EMcH 12

HOMEWORK ASSIGNMENT - 3

13.8 Angular velocity varies as $\dot{\theta} = 8 - 12t^2$ rad/s

To find $\vec{a}$ at $t=0$ and $t=1$ sec for point B

(i) At $t=0$:

$\dot{\theta} = 8$ rad/s

$\ddot{\theta} = -24t = 0$

$a_n = R\dot{\theta}^2 \qquad a_t = R\ddot{\theta} \qquad (\dot{r}=0$ for circular motion$)$

$a_n = (1\text{ m})(8\text{ rad/s})^2 = 64\text{ m/s}^2$

$a_t = 0$

$\therefore |\vec{a}| = 64\text{ m/s}^2$

(ii) At $t=1$ sec, $\quad \dot{\theta} = 8 - 12(1)^2 = -4$ rad/s

$\ddot{\theta} = -24(1) = -24\text{ rad/s}^2$

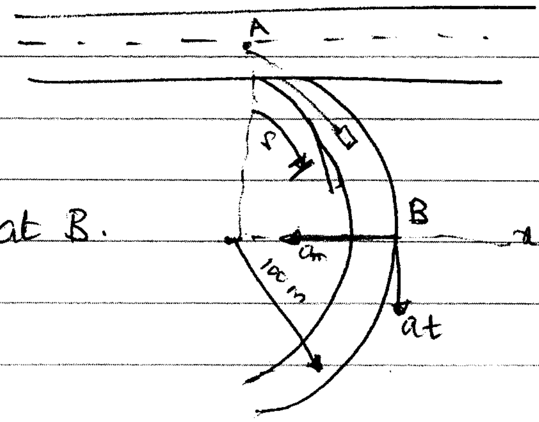
$a_n = R\dot{\theta}^2 = (1\text{ m})(-4\text{ rad/s})^2 = 16\text{ m/s}^2$

$a_t = R\ddot{\theta} = (1\text{m})(-24) = -24\text{ m/s}$

$|\vec{a}| = \sqrt{16^2 + (-24)^2} = 28.84\text{ m/s}^2$

13.14

$$a_t = \frac{(90+S)}{450} \ m/s^2$$



To find: Acceleration vector at B.

$$S \text{ from A to B} = \frac{\pi R}{2}$$

$$= \frac{\pi \times 100 \ m}{2} = 157.08 \ m$$

$$a_t(B) = \frac{(90+157.08)}{450} = 0.5491 \ m/s^2$$

$$a_n = \frac{V^2}{\rho}$$

To find v. $a_t = \frac{v\,dv}{ds}$

$$\Rightarrow \left(\frac{90+S}{450}\right) ds = v\,dv \qquad \text{[Plugg in the expression]}$$

Integrating both sides

$$\frac{S}{5} + \frac{S^2}{900} + C = \frac{V^2}{2}$$

At A, $S=0$, $V=0$. $\therefore C = 0$

$$\therefore \frac{2S}{5} + \frac{S^2}{450} = V^2$$

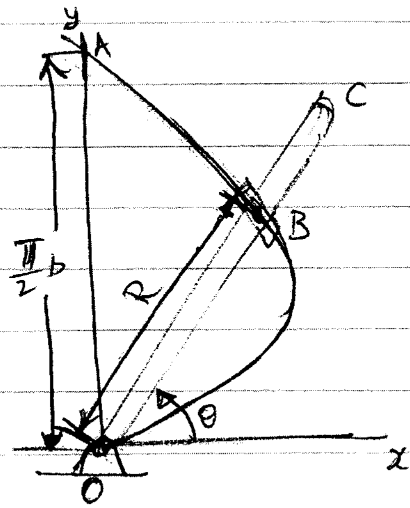
At $S = 157.08$ m (At B) $V = 10.85$ m/s

$$a_n(B) = \frac{V^2}{\rho} = \frac{(10.85)^2}{100} = 1.177 \ m/s^2$$

$\vec{a} = -1.177\,i - 0.5491\,j$ m/s^2 (Taking into account directions)

<u>13.30</u> Shape of spiral $R = b\theta$

Constant speed $= v_0$



<u>To find:</u> Angular speed $\dot\theta$ interms of v_0, b and θ.

We know that $V_R = \dot R$ and $V_\theta = R\dot\theta$

Also given $V = \sqrt{V_R^2 + V_\theta^2} = V_0 =$ constant

$$R = b\theta$$

$$V_R = \dot R = b\dot\theta$$

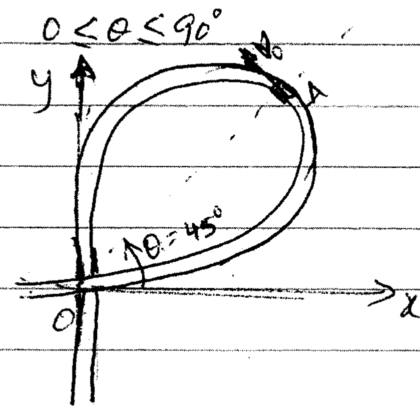
$$V = \sqrt{b^2\dot\theta^2 + R^2\dot\theta^2} = V_0$$

$$\therefore \dot\theta = \frac{v_0}{\sqrt{R^2 + b^2}} = \frac{V_0}{\sqrt{b^2\theta^2 + b^2}}$$

$$\therefore \dot\theta = \frac{V_0}{b\sqrt{\theta^2 + 1}}$$

13.34 Curve defined by $R^2 = b^2\sin 2\theta$ $\quad 0 \le \theta \le 90^\circ$

Constant speed = v_0



To find: Acceleration.

$$a_R = (\ddot{R} - R\dot{\theta}^2)e_R$$
$$a_\theta = (R\ddot{\theta} + 2\dot{R}\dot{\theta})e_\theta$$

$$R^2 = b^2 \sin 2\theta$$

Differentiating

$$2R\dot{R} = b^2 2\cos 2\theta \cdot \dot{\theta}$$

At $R = b$, $\theta = 45^\circ$, (for position A)

$$2R\dot{R} = 0 \quad \Rightarrow \quad \dot{R} = 0$$

$$\cancel{2}R\dot{R} = b^2 \cancel{2}\cos 2\theta\, \dot{\theta}$$

Differentiating

$$R\ddot{R} + \dot{R}^2 = -2b^2 \sin 2\theta\, \dot{\theta}^2 + b^2 \cos 2\theta\, \dot{\theta}^2$$

At $R = b$, $\theta = 45^\circ$

$$\ddot{R} = -2b\dot{\theta}^2$$

$$V = \text{constant} = \sqrt{\dot{R}^2 + R^2\dot{\theta}^2} = V_0 \qquad \text{At A, } \dot{R} = 0$$

$$\therefore R^2\dot{\theta}^2 = V_0^2$$

$$\dot{\theta}^2 = \frac{V_0^2}{R^2} = \frac{V_0^2}{b^2} \Rightarrow \boxed{\dot{\theta} = \frac{V_0}{b}}$$

$$|v| = \sqrt{\dot{R}^2 + \dot{R}^2 \cot^2\theta} = \dot{R}\,\mathrm{cosec}\,\theta.$$

Since the velocity is only is in the vertical direction ~~(~~only linear) the acceleration is the derivative of velocity

$$a = \dot{v} = (\dot{R}\,\mathrm{cosec}\,\theta)' = -\dot{R}\dot{\theta}\cot\theta\,\mathrm{cosec}\,\theta$$

$$= -\dot{R}\cdot\left(\frac{\dot{R}}{b\sec\theta\tan\theta}\right)\cot\theta\,\mathrm{cosec}\,\theta$$

$$= -\frac{\dot{R}^2}{b}\cot^3\theta.$$

Hence $\ddot{R} = -\frac{2bV_0^2}{b^2}$

To find $\ddot{\theta}$

$V_0^2 = \dot{R}^2 + R^2\dot{\theta}^2$

Differentiating

$0 = 2\dot{R}\ddot{R} + \dot{R}^2 2\dot{\theta}\ddot{\theta} + 2\dot{\theta}^2 R\dot{R}$

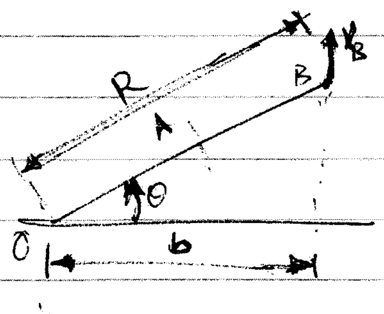
But since $\dot{R} = 0$, $\ddot{\theta}$ has to be zero.

Now $a_R = (\ddot{R} - R\dot{\theta}^2) = \left(-\frac{2V_0^2 b}{b^2} - \frac{bV_0^2}{b^2}\right) = -\frac{3V_0^2}{b}$

And $a_\theta = (R\ddot{\theta} + 2\dot{R}\dot{\theta}) = 0 + 0 = 0$

Acceleration $= -\frac{3V_0^2}{b}$

12.38 $\frac{b}{R} = \cos\theta$



To find: Expressions for magnitudes of velocity and acceleration

$R = b\sec\theta$

Differentiating: $\dot{R} = b\sec\theta\tan\theta\,\dot{\theta}$

$\Rightarrow \dot{\theta} = \frac{\dot{R}}{b\sec\theta\tan\theta}$

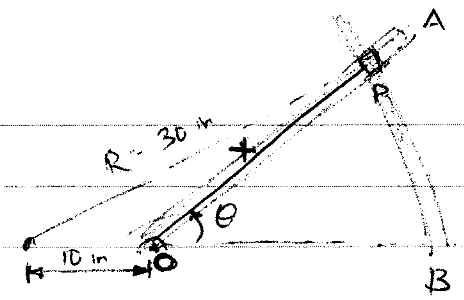
$V = \dot{R}e_R + R\dot{\theta}\hat{e}_\theta = \dot{R}\hat{e}_R + \cancel{b\sec\theta}\cdot\frac{\dot{R}}{\cancel{b\sec\theta}\tan\theta}\cdot\hat{e}_\theta$

$= \dot{R}e_R + \dot{R}\cot\theta\, e_\theta$

13.40

$\dot{\theta} = 2$ rad/s

To find Speed of P at $\theta = 60°$



We can find the components of velocity with respect to O

$$V_R = \dot{x} \quad ; \quad V_\theta = x\dot{\theta}$$

Using the cosine law

$$\cos(\pi - \theta) = \frac{x^2 + 10^2 - 30^2}{2(x)(10)}$$

$$\Rightarrow \quad -\cos\theta = \frac{x^2 + 10^2 - 30^2}{20x}$$

$$\Rightarrow \quad x^2 - 800 + 20x\cos\theta = 0 \quad -①$$ [simplifying the above expression)

At $\theta = 60°$, we can solve for x

$$x^2 + 10x - 800 = 0$$

Solving $x = 23.72$ in

Now we can differentiate ① to establish a relationship between $\dot{x}$ and $\dot{\theta}$.

$$2x\dot{x} + 20\dot{x}\cos\theta - 20x\sin\theta\,\dot{\theta} = 0$$

$$\Rightarrow \quad x\dot{x} + 10\dot{x}\cos\theta = 10x\sin\theta\,\dot{\theta}$$

Plugging in values of $x = 23.72$, $\dot{\theta} = 2$ rad/s & $\theta = 60°$

$$\dot{x}(23.72+5) = 10\times 23.72\times 0.866\times 2$$

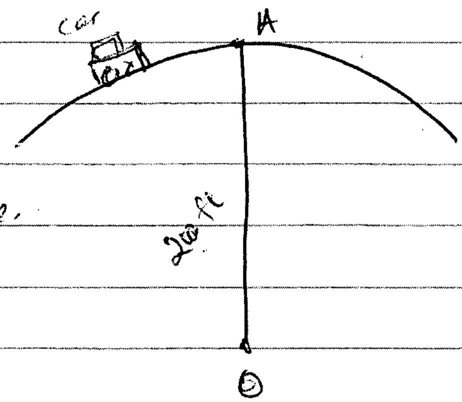
Solving for $\dot{x}$ we get $\dot{x} = 14.31$ in/s

Now $V_R = \dot{x} = 14.31$ in/s

$V_\theta = x\dot{\theta} = 23.72\times 2 = 47.44$ in/s.

$$\therefore |\vec{V}| = \sqrt{V_\theta^2 + V_R^2} = 49.54 \text{ in/s} = \text{Speed}.$$

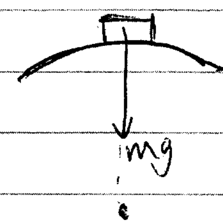
13.50 Weight of car = not specified



For the maximum speed v, the car is just in touch with the surface. Hence the car is on the point of losing contact - Hence the normal reaction tends to zero.

The acceleration of the car has two components

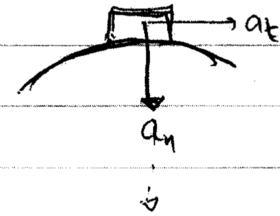
$$a_n = \frac{v^2}{R} \qquad a_t = \dot{v}$$



At point A,

The weight acts radially towards O

Using the diagrams we can relate



$$mg = ma_n$$

$$\therefore mg = \frac{mv^2}{R}$$

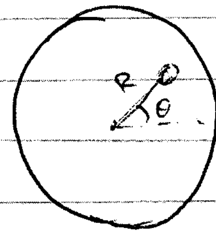
$\therefore V^2 = Rg$

$\therefore V = \sqrt{Rg} = \sqrt{200 \times 32.2} = 80.24$ ft/sec

<u>13.56</u> $R = 15$ in and $\theta = 0$

$\mu = 0.2$

$\ddot{\theta} = 1.5$ rad/s^2



<u>To find:</u> Angular velocity $\dot{\theta}$

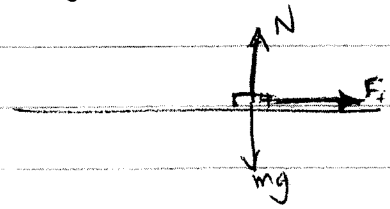
Since the coin is in a state of impending slip R is constant.

Hence $\dot{R} = \ddot{R} = 0$

We have the components of acceleration

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\hat{e}_\theta$$

However it simplifies to



$$\vec{a} = (-R\dot{\theta}^2)\hat{e}_R + (R\ddot{\theta})\hat{e}_\theta$$

The frictional force acts to oppose the net motion of the coin and is hence in a direction opposite to that of motion.

On the point of motion

$F_{fr} = \mu N = \mu mg$ (using the FBD)

Equating $\vec{F}_{fr} = m\vec{a}$

acceleration has a resultant magnitude

$\sqrt{R^2\dot{\theta}^4 + R^2\ddot{\theta}^2}$ along a direction opposite to that of friction

$$\therefore \mu \cancel{m} g = \cancel{m}\sqrt{R^2\dot{\theta}^4 + R^2\ddot{\theta}^2}$$

Plugging in the known values we can solve for $\dot{\theta}$ as

$$\dot{\theta} = \left[\frac{1}{R^2}\left\{(\mu g)^2 - R^2\ddot{\theta}^2\right\}\right]^{1/4}$$

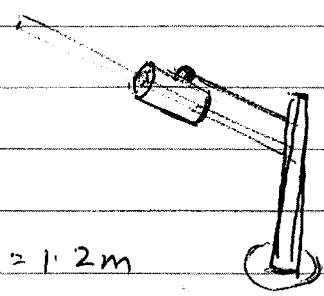
$$= 2.22 \text{ rad/s}$$

<u>13.58</u> mass = 2 kg

constant angular speed $\dot{\theta} = 6$ rad/s

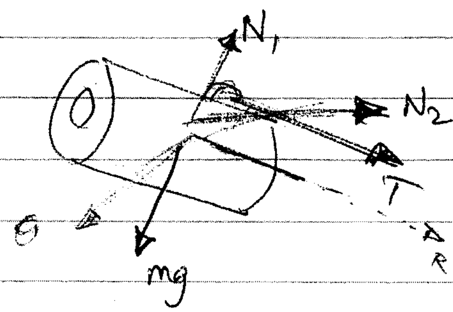
$\dot{r} = 0.8$ m/s

<u>To find</u>: Tension and contact force at $R_0 = 1.2$ m



From the FBD and MAD

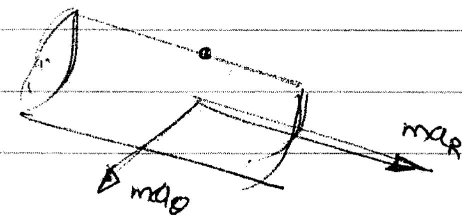
$N_1 = mg = 2 \times 9.8 = 19.6$ N



$F_\theta = N_2 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$\ddot{\theta} = 0$ — constant angular speed.

$\therefore$ Plugging in known values



we get $N_2 = 2\left(2(0.8)(6)\right) = 19.2\,N.$

The resultant contact force has a magnitude

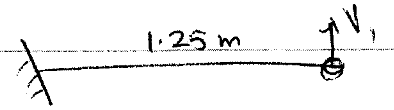
$$|\vec{N}| = \sqrt{N_1^2 + N_2^2} = \sqrt{(19.6)^2 + (19.2)^2} = 27.5\,N$$

Also T is the only radial force and so

$$F_R = T = m(\ddot{r} - r\dot{\theta}^2)$$
$$= 2(0 - 1.2 \times 36) = -86.4\,N.$$

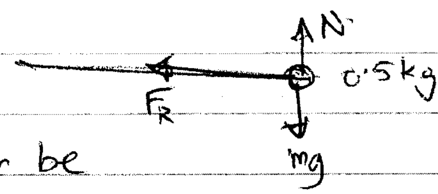
Hence T has a magnitude $= 86.4\,N$.

13.64 $k = 100\ N/m$



$L_0 = 0.75\ m$

To find : V_1 for a circular path.



The forces acting on the body can be resolved as Tangential and Normal.

From FBD we find $F_t = 0$

acceleration has components

$$a_t = \dot{v} \quad \text{and} \quad a_n = \frac{V^2}{R}$$

Using $\vec{F} = m\vec{a}$ $\quad \vec{F_t} = 0$ implies $m a_t = 0$ Hence $\dot{v} = 0$

V_1 is constant.

Also $F_R = k(L - L_0) = k(1.25 - 0.75) = m a_n.$

$\Rightarrow$ $100(0.5) = 0.5 \cdot \dfrac{V_1^2}{1.25}$ $\Rightarrow$ Solving we get $V_1 = 11.18\ m/s$