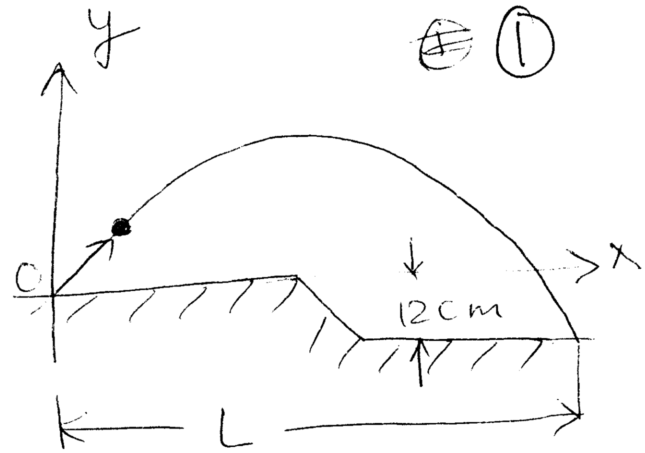
E Mech 12

HOME WORK 2

VIVEK VERMA
01/23/2003

①



12.9. Given

$x = 86t$

$y = 96t - 4.91t^2$

x, y in meters and

t in seconds

(a)

$\vec{r} = x\hat{i} + y\hat{j}$

$\therefore \vec{r} = 86t\,\hat{i} + (96t - 4.91t^2)\,\hat{j}$ m

velocity $\vec{v}(x,y) = \dfrac{d\vec{r}}{dt} = 86\,\hat{i} + (96 - 9.82t)\,\hat{j}$ m/s

acceleration $\vec{a} = \dfrac{d^2\vec{r}}{dt^2} = 0\,\hat{i} - 9.82\,\hat{j}$ m/s²

$$\therefore \boxed{\vec{a} = -9.82\,\hat{j} \text{ m/s}^2}$$

NOTE: acceleration due to gravity.

(b) velocity vector at O:

$\vec{v}(0,0) = ?$

$$\bar{v} = 86\hat{i} + (96 - 9.82\,t)\hat{j} \quad m/s.$$ ②

At 'O' i.e. at $(0,0)$ $t = 0$.

$$\therefore \bar{v}(0,0) = 86\hat{i} + (96 - 9.82(0))\hat{j} \; m/s$$

$$\therefore \boxed{\bar{v} = 86\hat{i} + 96\hat{j} \; m/s}$$

(c) Maximum height 'h':

Height from ground at any instant is given by the relation:

$$y = 96t - 4.91t^2. \qquad (1)$$

to find maximum height we can use maxima - minima.

Differentiating (1) with respect to time.

$$\frac{dy}{dt} = 96 - 9.82\,t \qquad (2)$$

For maxima or minima $\frac{dy}{dt} = 0$

$$\therefore \frac{dy}{dt} = 96 - 9.82\,t = 0$$

$$\therefore 9.82t = 96 \qquad \therefore t = 9.78 \text{ sec.}$$

To check whether we have maxima or minima differentiating (2) with respect to 't'.

$\therefore \frac{d^2y}{dt^2} = -9.82$ which is less than zero. $\therefore$ we have maximum 'y' at $t = 9.78$ seconds

maximum height $y_{max} = h = 96(9.78) - 4.91 * (9.78)^2$

$\therefore \boxed{h = 469.25 m}$

(d) Range L:

we have $y = -120$ m when the particle touches the ground.

Finding at what time the particle touched the ~~particle~~ ground:

$y = 96t - 4.91t^2 = -120\cancel{m}$

$\therefore 96t - 4.91t^2 = -120\cancel{m}$

$\therefore \; 4.91t^2 - 96t - 120m = 0$ (4)

Solving quadratically:

$$t = \frac{96 \pm \sqrt{96 + 4(4.91)(120)}}{2(4.91)}$$

$$= \frac{96 \pm 107.58}{9.82}$$

Since time is +ve (positive),

we have $t = 20.73$ seconds

$\therefore$ x at 20.73 seconds $=$ 'L'

$x = 86t = L$

$\therefore \; L = 86 * 20.73$ m

$\therefore$ $\boxed{L = 1782.9 \text{ m}}$

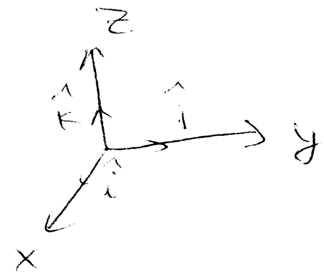
12.14. Given $x = R\cos\omega t$

$y = R\sin\omega t$

$z = \dfrac{-h}{2\pi}\,\omega t$.

To show:

velocity and acceleration have constant magnitudes



We can write:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j} - \frac{h}{2\pi}\omega t\,\hat{k}$$

$$\bar{v} = \frac{d\bar{r}}{dt} = -R\omega\sin\omega t\,\hat{i} + R\omega\cos\omega t\,\hat{j} - \frac{h\omega}{2\pi}\hat{k}$$

$$\equiv v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

~~ve~~

$$|\bar{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\therefore \cancel{|\bar{v}| = R^2\omega^2\sin^2\omega t + R^2\omega^2\cos^2}$$

$$\therefore |\bar{v}| = \sqrt{(-R\omega\sin\omega t)^2 + (R\omega\cos\omega t)^2 + \left(-\frac{h\omega}{2\pi}\right)^2}$$

$$= \left[R^2\omega^2\sin^2\omega t + R^2\omega^2\cos^2\omega t + \frac{h^2\omega^2}{4\pi^2}\right]^{1/2}$$

$$\therefore |\bar{v}| = \left[R^2\omega^2(\sin^2\omega t + \cos^2\omega t) + \frac{h^2\omega^2}{4\pi^2}\right]^{1/2}$$

$$\therefore |\bar{V}| = \left[R^2\omega^2 + \frac{h^2\omega^2}{4\pi^2}\right]^{1/2}$$ 6

= constant (no dependence on time)

Substituting values of R, ω & h

$$|\bar{V}| = \sqrt{227.39 + 225} \text{ m/s}$$

$$\therefore |\bar{V}| = 15.15 \text{ m/s}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2} = -R\omega^2\cos\omega t\,\hat{i} - R\omega^2\sin\omega t\,\hat{j}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

NOTE: Here $a_z\hat{k} = \bar{0}$

$$|\vec{a}| = \left[(-R\omega^2\cos\omega t)^2 + (-R\omega^2\sin\omega t)^2\right]^{1/2}$$

$$= \left[R^2\omega^4\cos^2\omega t + R^2\omega^4\sin^2\omega t\right]^{1/2}$$

$$= \left[R^2\omega^4(\cos^2\omega t + \sin^2\omega t)\right]^{1/2}$$

$$= (R^2\omega^4)^{1/2}$$

$$\therefore |\bar{a}| = R\omega^2$$

$$\therefore \boxed{|\bar{a}| = 189.5 \text{ m/s}^2}$$

12.16 $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \bar{r} = (3t^2 + 4t)\hat{i} + (-4t^2 + 3t)\hat{j} + (-6t + 9)\hat{k}$

(a) Find: velocity and acceleration vectors of the functions of time:

$$\bar{v}(t) = \frac{d\bar{r}(t)}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\therefore \bar{v} = \frac{d\bar{r}}{dt} = (6t + 4)\hat{i} + (-8t + 3)\hat{j} - 6\hat{k}$$

$$\bar{a}(t) = \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2} = 6\hat{i} - 8\hat{j}$$

(b) verify that the particle is undergoing plane motion: that is to show that unit vector perpendicular to the plane formed by $\bar{v}$ and $\bar{a}$ is constant.

This we can find by taking vector product of $\bar{v}$ and $\bar{a}$. By this we will get unit vector perpendicular to (8)

$\bar{v}$ and $\bar{a}$

$$\bar{v} \times \bar{a} = \{(6t+4)\hat{i} + (-8t+3)\hat{j} - 6\hat{k}\} \times (6\hat{i} - 8\hat{j})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6t+4 & -8t+3 & -6 \\ 6 & -8 & 0 \end{vmatrix}$$

$$\therefore \bar{v} \times \bar{a} = -48\hat{i} - 36\hat{j} - 50\hat{k}$$

Unit vector :

$$\hat{n} = \frac{\bar{v} \times \bar{a}}{|\bar{v} \times \bar{a}|} = \frac{-48\hat{i} - 36\hat{j} - 50\hat{k}}{\sqrt{(-48)^2 + (-36)^2 + (-50)^2}}$$

$$\therefore \hat{n} = \frac{-48\hat{i} - 36\hat{j} - 50\hat{k}}{\sqrt{6100}}$$

(no dependence on time).

it is a constant

12.28. Given $\bar{F} = -kmv^2\hat{i}$

k = constant m = mass of the ball

v = speed of the ball.

Also given

when $t=0$ $x=0$ and $\bar{v} = v_0\hat{i}$

Find

(a) Speed of ball as function of :

(a) x.

we know $\vec{F} = m\vec{a}$

$\therefore \vec{F} = -km\bar{v}^2\hat{i} = m\vec{a}\hat{i}$

$\therefore \vec{a} = -kv^2\hat{i}$

using $a = v\dfrac{dv}{dx}$

we get

$$v\frac{dv}{dx} = -kv^2$$

$$\therefore \frac{dv}{dx} = -kv \Rightarrow \frac{dv}{v} = -k\,dx$$

Integrating :

$$\int_{v_0}^{v} \frac{dv}{v} = -\int_0^x k\, x$$

$$\Rightarrow \ln v \Big|_{v_0}^{v} = -kx \Big|_0^x$$

$$\therefore \ln v - \ln v_0 = -k(x-0)$$

$$\therefore \ln \frac{v}{v_0} = -kx \Rightarrow \frac{v}{v_0} = e^{-kx}$$

$$\therefore \boxed{v = v_0 e^{-kx}}$$

(b) 't'.

We know $\frac{d\bar{v}}{dt} = \bar{a}$

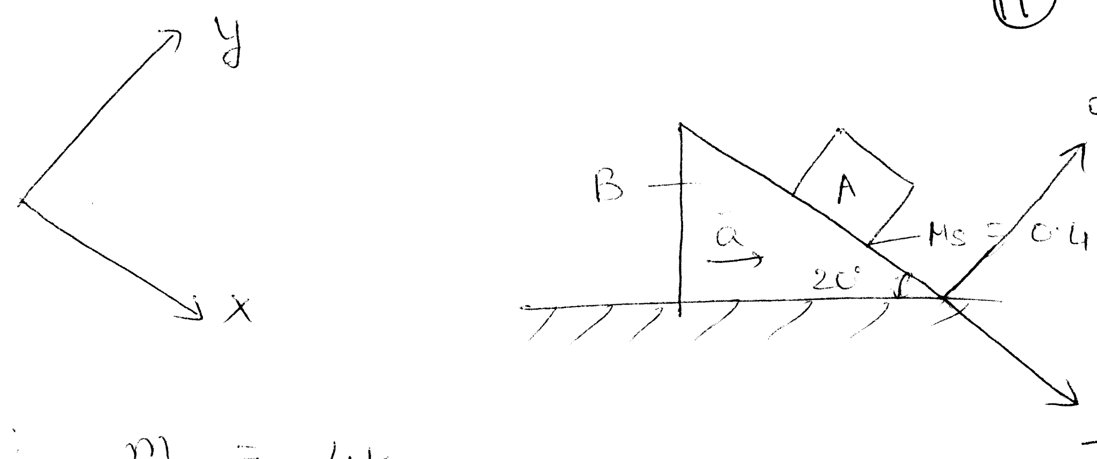
$$\therefore \frac{d\bar{v}}{dt} = -kv^2$$

$$\therefore -\int_{v_0}^{v} \frac{dv}{v^2} = +\int_0^t k\, t$$

$$\therefore \frac{1}{v} - \frac{1}{v_0} = kt \Rightarrow \frac{1}{v} = \frac{1}{v_0} + kt = \frac{1+v_0kt}{v_0}$$

$$\therefore \boxed{v = \frac{v_0}{1+v_0kt}}$$

12.34.



Given: $m_a = 4kg$

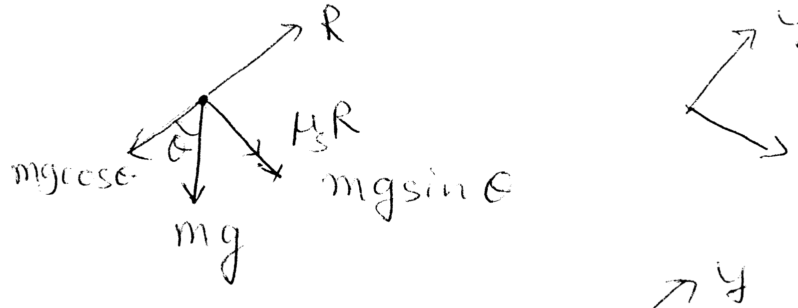
$\mu_s = 0.4$

$\theta = 20^\circ$

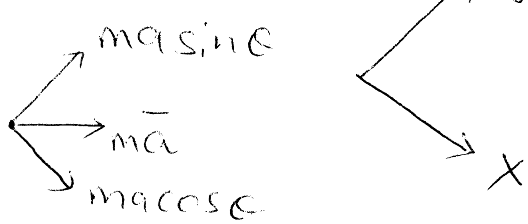
acceleration of $B = \bar{a}$

Find: smallest acceleration 'a' of B so that 'A' is at the verge of sliding up

FBD of A:



MAD of A:



$F_x = \quad mg\sin\theta + \mu R = ma\cos\theta$

$\therefore \quad mg\sin\theta + \mu R = ma\cos\theta$

$\therefore$ $mg\sin\theta - ma\cos\theta + \mu_s R = 0$ ①

$F_y = R - mg\cos\theta = ma\sin\theta$

$\therefore$ $R - mg\cos\theta = ma\sin\theta$

$\therefore$ $R = mg\cos\theta + ma\sin\theta$ ②

NOTE: 'R' is the reaction force on A from B.

Putting value of 'R' from ② into ① we get:

$$mg\sin\theta - ma\cos\theta + \mu_s(mg\cos\theta + ma\sin\theta) = 0$$

$$\therefore\ mg\sin\theta - ma\cos\theta + mg\mu_s\cos\theta + ma\mu_s\sin\theta = 0$$

$$\therefore\ ma(\cos\theta - \mu_s\sin\theta) = mg(\sin\theta + \mu_s\cos\theta)$$

$$\therefore\ \boxed{a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}$$

$$\Rightarrow\ a = \underline{\underline{8.76\ m/s^2}}$$

12.36. $\omega = 3000$ rev/min

$$x = R\left(1 + \frac{1}{2}\cos\omega t\right)$$

$R = 3$ inches

$W_A = 0.5$ lb

Find:

Smallest spring force that will maintain the contact when the follower is in its highest position

For that we need acceleration of the particle.

$$\bar{a} = \frac{d^2x}{dt^2}. \qquad x = R\left(1 + \frac{1}{2}\cos\omega t\right)$$

$$\therefore \frac{dx}{dt} = R\left(\frac{1}{2}\omega(-\sin\omega t)\right)$$

$$\frac{d^2x}{dt^2} = -\frac{R\omega^2}{2}\cos\omega t.$$

FBD. MAD

y, x

$F + 0.5$ $\downarrow$ $\uparrow$ ma

F = Force by spring

(4)

FBD = MAD.

$$\bar{F}_y = -(F_{spring})\hat{j} - 0.5\hat{j} = m\bar{a}$$

$$\therefore -F - 0.5 = \frac{0.5}{32.2} * \left[-\frac{1}{2} \frac{3}{12} \left(3000 \frac{2\pi}{60}\right)^2 \cos 0^\circ \right]$$

$$\therefore \boxed{F = 191.1 \text{ lb}}$$

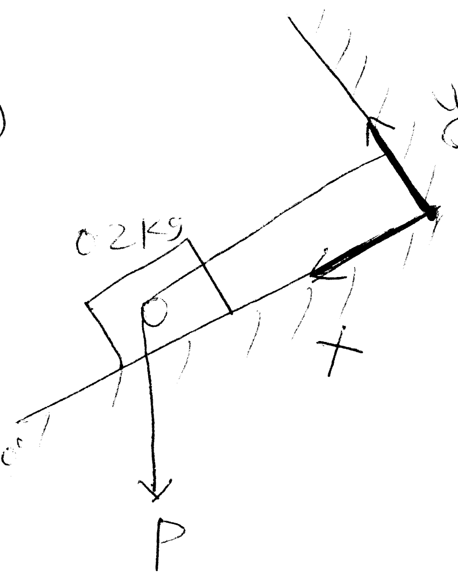
Note: 3000 rev/min was given. We need to convert to rad/sec so conversion factor of $\frac{2\pi}{60}$ was used.

12.42

$\mu_s = 0$ (No friction)

$\theta = 20^\circ$

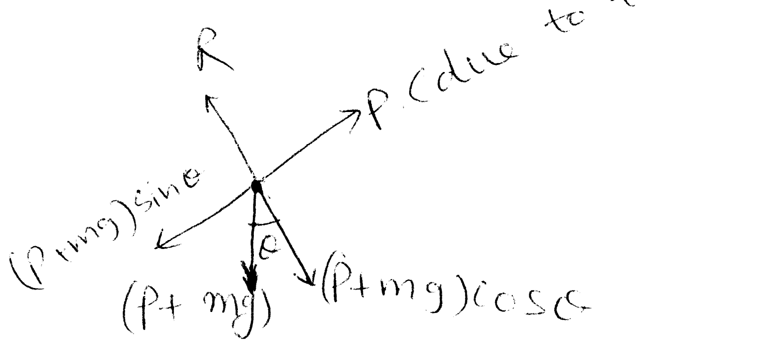
To find: P



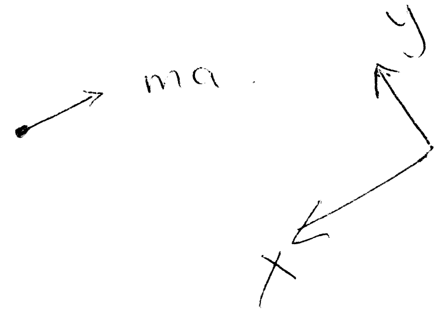
(a) block going up the inclined plane at $\bar{a} = 2 m/s^2$.

Thing to NOTE: 'P' is the tension in the rope. Since the mass of rope is assumed to be zero and no friction is present between rope an peg the tension will remain same throughout the rope.

FBD:



MAD



$$F_x: \quad -P + P\sin\theta + mg\sin\theta = -ma$$

$$\therefore \quad P - P\sin\theta = ma + mg\sin\theta$$

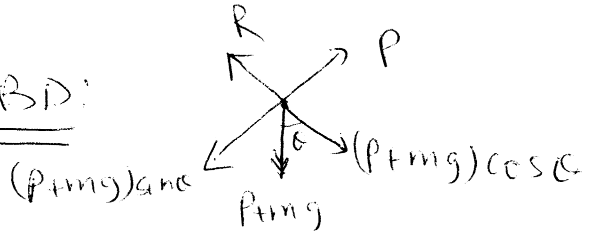
$$P(1-\sin\theta) = m(a+g\sin\theta)$$

$$\therefore \quad P = \frac{m(a+g\sin\theta)}{1-\sin\theta}$$

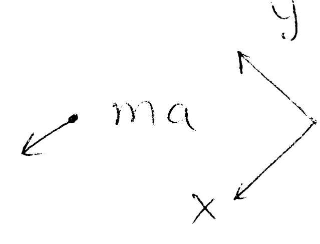
$$\therefore \quad P = 1.628\ N.$$

(b) Body doing down:

FBD:



MAD



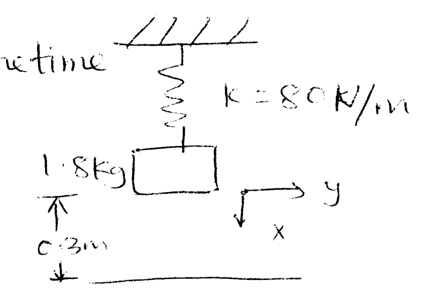
Fx: $P\sin\theta + mg\sin\theta - P = ma$

$\therefore P - P\sin\theta = mg\sin\theta - ma$

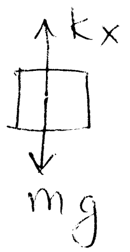
$$\therefore \boxed{P = \frac{m(g\sin\theta - a)}{1-\sin\theta}}$$

$\therefore P = 0.412\,N$

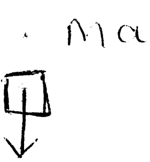
12.48. Chosing a moment sometime after the block was released and it was going up.

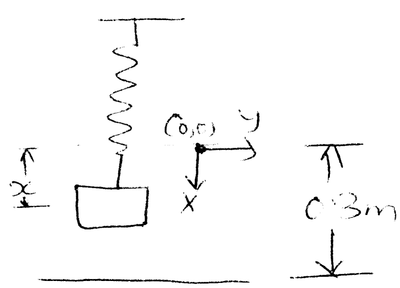


FBD:



MAD:





Fx: $mg - kx = ma$

$\therefore a = g - \frac{k}{m}x$

using $\boxed{a = \frac{v\,dv}{dx}}$

$$a = \frac{v\,dv}{dx} = g - \frac{k}{m}x$$

$$\therefore \int v\,dv = \int g\,dx - \int \frac{kx\,dx}{m}$$

$$\therefore \frac{v^2}{2} = gx - \frac{kx^2}{2m} + C$$

'C' is integration constant.

we know when $x = 0.3$ $v = 0$

$$\therefore 0 = 9.8(0.3) - \frac{80\,(0.3)^2}{2 \times 1.8} + C$$

$$\therefore \boxed{C = -0.94}.$$

$$\therefore \frac{v^2}{2} = 9.8x - 22.2x^2 - 0.94.$$

Rearranging terms and putting $v = 0$ for maximum height

$$22.2x^2 - 9.8x + 0.94 = 0$$

solving quadratically:

$$x = \frac{9.8 \pm \sqrt{9.8^2 - 4(0.94)(22.2)}}{2(22.2)}$$

$$\therefore x = \frac{9.8 \pm 3.53}{44.4} \text{ m}$$

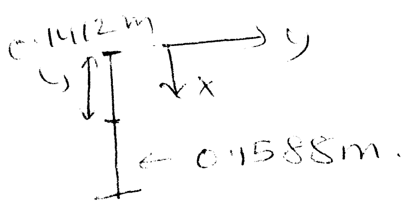
$$\therefore x = 0.3\text{ m},\ 0.1412 \text{ m}.$$

$x = 0.3$ (initial condition)

$x = 0.1412$ for maximum height.

NOTE: x is measured from original position of particle $\therefore$ height from ground

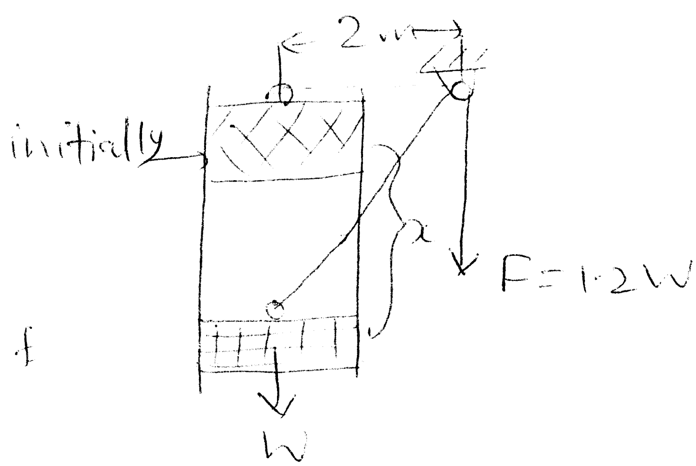
$$= (0.3 - 0.1412)\text{ m}$$
$$= 0.1588\text{ m}.$$



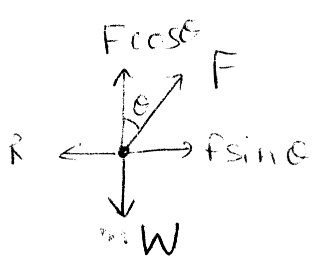
12.52

Tension constant

$F = 1.2W$.

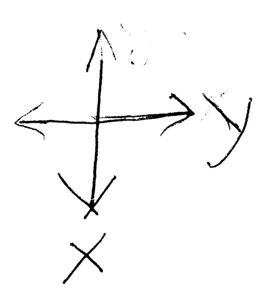


FBD:



MAD:

$\downarrow ma$



$$\vec{F}_x = -F\cos\theta\,\hat{i} + W\hat{i} = m\vec{a}.$$

$$\boxed{F = 1.2\,W}$$

$$\therefore\ W - 1.2W\cos\theta = ma = \frac{W}{g}\cdot a$$

$$\therefore\ a = g(1 - 1.2\cos\theta)$$

$$\Rightarrow \boxed{a = g\left(1 - \frac{1.2x}{\sqrt{x^2+4}}\right)}$$

(b) $$a = \frac{v\,dv}{dx} = g\left(1 - \frac{1.2x}{\sqrt{x^2+4}}\right)$$

$$\therefore \int v\,dv = g\int\left(1 - \frac{1.2x}{\sqrt{x^2+4}}\right)dx$$

$$\therefore\ \frac{v^2}{2} = \left(x - 1.2\sqrt{x^2+2}\right)g + C$$

C = integration constant.

$V=0$ $x=0$ (given)

$$\therefore\ C = 2.4g\ (= 2.4\times 9.8)$$

$$\therefore\ v^2 = 2g\left(X + 2.4 - 1.2\sqrt{x^2+4}\right)$$

NOTE: If 'v' is maximum for a given instance 'v^2' will also be maximum.

we have $v^2 = 2g(x + 2.4 - 1.2\sqrt{4+x^2})$

To find maximum of 'v' we can find maximum of 'v^2'. This will make differentiation easy.

for maximum or minimum $\frac{dv^2}{dx} = 0$

$$\frac{dv^2}{dx} = 2g\left(1 - \frac{1.2x}{\sqrt{4+x^2}}\right) = 0$$

$$\boxed{x = 3.015\,m}$$

Checking for maxima or minima:

$$\frac{d^2(v^2)}{dx^2} = -2g\left(\sqrt{4+x^2} * 1.2 + \frac{1.2\,x \cdot 2x}{(4+x^2)^{3/2}}\right)$$

Since all the terms inside the bracket greater than zero and we have a '-ve' sign outside it

$\therefore \dfrac{d^2(v^2)}{dx^2} < 0$

$\therefore$ maxima @ $x = 3.015m$

$\therefore V_{max}^2 = 2g\left(3.015 + 2.4 - 1.2\sqrt{4 + (3.015)^2}\right)$

$\therefore V_{max}^2 = 21.0681$

$\therefore \boxed{V_{max} = 4.59 \, m/s}$

12.56. $\vec{r} = x\hat{i} + y\hat{j}$

$\therefore \vec{r} = b\omega t \cos\omega t \, \hat{i} + b\omega t \sin\omega t \, \hat{j}$

$\bar{v} = \dfrac{d\bar{r}}{dt} = (b\omega \cos\omega t - b\omega^2 t \sin\omega t)\hat{i} +$

$(b\omega \sin\omega t + b\omega^2 t \cos\omega t)\hat{j}$

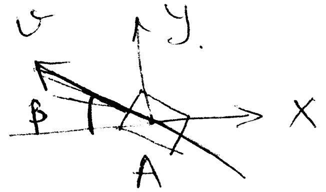
$\dfrac{d^2\bar{r}}{dt^2} = \bar{a} = (-b\omega^2 \sin\omega t - b\omega^2 \sin\omega t -$

$b\omega^3 t \cos\omega t)\hat{i} + (b\omega^2 \cos\omega t + b\omega^2 \cos\omega t -$

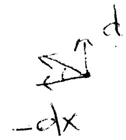
$- b\omega^3 t \sin\omega t)\hat{j}$

$$\therefore \bar{a} = (-2b\omega^2 \sin\omega t - b\omega^3 t \cos\omega t)\hat{i}$$
$$+ (2b\omega^2 \cos\omega t - b\omega^3 t \sin\omega t)\hat{j}$$

At A:



Let the collar make angle 'β' with 'x' line.



$$\tan\beta = \frac{+dy}{-dx}$$ (we want only the magnitude of 'β').

$$\therefore \tan\beta = \frac{dy}{-dx} = \frac{dy/dt}{-dx/dt} = \frac{v_y}{-v_x} = -\frac{v_y}{v_x}$$

$$\therefore \tan\beta = -\frac{v_y}{v_x} \quad \text{at } A.$$

at A: $\theta = \omega t = \pi/2$ $\therefore$ $\sin\theta = 1$, $\cos\theta = 0$

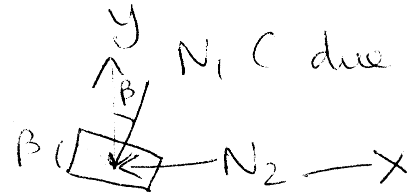
$$\therefore \bar{v} = (-b\omega^2 t)\hat{i} + (b\omega)\hat{j}$$
$$\equiv v_x\hat{i} + v_y\hat{j}$$

$$\therefore \tan\beta = -\frac{b\omega}{-b\omega^2 t} = \frac{1}{\omega t} = \frac{1}{\pi/2} = \frac{2}{\pi}.$$

$$\therefore \boxed{\beta = 32.48^\circ}$$

Forces:

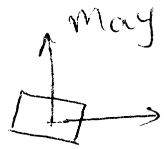
FBD:



N_1 (due to guide rod)

applied by slotted arm CC.

MAD



$$\bar{F}_x = (-N_2 - N_1 \sin\beta)\hat{i} = m\vec{a_x} \quad (1)$$

$$\bar{F}_y = -N_1 \cos\beta\,\hat{j} = m\vec{a_y} \quad (2)$$

$$\bar{a} = (-2b\omega^2 \sin\omega t - b\omega^3 t \cos\omega t)\hat{i}$$

$$+ (2b\omega^2 \cos\omega t - b\omega^3 t \sin\omega t)\hat{j}$$

{calculated earlier}

at A: $\omega t = \frac{\pi}{2}$ $\therefore$ $\sin\omega t = 1$

$\cos\omega t = 0$

$$\cancel{\bar{a} = (-2b\omega^2 - 2b\omega^3 t)\hat{i}}$$

$$\therefore \bar{a} = (-2b\omega^2)\hat{i} - (b\omega^3 t)\hat{j} \quad (3)$$

$$= a_x\hat{i} + a_y\hat{j}$$

$\therefore$ Putting a_x from 3 into 1

$$-N_2 - N_1 \sin\beta = -2m b\omega^2$$

$$\therefore 2mb\omega^2 = N_1 \sin\beta + N_2 \quad (4)$$

Putting a_y from 3 into 2

$$-N_1 \cos\beta = m \cdot (-b\omega^3 t)$$

$$\therefore N_1 = \frac{m b\omega^3 t}{\cos\beta} = \frac{m b\omega^2 \cdot \omega t}{\cos 32.48}$$

$$\therefore \boxed{N_1 = 1.862\, m b\omega^2} \quad (5)$$

Putting N_1 into 4

$$\therefore 2 m b\omega^2 = 1.862\, m b\omega^2 \sin 32.48 + N_2$$

$$\therefore \boxed{N_2 = m b\omega^2}$$