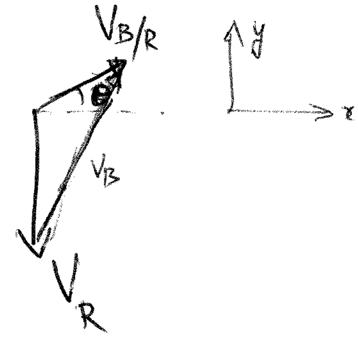
HOMEWORK-8 SOLUTIONS



15.2 V_R = Velocity of river = $-10j$ (given)

Using principles of relative motion

$$V_B = V_R + V_{B/R}$$

$$V_{B/R} = 26\cos\theta\,\hat{i} + 26\sin\theta\,\hat{j}$$

$$\therefore V_B = -10j + 26\cos\theta\,\hat{i} + 26\sin\theta\,\hat{j}$$

Since the boat travels in the 'x' direction from A to C it does not have a component of velocity along 'y'.

$$\therefore -10j + 26\sin\theta\,\hat{j} = 0$$

$$\Rightarrow \sin\theta = \frac{10}{26}$$

$$\text{or} \quad \theta = \sin^{-1}\left(\frac{5}{13}\right)$$

Hence velocity of boat = $V_B = 26\cos\theta\,\hat{i} = \underline{24 \text{ km/hr}}$

(b) Time needed to cross = $\frac{\text{Distance}}{\text{Speed}}$

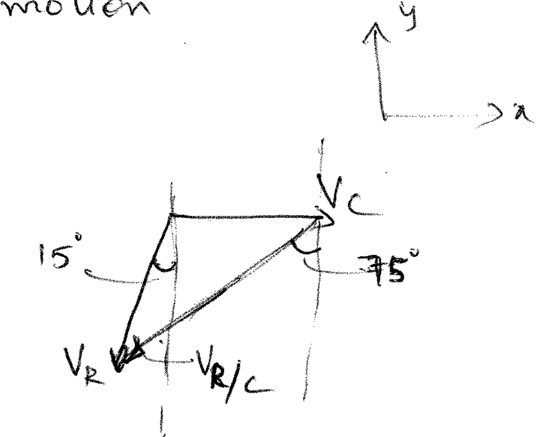
$$= \frac{4.8}{24} = \underline{12 \text{ min.}}$$

15.4

Velocity of rain = V_R.

Velocity of car = $V_C = 30$ mi/h

From the principles of relative motion and using figure



$$V_R = V_C + V_{R/C}$$

i.e $V_R(-\sin 15° \hat{i} - \cos 15° \hat{j})$

$$= 30\hat{i} + V_{R/C}(-\sin 75° \hat{i} - \cos 75° \hat{j})$$

Equating the 'x' & 'y' components

(i) $-V_R \sin 15° = 30 - V_{R/C} \sin 75°$

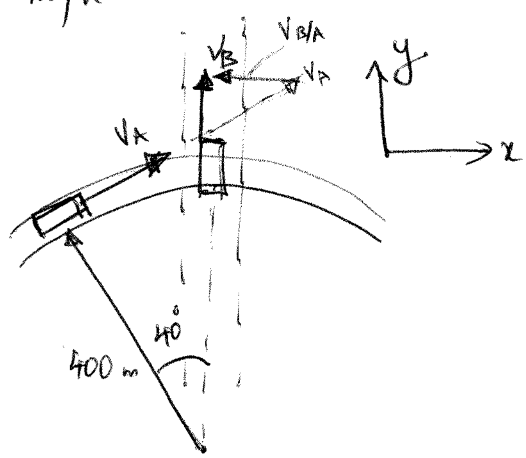
(i') $-V_R \cos 15° = -V_{R/C} \cos 75°$

Solving these simultaneous equations for V_R & $V_{R/C}$

we get

$V_{R/C} = 33.5$ mi/h & $V_R = 8.97$ mi/h.

15.12 From figure and principles of relative motion



$$V_B = V_A + V_{B/A}$$

Differentiating with respect to time

$$a_B = a_A + a_{B/A} \quad \text{—} (1)$$

As car A is travelling along the circular path ~~also~~ at a constant speed, it will have an acceleration in the radial direction.

$$a_A = \frac{V_A^2}{R}$$

Converting V_A to m/s we get $V_A = 22.222$ m/s.

Now

$$a_B = -2\hat{j} \text{ m/s}^2 \quad [\text{Decelerating at } 2 \text{ m/s}^2]$$

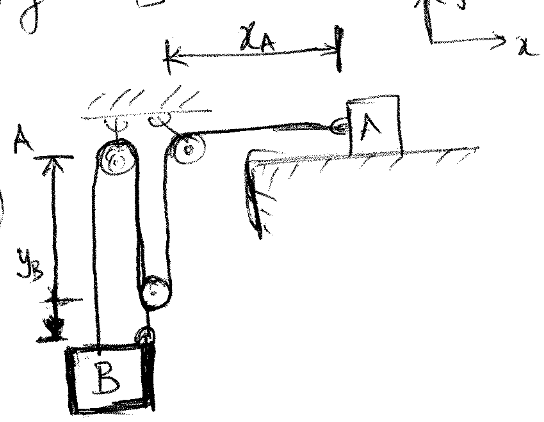
$$a_A = \frac{V_A^2}{R}\left[\sin 40\,\hat{i} - \cos 40\,\hat{j}\right] = 0.7935\,\hat{i} - 0.9457\,\hat{j} \text{ m/s}^2$$

$$\therefore a_{B/A} = a_B - a_A \quad (\text{from } (1))$$

$$= (-2\hat{j}) - (0.7935\,\hat{i} - 0.9457\,\hat{j})$$

$$= -0.7935\,\hat{i} - 1.054\,\hat{j} \text{ m/s}^2$$

15.14 To calculate the velocity of block A we need to relate it to the speed of B.



Since there is one inextensible string connecting blocks A & B we can use that:

Length of string $= L = x_A + 3y_B +$ (constant due to pulleys)

Since the length of string does not change

$$\frac{dL}{dt} = V_A + 3V_B = 0$$

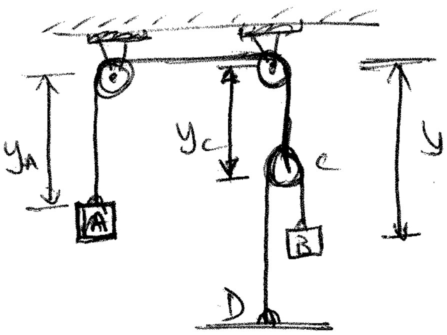
$$\Rightarrow V_A = -3V_B$$

$V_B = -0.2$ m/s [As the rope is decreasing in length]

$$\therefore V_A = -3(-0.2) = 0.6 \text{ m/s}$$

15.16

To determine the velocity of block B, we need to relate it to the motion of block A. This is done using the two inextensible strings.



String between block A & pulley C

$L_{AC} = y_A + y_C +$ constant (due to pulley)

String BD

$L_{BD} = (y_B - y_C) + (\text{constant} - y_C) +$ constant (due to pulley)

$(\text{Constant} - y_C) \Rightarrow$ (Distance between top wall & bottom wall)
— length of cable supporting C.

The constraints are obtained by differentiating with respect to time

$$\frac{dL_{AC}}{dt} = 0 = V_A + V_C \quad \text{— (1)}$$

$$\frac{dL_{BD}}{dt} = 0 = V_B - 2V_C \quad \text{— (2)}$$

Using 1 & 2 we get

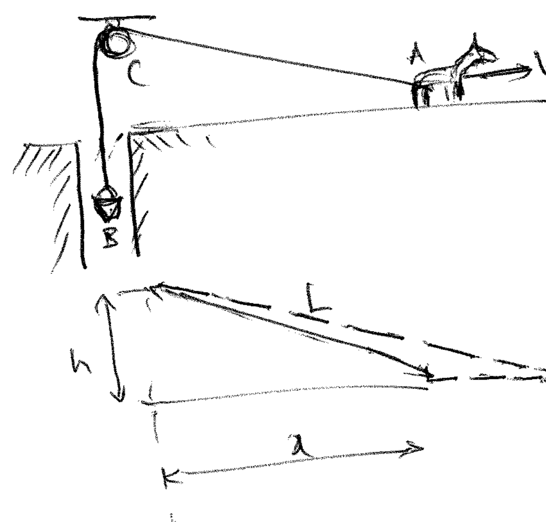
$$V_B = -2V_A$$

$$V_A = 24 \text{ in/s}$$

$$V_B = -2(24) = -48 \text{ in/s} = 48 \text{ in/s upwards.}$$

15.18

To relate the speed of the mule to velocity of the bucket, we use the inextensible string as a constraint.



Using pythagoras theorem

$$h^2 + x^2 = L^2 \quad \text{--- (1)}$$

V_A can be related as $\dot{x}$

V_B can be related to the change in length - $\dot{L}$.

h remains constant.

Differentiating (1) with respect to time

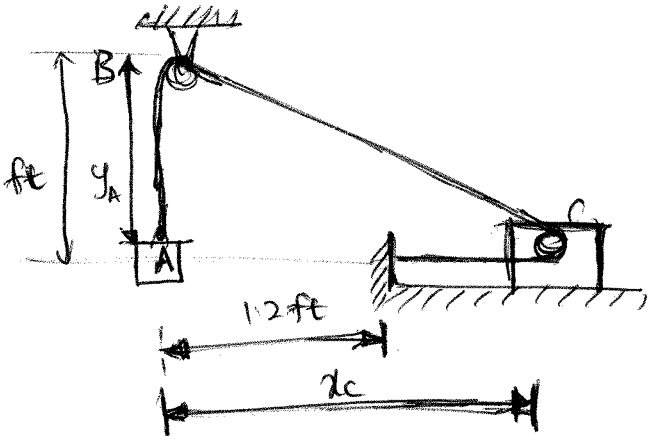
$$2x\dot{x} = 2L\dot{L}$$

$$\dot{x} = V_A.$$

$$\therefore \dot{L} = \frac{x\dot{x}}{L} = \frac{x}{L} \cdot V_A = \frac{x}{\sqrt{h^2+x^2}} \cdot V_A = V_B \text{ — directed upward.}$$

15.20

To relate the velocity of 'C' to velocity of A, we use the constant dimensions of the rope.



V_c ~~related~~ as $\dot{x}_c$

Length of rope $= (x_c - 1.2) + \sqrt{x_c^2 + 2^2} + y_A$

V_A related to $\dot{y}_A$

$$\frac{dL}{dt} = 0 = V_c + \frac{1}{2}\frac{(2x_c\dot{x}_c)}{\sqrt{x_c^2+2^2}} + V_A$$

$$\Rightarrow V_c + \frac{1}{\cancel{2}}\frac{\cancel{2}\cdot x_c V_c}{\sqrt{x_c^2+2^2}} = -V_A$$

When C is at position $x_c = 1.7$ ft, $V_c = 3$ft →.

$$\therefore -V_A = 3 + \frac{(1.7)(3)}{\sqrt{1.7^2+4}}$$

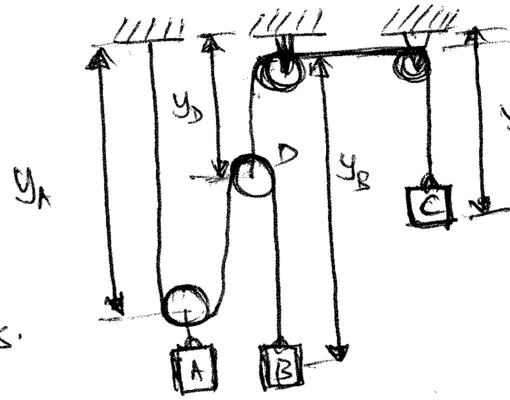
$$\Rightarrow V_A = -4.94 \text{ ft/s}$$

AS x_c increases y_A decreases. Hence opposite signs are used.

$\therefore V_A = 4.94$ ft/s (upwards).

15.24

$V_C = 6$ in/s (upwards)



Blocks A & B have the same elevation at all times.

There are two inextensible strings.

Length of cables connecting A & B and C & D are constant

$L_{AB} = y_A + (y_A - y_D) + (y_B - y_D)$ + constant (due to pulley)

$L_{DC} = y_D + y_C$ + constant (between pulleys)

Therefore differentiating both with respect to time

$$\frac{dL_{AB}}{dt} = 0 = 2V_A + V_B - 2V_D = 0 \quad \text{— (1)}$$

$$\frac{dL_{DC}}{dt} = 0 = V_C + V_D \quad \text{—— (2)}$$

$|V_C| = |V_D|$ but in opposite directions (from (2))

$V_C = -V_D$ — (3)

Also elevations of A & B are the same

$\therefore V_A = V_A$ (from $y_A = y_B$) — (4)

Using (3) & (4) in (1) we get

$3V_B + 2V_C = 0$

Since $V_C = -6$ in/s (y_C decreases), $V_B = 4$ in/s (y_B increases, downwards).