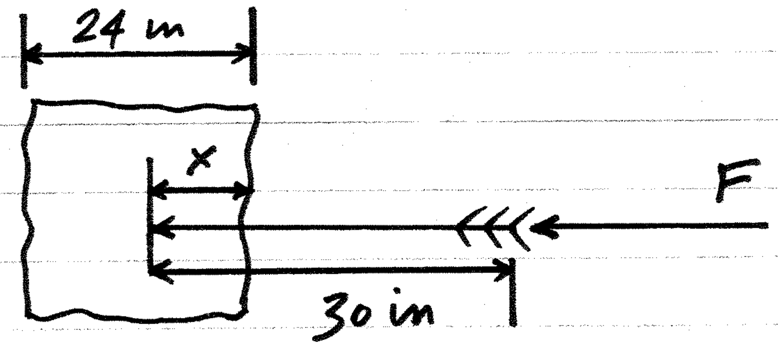
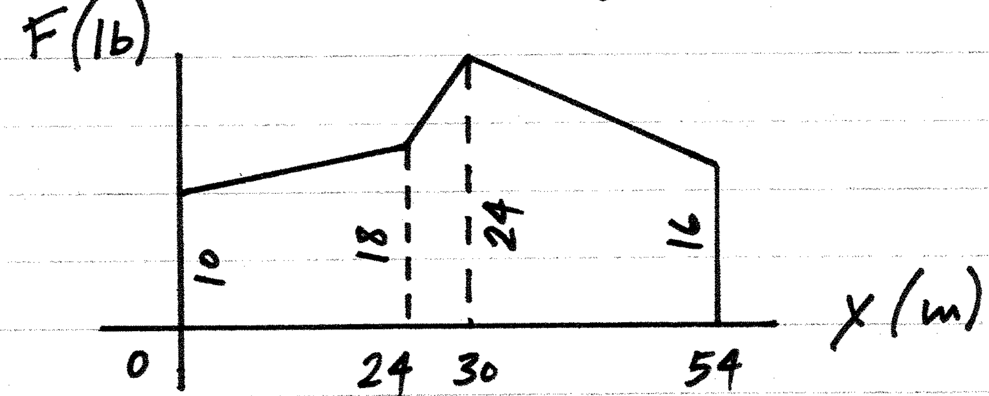
14.22 Given:





Find: the exit speed of a 3 oz arrow if its speed @ entry is 200 ft/s

Solution:

Start w/ WORK - ENERGY

$\rightarrow U_{1-2} = T_2 - T_1$

Define Positions

Position 1 - position @ entry ($x = 0$)

Position 2 - position @ exit ($x = 54$ in)

$$\rightarrow U_{1-2} = \int_0^{54} \vec{F} \cdot d\vec{x} = \int_0^{54} F\hat{\imath} \cdot -dx\,\hat{\imath}$$

Note: the force $(-F\hat{\imath})$ is the force the bail exerts on the arrow.

$$\rightarrow U_{1-2} = -\int_0^{54} F\,dx = -\text{AREA under the curve}$$

$$\rightarrow -\frac{ft.}{12in}\left(\frac{10+18}{2}(24)+\frac{18+24}{2}(6)+\frac{24+16}{2}(24)\right)$$

$$=\frac{1}{2}\left(\frac{3}{16}\right)\left(\frac{1}{32.2}\right)v_2^2-\frac{1}{2}\left(\frac{3}{16}\right)\left(\frac{1}{32.2}\right)(200)^2$$

$$\rightarrow -78.5\ lb\cdot ft = 0.00291\,v_2^2 - 116.460\ lb\cdot ft$$

$$\Longrightarrow v_2 = 114.184\ ft/s$$

$\boxed{14.26}$ Given: 2 Springs w/ stiffness k, $m = 20$ Mg railroad car traveling @ 8 km/h

Find: k if the springs are designed to stop 300 mm after impact.

Solution:

Start w/ conservation of Mechanical energy.

Define Positions

Position 1 - Before the car impacts the longer spring

Position 2 - When the springs stop deforming.

$$\rightarrow V_1 + T_1 = V_2 + T_2$$

$$\rightarrow \tfrac{1}{2}k\cancel{\delta_1^2} + \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}k\delta_2^2 + \tfrac{1}{2}m\cancel{v_2^2}$$

0 before impact, springs have not deformed yet.

0 system comes to a stop

$$\rightarrow \tfrac{1}{2}(20000\,\text{kg})\left(\frac{8000\,\text{m}}{3600\,\text{s}}\right)^2 = \tfrac{1}{2}k\left(0.100^2 + 0.300^2\right)$$

the deformation in position 2 is the combined deformation of the two springs the longer spring deforms 0.300 m after impact and the shorter spring deforms 0.100 m after impact $\Rightarrow k = 987654$ N/m

$\boxed{14.32}$ Given: Weight of collar = 6 oz, Platform is traveling to the right w/ speed v_0, Neglect friction.

Find: Speed v_0 after the platform stops and the collar slides up the rod to its highest position.

Solution:

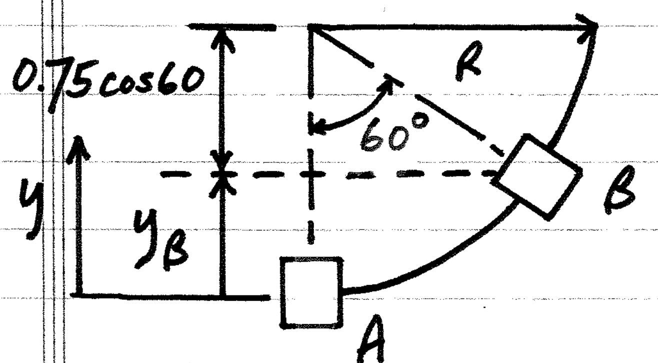
Start w/ Conservation of Mechanical energy.

$$\longrightarrow V_A + T_A = V_B + T_B$$

$$\longrightarrow mg\cancel{y_A} + \tfrac{1}{2}mv_0^2 = mgy_B + \tfrac{1}{2}m\cancel{v_B^2}$$

0 see figure below

0 collar comes to a stop



$R = 9\text{ in} = 0.75\text{ ft}$

$$\longrightarrow \frac{1}{2}\left(\frac{6}{16}\right)\left(\frac{1}{32.2}\right)v_0^2 = \left(\frac{6}{16}\right)\overbrace{\left(0.75 - 0.75\cos 60^\circ\right)}^{y_B}$$

$$\Longrightarrow v_0 = 4.914\ \text{ft/s}$$

$\boxed{14.34}$ Given: Mass of bob = 0.5 kg

Find: Smallest v_A if the bob is suspended from

a) a rigid rod w/ negligible mass

b) a string

Solution:

Start w/ conservation of mechanical energy.

Define Positions

Position A - bottom of circle

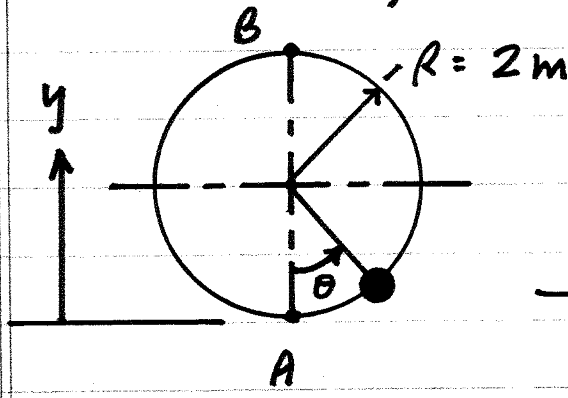
Position B - top of circle

$\longrightarrow V_A + T_A = V_B + T_B$

$\longrightarrow mg\cancel{y_A} + \frac{1}{2} m v_A^2 = m g y_B + \frac{1}{2} m \cancel{v_B^2}$

0 see figure below   0 Why?

To minimize velocity @ A, the velocity @ B must be negligible



$\longrightarrow \frac{1}{2} \cancel{m} v_A^2 = \cancel{m} g y_B$

$\longrightarrow v_A^2 = 2 g y_B$ where $y_B = 2R = 4\text{ m}$

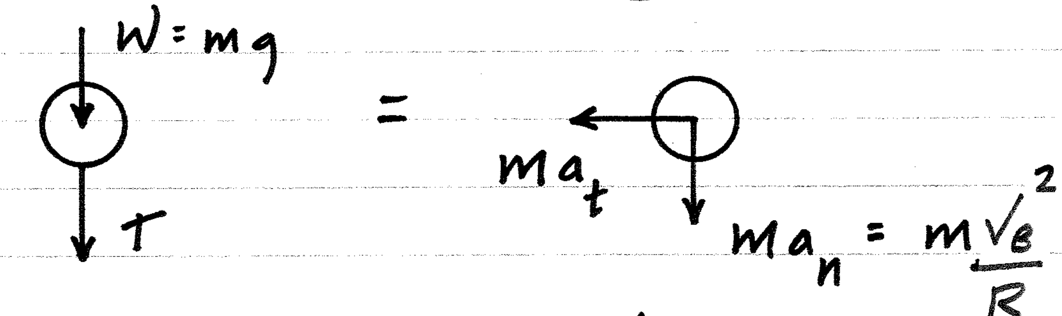
$\Longrightarrow v_A = \sqrt{2(9.81)(4)} = 8.859\text{ m/s}$ (ROD)

PART B

$$\underbrace{mgy_A}_{0} + \frac{1}{2}mv_A^2 = mgy_B + \frac{1}{2}mv_B^2$$

V_B is no longer negligible @ B because the bob would not be able to complete the circle w/ no velocity @ B (the string would be lump)
So, what is V_B?

FBD MAD



$$+\downarrow \Sigma F_n = ma_n : \quad mg + \cancel{T} = \frac{mv_B^2}{R}$$

negligibe to minimize V_B

$$\longrightarrow V_B = \sqrt{Rg} = \sqrt{(2)(9.81)} = 4.429 \text{ m/s}$$

$$\therefore \frac{1}{2}\cancel{m}v_A^2 = \cancel{m}(9.81)(4) + \frac{1}{2}\cancel{m}(4.429)^2$$

$$\Longrightarrow V_A = 9.905 \text{ m/s (STRING)}$$

14.36 Given: Mass m is at rest at A, Neglect friction.

Find: the speed of m as a function of θ and the value of θ when m loses contact w/ the surface.

Solution:

Start w/ conservation of Mechanical energy.

Define Positions

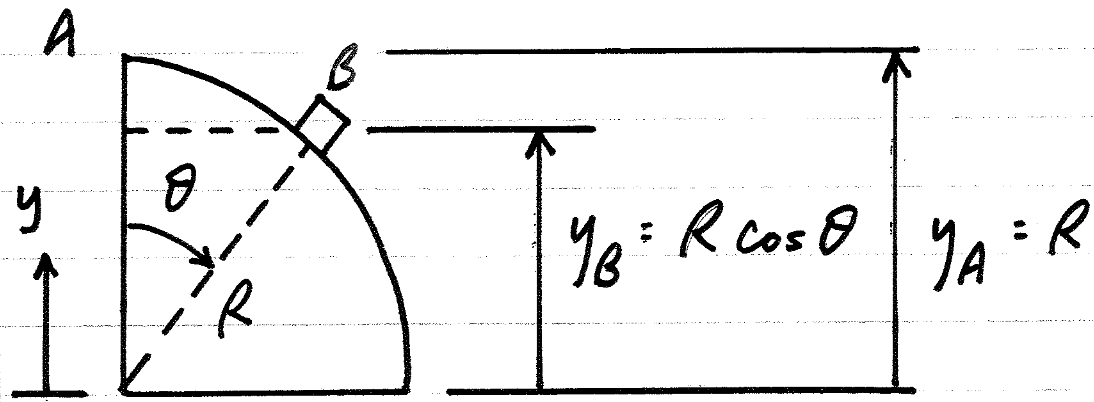
Position A - at rest at A.

Position B - arbitrary position on cylindrical surface.

$$\longrightarrow V_A + T_A = V_B + T_B$$

$$\longrightarrow mgy_A + \tfrac{1}{2}m\cancel{v_A^2} = \tfrac{1}{2}mv_B^2 + mgy_B$$

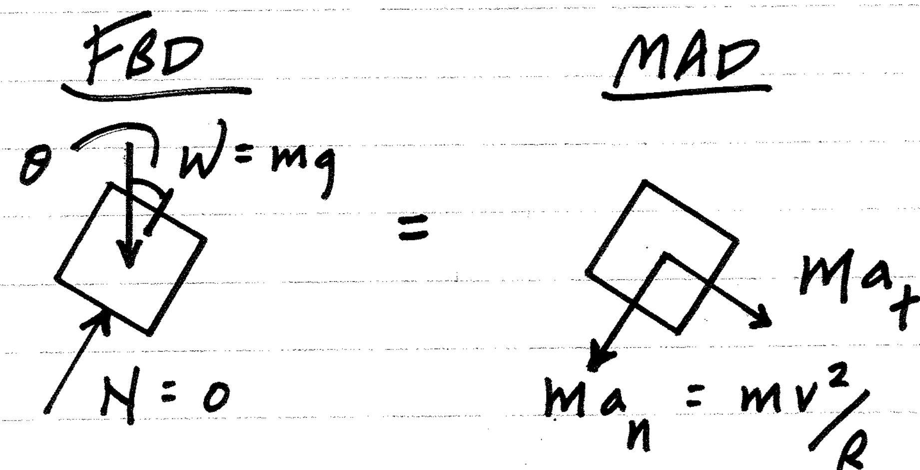
0 at rest



$$\longrightarrow mgR = \tfrac{1}{2}mv_B^2 + mg(R\cos\theta)$$

$$\Rightarrow v_B = \sqrt{2Rg(1-\cos\theta)} \quad \text{(any pt. on surface)}$$

to find the value of θ when m loses contact w/ the surface, we need to find the velocity when the normal force (contact force) is zero and equate this expression to v_B



$$\overset{+}{\swarrow} \Sigma F_n = ma_n : \quad mg\cos\theta = m\frac{v^2}{R}$$

$$\longrightarrow v = \sqrt{Rg\cos\theta} \quad (v \text{ @ loss of contact})$$

$$\therefore \sqrt{2Rg(1-\cos\theta)} = \sqrt{Rg\cos\theta}$$

$$\longrightarrow 2Rg = 3Rg\cos\theta$$

$$\Rightarrow \theta = \cos^{-1}(2/3) = 48.19^\circ$$

$\boxed{14.40}$ Given: $W = mg = 40$ lb sliding panel, spring attached to rope ABC w/ $k = 1.0$ lb/ft, Spring is undeformed when $x = 0$, panel starts from rest @ $x = 8$ ft.

Find: Speed when $x = 0$.

Solution:

Start w/ Conservation of Mechanical energy.

Define Positions

Position 1 - Panel @ $x = 8$ ft (Rest)

Position 2 - Panel @ $x = 0$

$$V_1 + T_1 = V_2 + T_2$$

$$\tfrac{1}{2}k\delta_1^2 + \underbrace{\cancel{\tfrac{1}{2}mv_1^2}}_{0 \text{ @ rest}} = \underbrace{\cancel{\tfrac{1}{2}k\delta_2^2}}_{0 \text{ undeformed @ } x=0} + \tfrac{1}{2}mv_2^2$$

What is the deformation of the spring in position 1? (δ_1)

$\delta_1 = L - L_0$ where L is the length of the string from pt. A to B @ $x = 8$ ft, and L_0 is the length of the string from pt. A to B in the undeformed position.

$$\delta_1 = \sqrt{8^2 + 6^2} - 6 = 10 - 6 = 4 \text{ ft}$$

$$\therefore \ \tfrac{1}{2}(1.0)(4)^2 = \tfrac{1}{2}\left(\frac{40.0}{32.2}\right) V_2^2$$

$$\Longrightarrow V_2 = 3.589 \text{ ft/s}$$

Note : W (weight) has no potential energy because there is no elevation change

$\boxed{14.44}$ Given: $W = 0.25$ lb collar slides on the circular rod AB, the spring attached to the collar has a free length of 9 in. and $k = 0.1$ lb/in

Find: the speed of the collar @ B if released from rest @ A.

Solution:

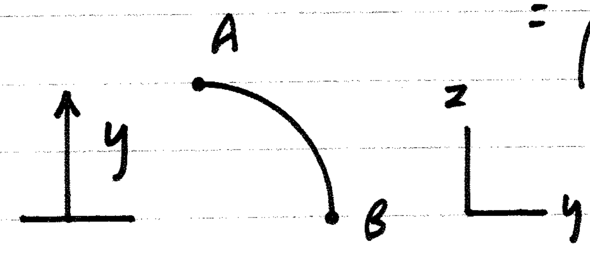
Start w/ Conservation of Mechanical energy.

$$\rightarrow V_A + T_A = V_B + T_B$$

there are two sources of potential energy, the weight (mg) and the spring force.

$$\rightarrow \left(\tfrac{1}{2}k\delta_A^2 + mgy_A\right) + \tfrac{1}{2}m\underbrace{v_A^2}_{0 \text{ (Rest)}}$$

$$= \left(\tfrac{1}{2}k\delta_B^2 + mg\underbrace{y_B}_{0 \text{ see figure}}\right) + \tfrac{1}{2}mv_B^2$$



What are the defomations? (δ_A and δ_B)

$\delta_A = L_A - L_0$ where L_A is the length of the spring @ A and $L_0 = 9$ in

$$\rightarrow \delta_A = \sqrt{3^2 + 4^2 + 12^2} - 9 \text{ in} = 13 - 9 = 4 \text{ in}$$

$$= 0.3333 \text{ ft}$$

$$\delta_B = L_B - L_0 = \sqrt{4^2 + (12-3)^2} - 9\text{ in} = 0.849\text{ in}$$

$$= 0.07075\text{ ft}$$

$$\therefore \tfrac{1}{2}\left(0.1\ \text{lb/in}\right)\left(12\ \text{in/ft}\right)(0.3333)^2 + (0.25)(1\text{ ft})$$

$$= \tfrac{1}{2}\left(0.1\ \text{lb/in}\right)\left(12\ \text{in/ft}\right)(0.07075)^2 + \tfrac{1}{2}\left(\frac{0.25}{32.2}\right)V_B^2$$

$$\Longrightarrow V_B = 8.989\text{ ft/s}$$