$\therefore \ \delta_3 = 2.158 \text{ ft}$$

$\boxed{16.108}$ Given: $\vec{V}_D = -8.0\,\hat{i}$ ft/s

Find: V_A

Solution: All ang. velocities are assumed to be positive

$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$$

$$\vec{V}_A = \vec{V}_B + (\vec{w}_{AB} \times \vec{r}_{A/B})$$

$$-V_A\hat{i} = \vec{V}_B + (w_{AB}\hat{k} \times (-2\cos 60°\,\hat{i} - 2\sin 60°\,\hat{j}))$$

$$-V_A\hat{i} = \vec{V}_B - w_{AB}\hat{j} + 1.732\,w_{AB}\,\hat{i} \quad (1)$$

$$\vec{V}_B = \cancel{\vec{V}_C}^{\,0} + \vec{V}_{B/C}$$

$$\vec{V}_B = \vec{w}_{BC} \times \vec{r}_{B/C}$$

$$\vec{V}_B = w_{BC}\hat{k} \times (-2\cos 60°\,\hat{i} + 2\sin 60°\,\hat{j})$$

$$\vec{V}_B = -w_{BC}\hat{j} - 1.732\,w_{BC}\,\hat{i} \quad (2)$$

$$\vec{V}_B = \vec{V}_D + \vec{V}_{B/D}$$

$$\vec{V}_B = \vec{V}_D + (\vec{w}_{BD} \times \vec{r}_{B/D})$$

$$\vec{V}_B = -8.0\,\hat{i} + (w_{BD}\hat{k} \times -1.2\,\hat{i})$$

$$\vec{V}_B = -8.0\,\hat{i} - 1.2\,w_{BD}\,\hat{j} \quad (3)$$

Equating 2 and 3

$$-w_{BC}\hat{j} - 1.732\,w_{BC}\,\hat{i} = -8.0\,\hat{i} - 1.2\,w_{BD}\,\hat{j}$$

Components

$\hat{i}$: $-1.732\,w_{BC} = -8.0 \longrightarrow w_{BC} = 4.619$ rad/s

$\hat{j}$: $-w_{BC} = -1.2\,w_{BD} \longrightarrow w_{BD} = 3.849$ rad/s

Using 2 w/ $w_{BC} = 4.619$ rad/s

$$\vec{V}_B = -4.619\hat{j} - 1.732(4.619)\hat{i}$$
$$\vec{V}_B = -4.619\hat{j} - 8.0\hat{i} \quad (4)$$

Using 1 and 4

$$-V_A\hat{i} = (-4.619\hat{j} - 8.0\hat{i}) - w_{AB}\hat{j} + 1.732w_{AB}\hat{i}$$

Components

$$\hat{i}: -V_A = -8.0 + 1.732w_{AB}$$
$$\hat{j}: 0 = -4.619 - w_{AB} \longrightarrow w_{AB} = -4.619\ \frac{rad}{s}$$

$$\therefore -V_A = -8.0 + 1.732(-4.619)$$

$\therefore V_A = 16.0$ ft/s to the left

$\boxed{16.112}$ Given: $\vec{w}_{AB} = 3\hat{k}$ rad/s

Verify: $\vec{w}_{BC} = 1.2\hat{k}$ rad/s, $\vec{w}_{CD} = -3k$ rad/s

Find: α_{BC} and α_{CD}

Solution: All ang. velocities and ang. accelerations are assumed to be positive.

a)

$$\vec{V}_B = \underbrace{\cancel{\vec{V}_A}}_{0} + \vec{V}_{B/A}$$

$$\vec{V}_B = \vec{w}_{AB} \times \vec{r}_{B/A}$$

$$\vec{V}_B = 3\hat{k} \times (80\hat{\imath} + 60\hat{\jmath})$$

$$\vec{V}_B = 240\hat{\jmath} - 180\hat{\imath} \text{ ft/s}$$

$$\vec{V}_B = \vec{V}_C + \vec{V}_{B/C}$$

$$\vec{V}_B = \underbrace{\underbrace{\cancel{\vec{V}_D}}_{0} + \vec{V}_{C/D}}_{\vec{V}_C} + \vec{V}_{B/C}$$

$$\vec{V}_B = (\vec{w}_{CD} \times \vec{r}_{C/D}) + (\vec{w}_{BC} \times \vec{r}_{B/C})$$

$$240\hat{\jmath} - 180\hat{\imath} = (w_{CD}\hat{k} \times -80\hat{\imath}) + (w_{BC}\hat{k} \times 150\hat{\jmath})$$

$$240\hat{\jmath} - 180\hat{\imath} = -80w_{CD}\hat{\jmath} - 150w_{BC}\hat{\imath}$$

equating components

$\hat{\imath}: -180 = -150\, w_{BC} \longrightarrow w_{BC} = 1.2$ rad/s CCW

$\hat{\jmath}: 240 = -80w_{CD} \longrightarrow w_{CD} = -3$ rad/s $= 3$ rad/s CW

b) $\vec{a}_B = \underbrace{\cancel{\vec{a}_A}}_{0} + \vec{a}_{B/A}$

$\vec{a}_B = (\underbrace{\cancel{\vec{\alpha}_{AB}}}_{0} \times \vec{r}_{B/A}) + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A})$

(Constant ω_{AB})

$\vec{a}_B = 3\hat{k} \times (3\hat{k} \times (80\hat{\imath} + 60\hat{\jmath}))$

$\vec{a}_B = -720\hat{\imath} - 540\hat{\jmath}$

$\vec{a}_B = \vec{a}_C + \vec{a}_{B/C}$

$\vec{a}_B = \underbrace{\cancel{\vec{a}_D}}_{0} + \vec{a}_{C/D} + \vec{a}_{B/C}$

$$\vec{a}_B = (\vec{\alpha}_{CD} \times \vec{r}_{C/D}) + \vec{\omega}_{CD} \times (\vec{\omega}_{CD} \times \vec{r}_{C/D}) + (\vec{\alpha}_{BC} \times \vec{r}_{B/C}) + \vec{\omega}_{BC} \times (\vec{\omega}_{BC} \times \vec{r}_{B/C})$$

$$-720\hat{\imath} - 540\hat{\jmath} = (\alpha_{CD}\hat{k} \times -80\hat{\imath}) + \omega_{CD}\hat{k} \times (\omega_{CD}\hat{k} \times -80\hat{\imath}) + (\alpha_{BC}\hat{k} \times 150\hat{\jmath}) + \omega_{BC}\hat{k} \times (\omega_{BC}\hat{k} \times 150\hat{\jmath})$$

$$-720\hat{\imath} - 540\hat{\jmath} = -80\alpha_{CD}\hat{\jmath} + 80\omega_{CD}^2\hat{\imath} - 150\alpha_{BC}\hat{\imath} - 150\omega_{BC}^2\hat{\jmath}$$

equating components

$\hat{\imath}: \; -720 = 80\omega_{CD}^2 - 150\alpha_{BC}$ (1)

$\hat{\jmath}: \; -540 = -80\alpha_{CD} - 150\omega_{BC}^2$ (2)

$\vec{\omega}_{CD} = -3\hat{k} \Rightarrow \alpha_{BC} = 9.6$ rad/s^2 CCW from (1)

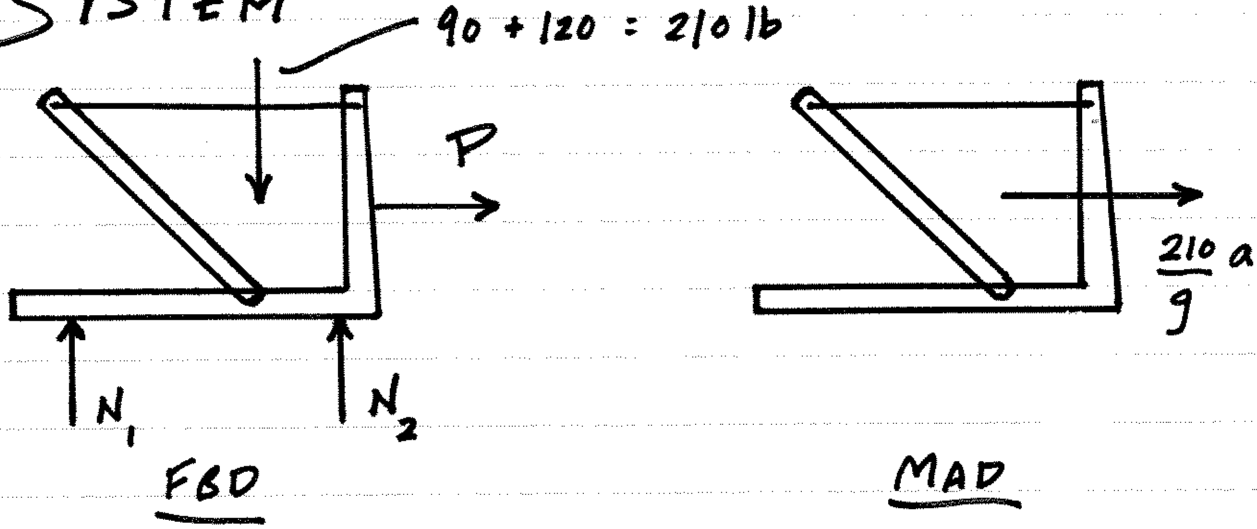
$\vec{\omega}_{BC} = 1.2\hat{k} \Rightarrow \alpha_{CD} = 4.05$ rad/s^2 CCW from (2)

17.20 Given: $W_{AB} = 90\ lb$, $W_C = 120\ lb$
$T_{CORD} = 60\ lb$

Find: Horizontal Force P.

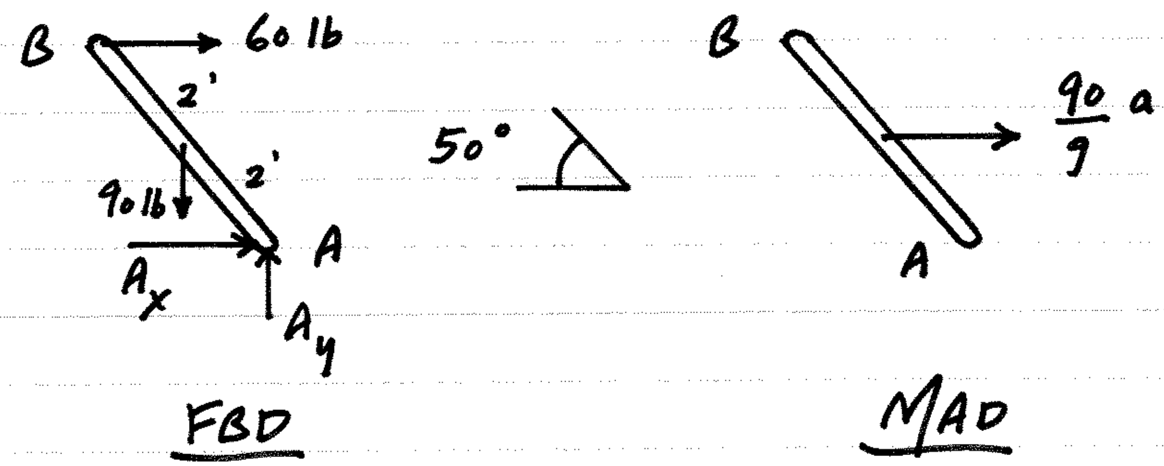
Solution:

SYSTEM



$$\xrightarrow{+} \Sigma F_x = m a_x : \quad P = \frac{210}{32.2} a$$

BAR AB



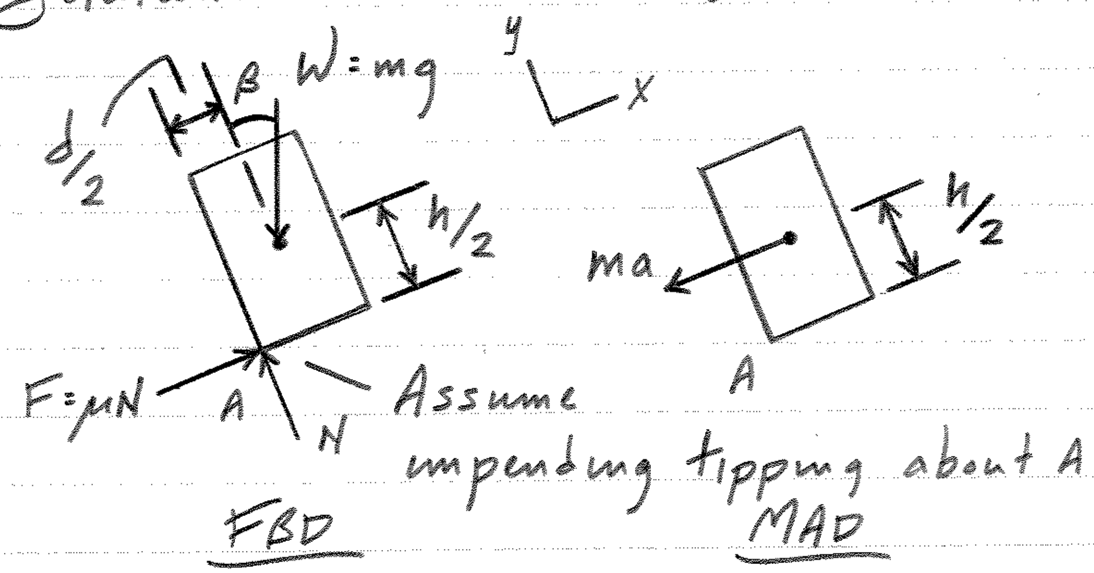
$$\circlearrowleft + \; (\Sigma M_A)_{FBD} = (\Sigma M_A)_{MAD}$$

$$90(2\cos 50°) - 60(4\sin 50°) = -\frac{90}{32.2}a(2\sin 50°)$$

$$\longrightarrow a = 15.914 \text{ ft/s}^2$$

$$\therefore P = \frac{210}{32.2}(15.914) = 103.8 \text{ lb}$$

$\boxed{17.24}$ Given: Cylinder of mass m slides down the incline of slope angle β.
μ_K between cylinder and incline.
Find: smallest ratio d/h for which cylinder will (not tip) – $\alpha_{cyl} = 0$
Solution:



3 Unknowns - N, a, d/h $\Rightarrow$ need 3 eqns.

$\circlearrowleft + \ (\Sigma M_A)_{FBD} = (\Sigma M_A)_{MAD}$:

$$-mg\cos\beta\left(\frac{d}{2}\right) + mg\sin\beta\left(\frac{h}{2}\right) = ma\left(\frac{h}{2}\right)$$

$$-gd\cos\beta + gh\sin\beta = ah \quad (1)$$

$+\nearrow \Sigma F_x = ma_x : \ \mu N - mg\sin\beta = -ma \quad (2)$

$+\nwarrow \Sigma F_y = ma_y : \ -mg\cos\beta + N = 0$

$$N = mg\cos\beta \quad (3)$$

Plug (3) into (2)

$$\mu m g \cos\beta - m g \sin\beta = -m a$$

$$a = g\sin\beta - \mu g \cos\beta \quad (4)$$

Plug (4) into (1)

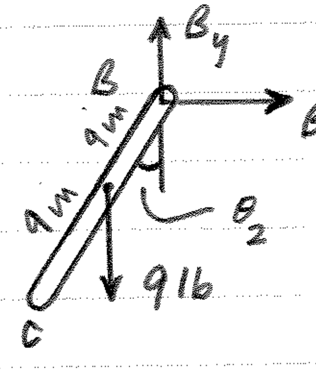
$$-g d \cos\beta + g h \sin\beta = g h \sin\beta - \mu g h \cos\beta$$

$$\Longrightarrow \frac{d}{h} = \mu$$

17.26 Given: $a_{collar} = 8.05\ ft/s^2 \rightarrow$, θ_1 and θ_2 remain constant

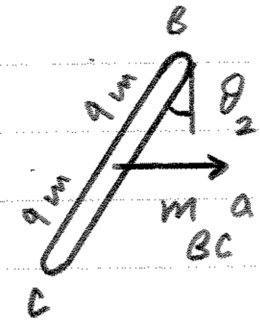
Find: θ_1 and θ_2

Solution:



FBD

=



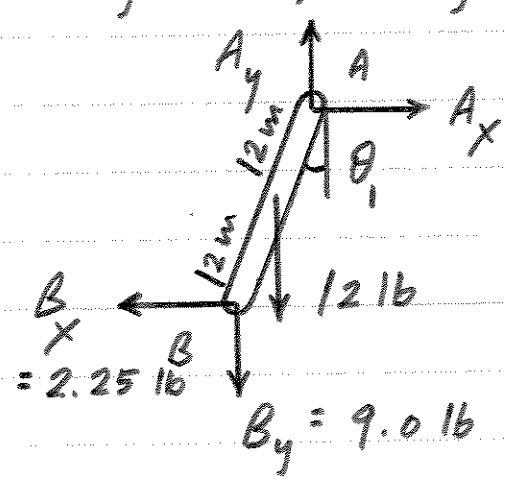
MAD

$\circlearrowleft + \ (\Sigma M_B)_{FBD} = (\Sigma M_B)_{MAD}$:

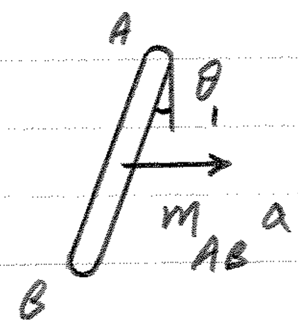
$$9(9\sin\theta_2) = \left(\frac{9}{32.2}\right)(8.05)(9\cos\theta_2) \longrightarrow \theta_2 = 14.04^\circ$$

$\xrightarrow{+}\ \Sigma F_x = ma_x$: $\quad B_x = \left(\frac{9}{32.2}\right)(8.05) = 2.25\ lb$

$+\uparrow\ \Sigma F_y = ma_y$: $\quad B_y - 9 = 0 \longrightarrow B_y = 9.0\ lb$



FBD



MAD

$\circlearrowleft^{+}$ $(\Sigma M_A)_{FBD} = (\Sigma M_A)_{MAD}$:

$$9(24\sin\theta_1) - 2.25(24\cos\theta_1) + 12(12\sin\theta_1)$$
$$= \left(\frac{12}{32.2}\right)(8.05)(12\cos\theta_1)$$

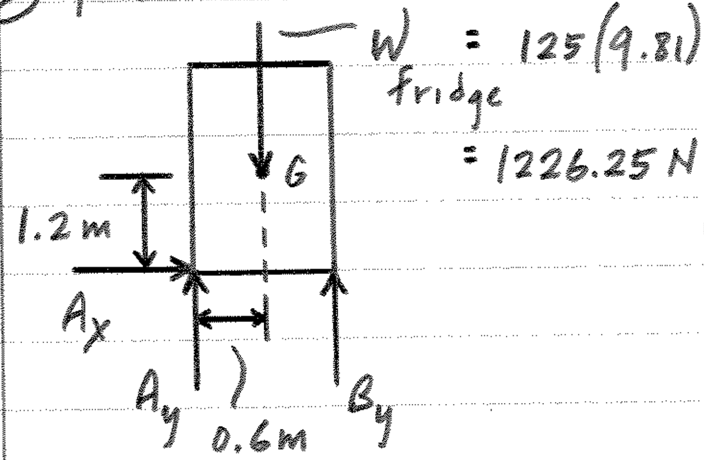
$$\longrightarrow \theta_1 = 14.04°$$

17.28 Given: $m_{fridge} = 125\ kg$, $\theta = 0°$,

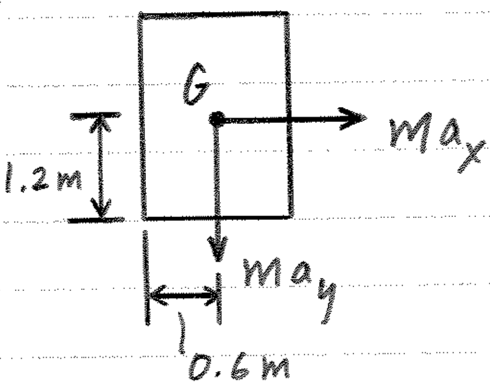
$\omega = 1.2\ rad/s$, $\alpha = 1.6\ rad/s^2$

Prove: The refrigerator does not tip.

Solution:



FBD



MAD

The refrigerator translates, it does not rotate $\longrightarrow$ No inertia couple ($\alpha = 0$ for fridge)

$a_x = r\alpha = (2)(1.6) = 3.2\ m/s^2$

$a_y = r\omega^2 = (2)(1.2^2) = 2.88\ m/s^2$

$\circlearrowleft + \ (\Sigma M_A)_{FBD} = (\Sigma M_A)_{MAD}$:

$$-(1226.25)(0.6) + (1.2)(B_y) = -(125)(3.2)(1.2) - (125)(2.88)(0.6)$$

$\longrightarrow B_y = 33.1\ N$

$+\uparrow \ \Sigma F_y = ma_y$: $A_y + B_y - 1226.25 = -(125)(2.88)$

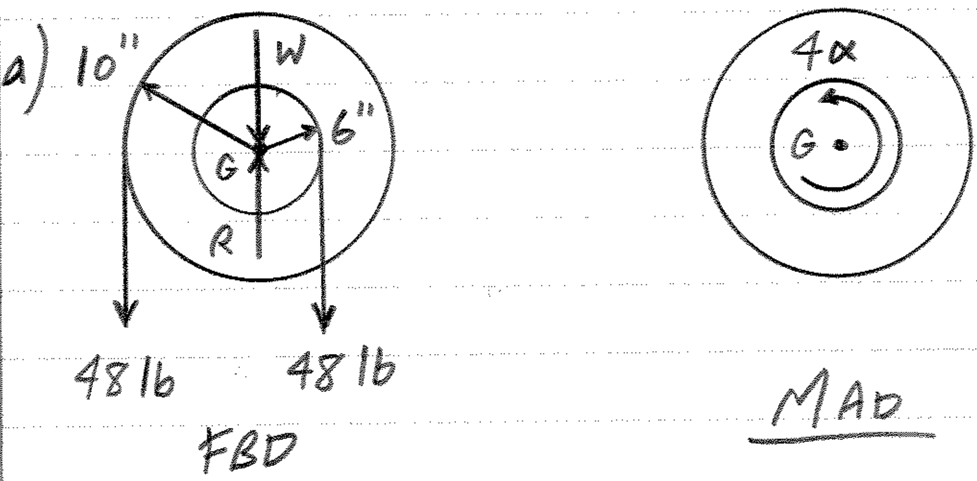
$\longrightarrow A_y = 833\ N$

Since $A_y > 0$, $B_y > 0$, the fridge will maintain contact @ A and B. No TIP

17.30 Given: The mass moment of inertia for each pulley about its mass center G is 4 $slug \cdot ft^2$.

Find: Angular accelerations of the homogeneous pulleys a) and b)

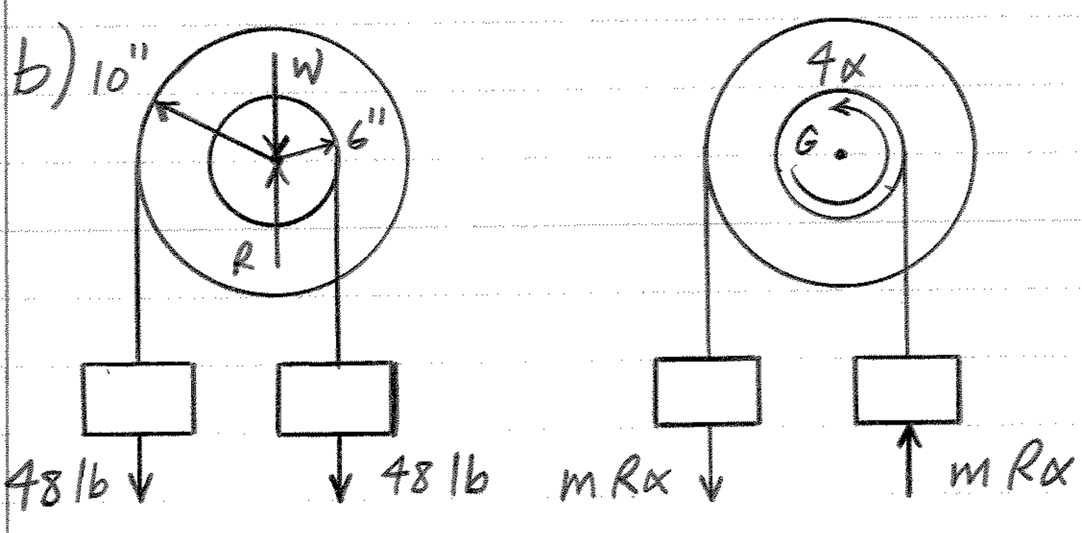
Solution:



$$\overset{+}{\circlearrowleft}\ (\Sigma M_G)_{FBD} = (\Sigma M_G)_{MAD}:$$

$$48\left(\frac{10}{12}\right) - 48\left(\frac{6}{12}\right) = 4\alpha$$

$$\longrightarrow \alpha = 4.00\ rad/s^2\ CCW$$



$\circlearrowleft + \; (\Sigma M_G)_{FBD} = (\Sigma M_G)_{MAD}$:

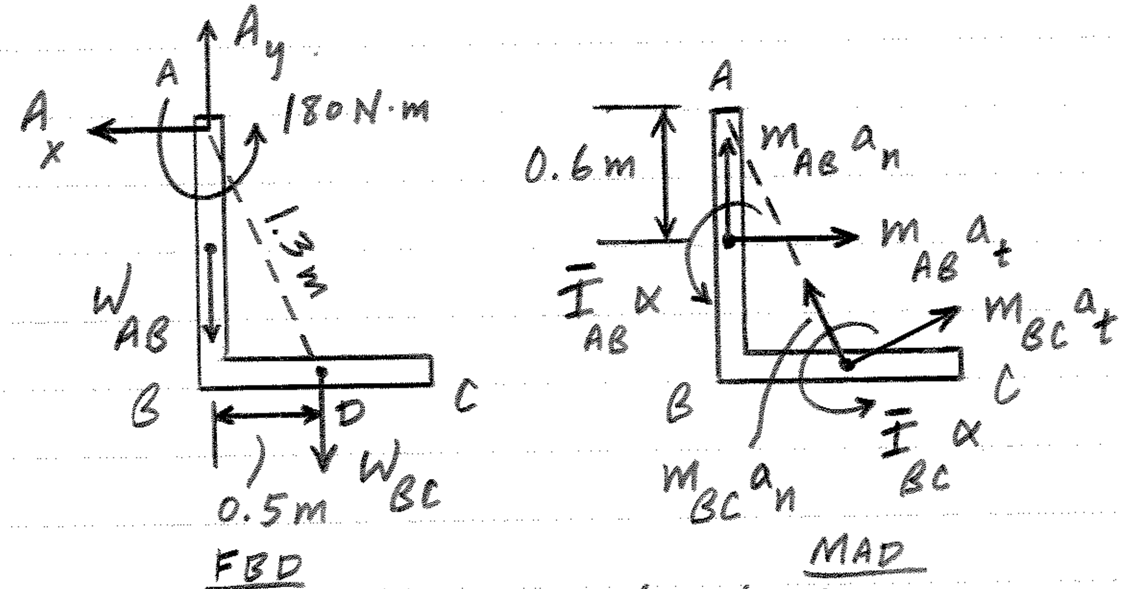
$$48\left(\frac{10}{12}\right) - 48\left(\frac{6}{12}\right) = 4\alpha + \left[\frac{48}{32.2}\left(\frac{10}{12}\right)\alpha\right]\left(\frac{10}{12}\right) + \left[\frac{48}{32.2}\left(\frac{6}{12}\right)\alpha\right]\left(\frac{6}{12}\right)$$

$\longrightarrow \alpha = 2.96 \text{ rad/s}^2$ CCW

17.40 Given: 180 N·m CCW couple acting on the L-shaped rod, $m_{AB} = 24\ kg$, $m_{BC} = 20\ kg$

Find: α of the rod.

Solution:



ROD AB: $W_{AB} = m_{AB}g = (24)(9.81) = 235.44\ N$

$$\bar{I}_{AB} = \left(\frac{1}{12}\right) m_{AB} L_{AB}^2 = \left(\frac{1}{12}\right)(24)(1.2^2) = 2.88\ kg \cdot m^2$$

$$m_{AB}a_t = m_{AB}\left(\frac{L}{2}\alpha\right) = (24)(0.6\alpha) = 14.4\alpha$$

$$m_{AB}a_n = m_{AB}\left(\frac{L}{2}w^2\right) = (24)(0.6w^2) = 14.4w^2$$

ROD BC: $W_{BC} = m_{BC}g = (20)(9.81) = 196.2\ N$

$$\bar{I}_{BC} = \left(\frac{1}{12}\right) m_{BC} L_{BC}^2 = \left(\frac{1}{12}\right)(20)(1.0^2) = 1.667\ kg \cdot m^2$$

$$m_{BC}a_t = m_{BC}(\overline{AD}\alpha) = (20)(1.3\alpha) = 26.0\alpha$$

ROD BC cont'd

$$m_{BC}\, a_n = m_{BC}\left(\overline{AD}\, w^2\right) = (20)(1.3 w^2) = 26.0 w^2$$

$$\circlearrowleft + \; (\Sigma M_A)_{FBD} = (\Sigma M_A)_{MAD}$$

$$180 - (196.2)(0.5) = 2.88\alpha + (14.4\alpha)(0.6) + 1.667\alpha + (26.0\alpha)(1.3)$$

$$\Rightarrow 81.9 = 46.987\alpha$$

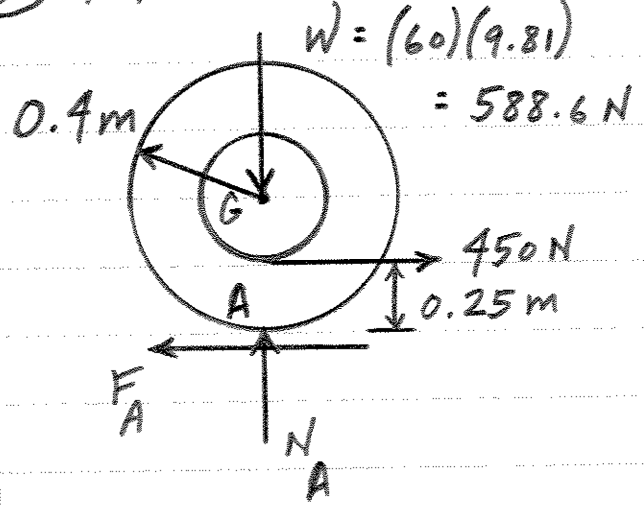
$$\therefore \alpha = 1.743 \text{ rad/s}^2$$

17.48 Given: $\bar{I} = 1.35\ kg \cdot m^2$, $\mu_s = 0.30$

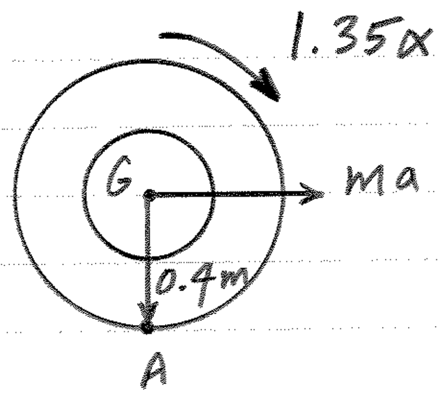
$\mu_k = 0.27$, $P = 200\ N$

Find: Acceleration of the center of the spool.

Solution:



FBD



MAD

Assume that the spool does not slip

- Check assumption $F_A \le F_{A\,max} = \mu_s N$

Note: no slip $\Longrightarrow a = R\alpha$

$+\uparrow \Sigma F_y = ma_y : \quad N_A - 588.6 = 0 \longrightarrow N = 588.6\ N$

$\xrightarrow{+} \Sigma F_x = ma_x : \quad -F_A + 450 = 60(0.4\alpha)$

$\circlearrowleft + \ (\Sigma M_A)_{FBD} = (\Sigma M_A)_{MAD} :$

$$-450(0.25) = -1.35\alpha - [60(0.4\alpha)](0.4)$$

$\longrightarrow \alpha = 10.274\ rad/s^2$

$F_A = 203.4\ N$

$F_{A\,max} = (0.30)(588.6\text{ N}) = 176.58\text{ N}$

Since $F_A > F_{A\,max}$, the spool must be slipping and a is independent of α

$$\therefore F_A = \mu_k N = 0.27(588.6\text{ N}) = 158.92\text{ N}$$

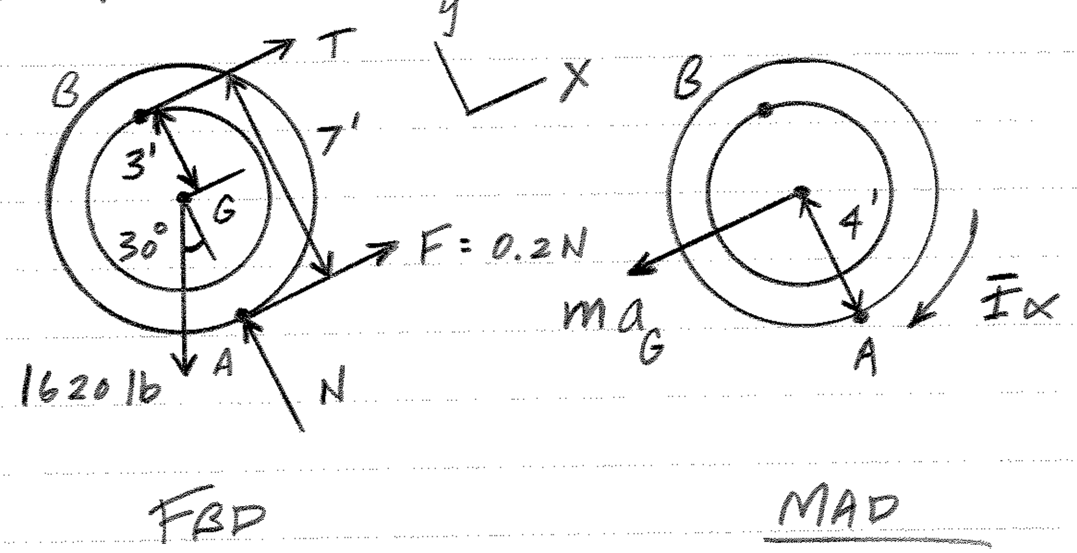
$$\xrightarrow{+} \Sigma F_x = m a_x : \; 450 - \underset{(F_A)}{158.92} = 60\,a$$

$$\longrightarrow a = 4.85\text{ m/s}^2$$

$\boxed{17.56}$ Given: $W_{spool} = 1620\ lb,\ k = 1.75\ ft$

Find: the angular acceleration and the tension in the cable.

Solution:



$$\bar{I} = mk^2 = \left(\frac{1620}{32.2}\right)(1.75^2) = 154.076\ slug\cdot ft^2$$

The spool rolls w/o slip @ B. $(\vec{a}_B = -R_{GB}\,w^2\hat{j})$

$$\vec{a}_G = \vec{a}_B + \vec{a}_{G/B}$$

$$-a_G\hat{i} = -R_{GB}w^2\hat{j} + (-\alpha\hat{k} \times -R_{GB}\hat{j}) + (-w\hat{k}) \times (-w\hat{k} \times -R_{GB}\hat{j})$$

$$-a_G\hat{i} = -R_{GB}w^2\hat{j} - R_{GB}\alpha\hat{i} + R_{GB}w^2\hat{j}$$

$\longrightarrow a_G = R_{GB}\,\alpha$

3 Unknowns - T, N, α, 3 eqns

$+\nwarrow \Sigma F_y = ma_y : N - 1620\cos 30° = 0$

$\longrightarrow N = 1402.96 \text{ N}$

$\circlearrowleft + \ (\Sigma M_B)_{FBD} = (\Sigma M_B)_{MAD}$:

$$-1620 \sin 30°(3) + 0.2N(7) = -154.07\alpha - \left[\frac{1620}{32.2}(3)(\alpha)\right](3)$$

$\longrightarrow -465.856 = -606.865\alpha$

$\alpha = 0.7676 \text{ rad/s}^2$

$+\nearrow \Sigma F_x = ma_x : \ T + 0.2N - 1620 \sin 30° = -\frac{1620}{32.2}(3)(\alpha)$

$\longrightarrow T = 413.553 \text{ lb}$