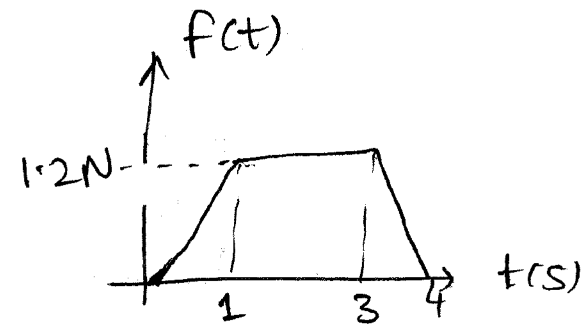
VIVEK VERMA

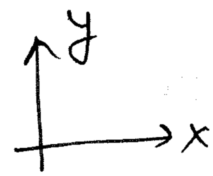
EMch 12 HOME WORK #7 SOLUTIONS.



14.64. $m = 0.2$ kg

$t=0 \quad \vec{v}_1 = 8\hat{j}$ m/s

$\vec{F} = f(t)\,\hat{i} - mg\,\hat{j}$



$\vec{v}$ at $t = 4s = \vec{v}_2 = ?$

Using Impulse-momentum principle:

$$L_{1-2} = p_2 - p_1.$$

$$L_{1-2} = \int_{t_1=0}^{t_2=4} f(t)\,dt\,\hat{i} + \int_{t_1=0}^{t_2=4} (-mg)\,dt\,\hat{j}$$

$\Downarrow$

area under F-t curve

$$\therefore \int f(t)\,dt\,\hat{i} = \frac{1}{2}(2+4) * 1.2\hat{i}$$

$$= 3.6\,i \text{ N-s}$$

$$\int_{t_1=0}^{t_2=4} -mg\,dt\,\hat{j} = -0.2 * 9.8 * 4j \text{ N-s}$$

$$= -7.848\hat{j} \text{ N-s}.$$

$$L_{1-2} = P_2 - P_1$$

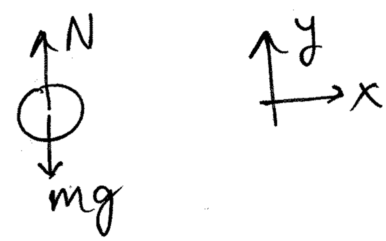
$$\therefore \quad 3.6\hat{i} - 7.848\hat{j} = 0.2(\vec{v}_2) - (8\hat{j})\,0.2$$

$$\therefore \quad \boxed{\vec{v}_2 = 1.8\hat{i} - 31.2\hat{j}}$$

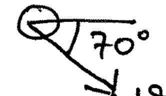
14.68. $m = 2.5\text{-oz}$, $v_1 = 30\ \text{ft/s}$ $\quad v_2 = ?$

$\theta_1 = 70°$ $\quad \theta_2 = 62°$.

FBD during impact:

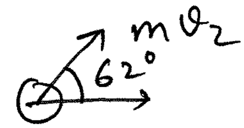


Momentum just before impact:



$\vec{v}_1 = v_1 \cos 70\,\hat{i} - v_1 \sin 70$

Momentum just after impact



$\vec{v}_2 = v_2 \cos 62°\,\hat{i} + v_2 \sin 62\,\hat{j}$

Using Impulse-momentum principle in x-direction:

$$(L_{1-2})_x = (P_2)_x - (P_1)_x$$

$F_x = 0 \quad \therefore (L_{1-2})_x = 0$

$$\Rightarrow (P_1)_x = (P_2)_x \quad (1)$$

$$(P_2)_x = m\,v_2 \cos\theta_2 \,\hat{i}$$

$$(P_1)_x = m\,v_1 \cos\theta_1 \,\hat{i}$$

$\therefore$ substituting in 1

$$m\,v_2 \cos\theta_2 = m\,v_1 \cos\theta_1$$

$$\Rightarrow v_2 = \frac{v_1 \cos\theta_1}{\cos\theta_2} = 30(\text{ft/s}) \cdot \frac{\cos 70}{\cos 62}$$

$$\therefore v_2 = 21.86 \text{ ft/s.}$$

(b) Impulse - momentum principle in y direc^n:

$$(L_{1-2})_y = (P_2)_y - (P_1)_y$$

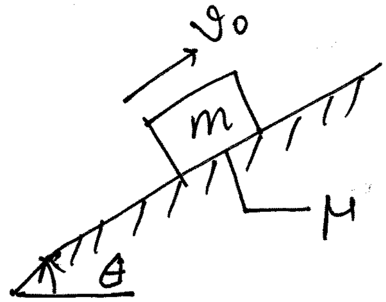
$$= m\,v_2 \sin\theta_2 - (-m\,v_1 \sin\theta_1)$$

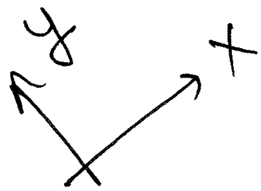
$$\therefore (L_{1-2})_y = m(v_2 \sin\theta_2 + v_1 \sin\theta_1)$$

$\therefore$ Resultant impulse acting on the ball

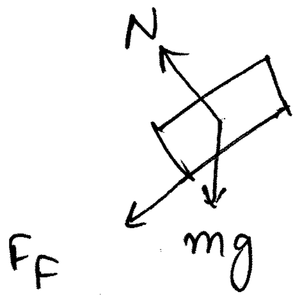
$$= (L_{1-2})_y = 0.230 \text{ lb-s.}$$

14.70.





FBD:



$N =$ surface reaction force

$= mg\cos\theta$

$F_F =$ frictional force $= \mu N$

$= \mu mg\cos\theta$

1: mass is launched with ~~velocity~~ speed v_0

2: mass comes to rest.

Impulse-momentum principle in x-direction:

$$\int_{t_1}^{t_2} F_x\,dt = p_2 - p_1$$

F_x: $\cancel{-mg\sin\theta}$ $-mg\sin\theta\,\hat{i} - \mu mg\cos\theta\,\hat{i}$

$= -mg(\sin\theta + \mu\cos\theta)\hat{i}$

$\bar{p}_2 = 0$ (since $\bar{v} = 0$)

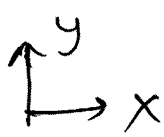
$$\bar{P}_1 = m v_0 \hat{i}$$

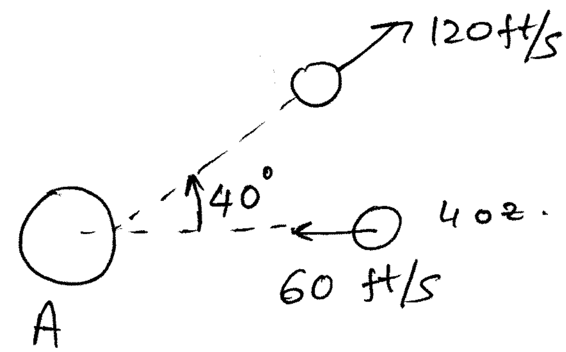
$$\therefore \int_{t_1}^{t_2} - mg(\sin\theta + \mu\cos\theta)\,dt = 0 - m v_0$$

$$\therefore -mg(\sin\theta + \mu\cos\theta)\int_{t_1}^{t_2} dt = -m v_0$$

$$\therefore \Delta t = t_2 - t_1 = \frac{v_0}{g(\sin\theta + \mu\cos\theta)}$$

14.72





Assuming no impulse due to the weight of the ball.

$$\vec{v}_1 = -60\hat{i} \text{ ft/s}$$

$$\vec{v}_2 = 120\cos 40\hat{i} + 120\sin 40\hat{j}$$

Let average force applied be F

Using Impulse - momentum principle

$$\bar{L}_{1-2} = \bar{p}_2 - \bar{p}_1$$

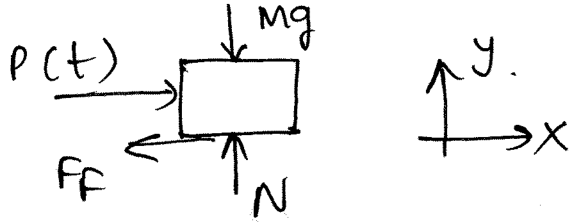
$$\Rightarrow \bar{F}_{av}.\,\Delta t = m\bar{v}_2 - m\bar{v}_1$$

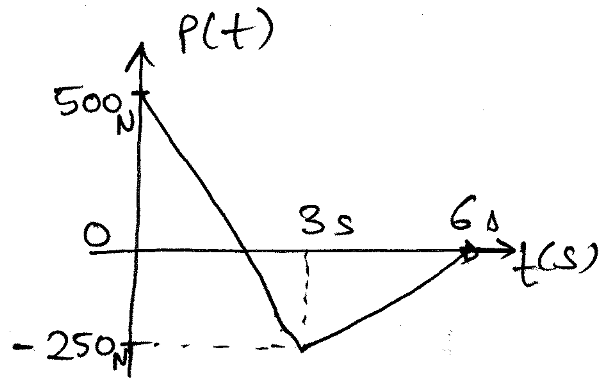
$$\Rightarrow (F_{av})(0.02) = \frac{0.25}{32.2}\left[120\cos 40\,\hat{i} + 120\sin 40\,\hat{j}\right] - \frac{0.25}{32.2}\left[-60\hat{i}\right]$$

$$\therefore \vec{F}_{av} = 58.98\hat{i} + 29.94\hat{j} \text{ lb.}$$

$$\therefore F_{av} = 66.1 \text{ lb.}$$

74. $m = 100$ kg.





~~N=~~ +

F_y: $N - mg = 0 \Rightarrow N = mg. \therefore F_f = \mu mg$

F_x: $P(t) - F_f$

(a) Maximum velocity of the block and corresponding time.

The block will accelerate in x-direction ~~x~~ while $P(t) > f_f$.

Once $P(t) \leq f_f$ the block will experience a net force in $-ve$ x-direction So its velocity in $+ve$ x-direction will decrease once $P(t) = f_f$.

Finding f_f:

Analogous to $y = mx + c$ (Equation of straight line) we have

$$\boxed{P(t) = mt + c}$$

at $t = 0$ $P(t) = 500$

$\therefore$ $500 = m(0) + c \Rightarrow c = 500\ N$

at $t = 3s$ $P = -250\ N$

$\therefore$ $-250 = m(3) + 500$

$\therefore$ $m = -250\ N/s$ $\Rightarrow \boxed{P(t) = -250t + 500}$

$\therefore$ ~~$P(t) = \mu_k mg$~~

$\therefore \quad P(t) = f_f$

when:

$$-250t + 500 = 0.3 * 100 * 9.81$$

$$\therefore \quad \boxed{t = 0.8228 \text{ seconds}}$$

Using impulse - momentum theorem in x- direction:

$$L_{1-2} = P_2 - P_1$$

1- when $t=0$

2. when $v =$ maximum v, i.e. $t = 0.8228s$.

$$\therefore \int_{t=0}^{t=0.8228s} F_x \, dt = m v_{max} - m(0)$$

$$\therefore \quad m v_{max} = \int_0^{0.8228} (-250t + 500 - f_f) \, dt$$

$$\therefore \quad \boxed{v_{max} = 0.846 \text{ m/s}}$$

(b) when the block come to rest $t = ??$

Using impulse-momentum principle in x-direction:

$$F_x = P(t) - F_f = -250t + 500 - 294.3$$

$\cancel{= -250\,m\,t}$

$$\therefore F_x = -250t + 205.7$$

$$L_{1-2} = P_2 - P_1$$

2: block comes to rest $\Rightarrow$ $v_2 = 0$

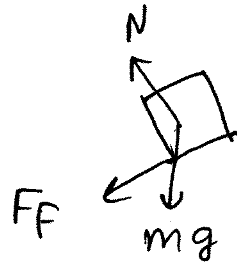
1: initially, $t = 0$ $\quad v_1 = 0$

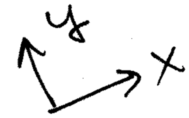
$$\therefore \int_0^t F_x \, dt = m(0) - m(0)$$

$$\therefore \int_0^t (-250t + 205.7)\,dt = 0$$

$$\therefore \boxed{t = 1.646\,s}$$

78. FBD:





F_y: $N - mg\cos\theta = 0$

$\therefore N = mg\cos\theta$

$F_F = \mu N = \mu mg\cos\theta$

F_x: $-mg\sin\theta - \mu mg\cos\theta$.

Applying Impulse momentum principle in x direction

$$L_{1-2} = mv_2 - mv_1$$

$$\therefore \int_{t_1}^{t_2} -(mg\sin\theta + \mu mg\cos\theta)\,dt = m(0) - mV_0$$

1: particle starts up the inclined portion

2: particle reaches maximum height. (its velocity $= 0$).

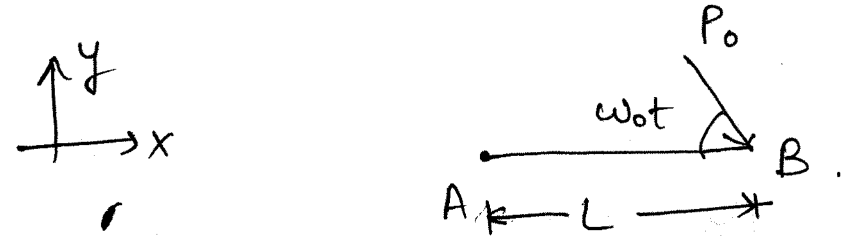
$t_2 - t_1 = \Delta t$

$$\therefore \left[mg(\sin\theta) + \mu mg(\cos\theta)\right]\Delta t = mV_0$$

$$\therefore \mu = \tan\theta - \frac{v_0}{g\,\Delta t \cos\theta}$$

$$\therefore \boxed{\mu = 0.298}$$

82.

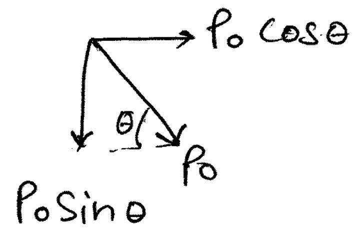


Angular impulse of force about A:

$$(A_A)_{1-2} = \int_{t_1}^{t_2} M_A \, dt$$

1: when $\theta = 0$

2: when $\theta = 90°$



$P_0 \cos\theta$ in the direction of $\vec{r}$ $\therefore$ moment about A due to $P_0 \cos\theta = 0$

$$\therefore M_A = P_0 \sin\theta \, L$$

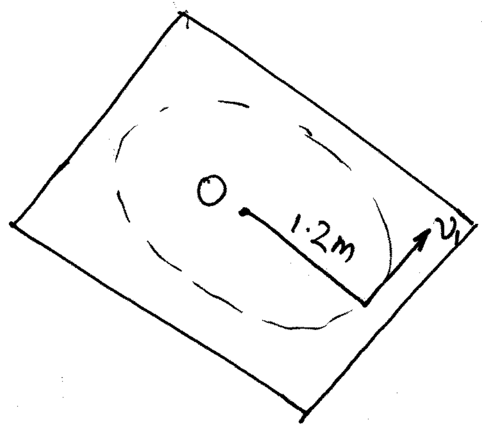
$$\therefore (A_A)_{12} = \int_{t_1}^{t_2} P_0 \sin \omega_0 t \cdot L \, dt$$

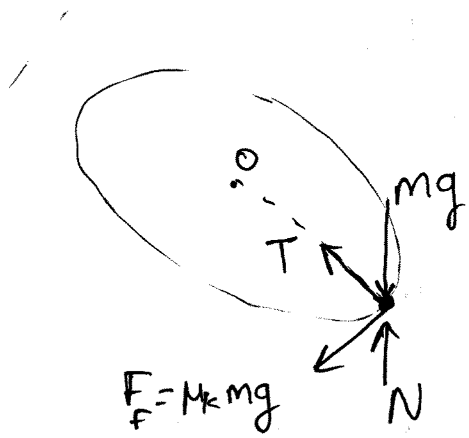
$\theta = 0 \quad \therefore t_1 = 0 \qquad \theta = \frac{\pi}{2} \quad \therefore t_2 = \frac{\pi}{2\omega_0}$

$$\therefore (A_A)_{1-2} = \int_0^{\pi/2\omega_0} P_0 L \sin\omega_0 t \, dt$$

$$\therefore (A_A)_{1-2} = \frac{P_0 L}{\omega_0}.$$

86. $v_1 = 8 \text{ m/s}$.
$\mu_K = 0.15$.





FBP

Applying momentum-angular impulse relationship:

$$(\bar{A}_O)_{1-2} = (\bar{h}_O)_2 - (\bar{h}_O)_1$$

1: $v_1 = 8 \text{ m/s}$

2: $v_2 = 0$.

$$(\circlearrowleft +): \quad \int_{t_1}^{t_2} M_o\,dt = 0 - (m v_1) R$$

$$M_o = -\mu_k m g R$$

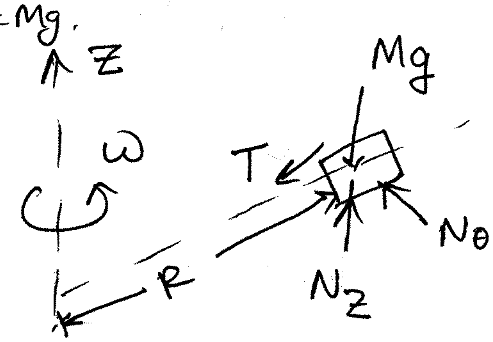
$$t_2 - t_1 = \Delta t$$

$$\therefore -\mu_k m g R \cdot \Delta t = 0 - m v_1 R$$

$$\therefore \Delta t = \frac{v_1}{\mu_k \cdot g} \qquad \therefore \boxed{\Delta t = 5.448}$$

14.90 $\quad F_z: \; N_z - Mg = 0 \Rightarrow N_z = Mg.$

Angular momentum in z direction:



FBD:

$$(\circlearrowleft +) \quad h_z = (M v_\theta) R = (M R \omega) R$$

$$\therefore h_z = M \omega R^2 = M_z$$

$$\therefore \dot{h}_z = 2 M \omega R \dot{R} = M_z$$

also $\quad M_z = R \cdot N_\theta$

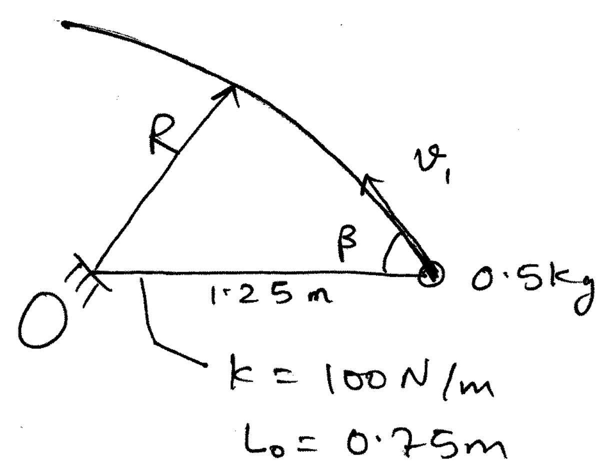
$$\therefore R N_\theta = 2 M \omega R \dot{R}$$

$$\therefore N_\theta = 2 M \omega \dot{R} = 8\,\text{N}.$$

∴ contact force between the slider and the rod $(N_A) = ?$

$$N_A = \sqrt{N_\theta^2 + N_z^2} = \sqrt{8^2 + (mg)^2}$$

$$\therefore \boxed{N_A = 21.2\ N}$$



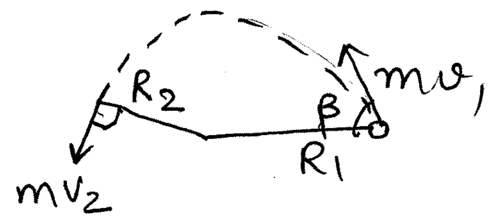
94. $v_1 = 15 m/s$
$\beta = 70°$

R_{max}, R_{min} :

when we have the condition of R_{max} or R_{min} that means we have maximum restoring force from the spring. In that case there won't be any velocity component in the direction of cord, i.e. velocity $\vec{v}$ will be perpendicular to the cord when $R = R_{min}$ or $R = R_{max}$.

Applying conservation of angular momentum about O:

$(h_o)_1 = (h_o)_2$



1: initial condition

2: when we have R_{max} or R_{min}.

$\therefore m v_1 \sin\beta \cdot R_1 = m V_2 R_2$

$\therefore V_2 = \dfrac{V_1 R_1}{R_2} \sin\beta \Rightarrow V_2 = \dfrac{17.619}{R_2}$

Applying conservation of mechanical energy at point ① and ②.

$T_1 + (V_e)_1 = T_2 + (V_e)_2$:

$\therefore \frac{1}{2} m v_1^2 + \frac{1}{2} k (R_1 - L_0)^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k (R_2 - L_0)^2$

$\therefore (0.5)(15)^2 + 100(1.25 - 0.75)^2 =$

$(0.5)\left(\dfrac{17.619}{R_2}\right)^2 + 100(R_2 - 0.75)^2$

$\therefore 137.5 R_2^2 - 155.21 = 100 R_2^2 (R_2 - 0.75)^2$

$\therefore\ R_2 = 1.119\,m = R_{minimum}$

$1.645\,m = R_{maximum}.$