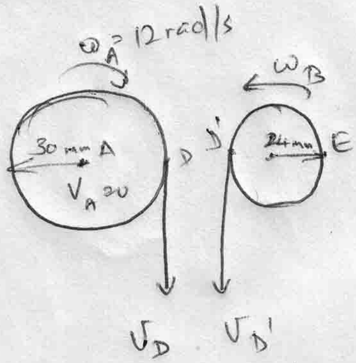
HOMEWORK 10 SOLUTIONS

16.22



$v_E = 0 \quad v_A = 0$

$\vec{\omega_A} = 12k$ rad/s $\quad \vec{\omega_B} = \omega_B k$ rad/s

$\vec{\omega}_{AB} = \omega_{AB} k$ rad/s

When there is no slip $V_D' = V_D$

$V_D = V_A + V_{D/A}$

$= 0 + (\vec{\omega_A} \times r_{D/A}) = -12k \times 30i = -360j$ mm/s — ①

$V_D' = V_D = V_E + V_{D'/E} = 0 + \vec{\omega_B} \times (-48i)$ — ②

$\therefore$ Equating $\quad -360j = \omega_B k \times (-48i)$

$\Rightarrow \omega_B = \dfrac{360}{48} = 7.50$ rad/s (ccw)

To calculate ω_{AB}, we need to relate the motion of B to the fixed point A.

V_B can expressed in terms of POINT E

$\therefore V_B = V_E + V_{B/E}$

$= 0 + (\vec{\omega_B} \times r_{B/E})$. — ③

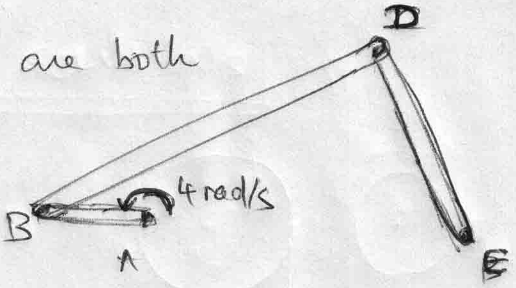
Now V_B in terms of A

$V_B = V_A + V_{B/A} = 0 + (\vec{\omega_{AB}} \times r_{B/A})$ — ④

Equating we get $-24\,\omega_B = 54\,\omega_{AB} \Rightarrow \omega_{AB} = -3.33$ rad/s (or CW)

16.26

Assuming that ω_{DE} & ω_{BD} are both counterclockwise



Relating the point B to A.

$$V_B = V_A + V_{B/A}$$
$$= 0 + (\vec{\omega}_{BA} \times r_{B/A}) \qquad (\Rightarrow V_A = 0)$$
$$= [4k \times (-6i)] = -24j \ \text{in/s}$$

Now relating point D to point B, we get

$$V_B = V_D + V_{B/D}$$
$$\Rightarrow (\omega_{AB} \times r_{B/A}) = V_D + (\vec{\omega}_{BD} \times r_{B/D})$$
$$\Rightarrow -24j = V_D + (\vec{\omega}_{BD} \times r_{B/D}) \quad \text{—— (1)}$$

Now V_D can be obtained in terms of the point E

$$\Rightarrow V_D = V_E + V_{D/E}$$
$$V_E = 0$$
$$\Rightarrow V_D = (\vec{\omega}_{DE} \times r_{D/E}) \quad \text{—— (2)}$$

Plugging in (2) in (1)

$$-24j = (\vec{\omega}_{DE} \times (-6i + 8j)) + [\vec{\omega}_{BD} \times (-15i - 8j)]$$
$$= (-6\omega_{DE}\hat{j} - 8\omega_{DE}\hat{i}) + (-15\omega_{BD}\hat{j} + 8\omega_{BD}\hat{i})$$

Equating $\hat{i}$ & $\hat{j}$ components & solving gives $\omega_{BD} = \omega_{DE} = 1.143$ rad/s

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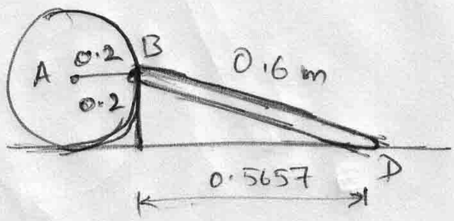
Given that V_A is to the left

∴ Assume V_D is directed to the left too.



A is point whose motion is known.

Hence relate B to A.

$$V_B = V_A + V_{B/A}$$

$$= -0.6\hat{i} + (\vec{\omega}_{AB} \times r_{B/A})$$

Now ω_{AB} can be obtained from dynamics on curvilinear coordinates.

Point B has ω_{AB} such that

$$\omega_{AB} = \frac{V_A}{R} = \frac{0.6}{0.2} = 3 \text{ rad/s}$$

Now $V_B = -0.6i + (3\hat{k} \times 0.2\hat{i})$

Now relating D to B

$$V_D = V_B + V_{D/B}$$

$$-V_D\hat{i} = -0.6i + (0.6\hat{j}) + (\vec{\omega}_{BD} \times r_{D/B})$$

$$-V_D\hat{i} = -0.6i + 0.6j + 0.5657\omega_{BD}\, j + 0.2\omega_{BD}\, i$$

Equating $\hat{i}$ & $\hat{j}$ components and solving gives

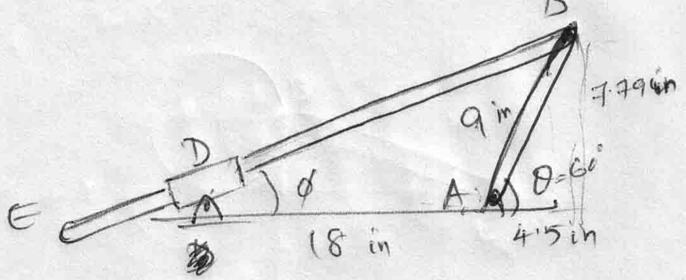
$$\omega_{BD} = -1.0607 \text{ rad/s}$$

$$V_D = 0.812 \text{ m/s.}$$

16.32

Using trigonometry we get

$\theta = 60° \Rightarrow \phi = 19.11°$



D is a point on the rod at the collar. Hence D is not a fixed point.

Relating point B to A.

$$V_B = V_A + V_{B/A}$$
$$= 0 + (\omega_{AB} \times r_{B/A}) \qquad (V_A = 0)$$
$$= 16k \times (4.5\hat{i} + 7.794\hat{j})$$

Now relating the point D to E.

[We relate D, as we do not have lengths to associate E and BD has the same motion as BE]

$$V_D = V_B + V_{D/B}$$

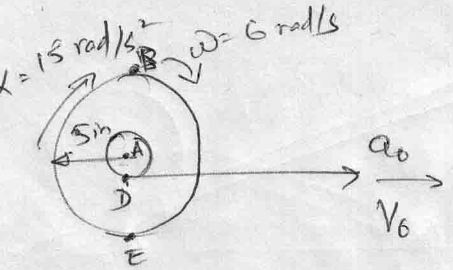
Point D on the rod moves through the collar and hence has a velocity V_D directed along the rod.

$$\therefore V_D\left[-\cos 19.11\,\hat{i} - \sin 19.11\,\hat{j}\right] = 16k \times (4.5i + 7.794j) + (\omega_{BD} \times r_{D/B})$$
$$= 16k \times (4.5i + 7.794j) + (\omega_{BD}\hat{k} \times (-22.5i - 7.794j))$$

Equating $\hat{i}$ & $\hat{j}$ components & solving.

$$\vec{\omega}_{BD} = \vec{\omega}_{BE} = \underline{4.57 \text{ rad/s}}$$

16.56



As there is no slip at the point E, we can calculate a_A from α as $a_A = R\alpha = 75$ in/s^2.

(a) Relating point D to point A.

$$a_D = a_A + a_{D/A}$$

$$= 75\hat{i} + (\alpha_{AD} \times r_{D/A}) + \omega_{AD} \times (\omega_{AD} \times r_{D/A})$$

$$= 75\hat{i} + (-15k \times -2j) + (-6k) \times [(-6k) \times (-2j)]$$

$$a_D = 45i + 72j \text{ in/s}^2$$

(b) Relating point B to point A

$$a_B = a_A + a_{B/A}$$

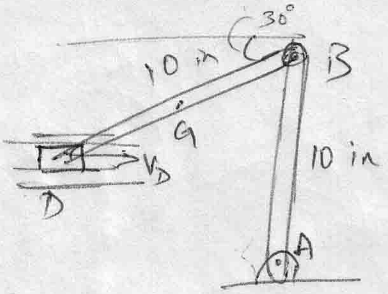
$$= 75i + (\alpha_{AB} \times r_{B/A}) + \{\omega_{BA} \times [(\omega_{AB} \times r_{B/A})]$$

$$= 75\hat{i} + (-15k) \times 5j + (-6k) \times [-6k \times 5j]$$

$$a_B = 150i - 180j \text{ in/s}^2$$

(c) The acceleration at the end of the string equals the horizontal component of acceleration at D $= 45i$ in/s^2.

16.62



Starting from point A we can relate points in order.

Point B

$$V_B = V_A + V_{B/A}$$

At the point ~~shown~~ V_D & V_B are parallel ~~equal~~ as they are both moving horizontally.

$$V_B = 0 + (\omega_{AB} \times r_{B/A}) \qquad (V_A = 0)$$

$$\Rightarrow V_B = (\omega_{AB} \times r_{B/A}) \quad \text{— (1)}$$

Relating D to B

$$V_D = V_B + V_{D/B}$$

$$V_D = V_B + (\omega_{DB} \times r_{D/B})$$

As V_D and V_B are parallel, the rod is moving translationally only. ∴ $\omega_{BD} = 0$

$$\Rightarrow \underline{V_D = V_B} = 2.4 \text{ ft/s} \quad \text{— (2)}$$

Using (2) in (1) we get

$$\omega_{AB} \times (10j) = 2.4i \quad \Rightarrow \underline{\omega_{AB} = -2.88\hat{k} \text{ rad/s}}$$

Proceeding to acceleration.

$$a_B = a_A + a_{B/A}$$

$$\Rightarrow a_B = 0 + \alpha_{AB} \times r_{B/A} + \omega_{AB} \times (\omega_{AB} \times r_{B/A})$$

$$= \alpha_{AB}\hat{k} \times 10\hat{j} + (-2.88k) \times [(-2.88k) \times 10j]$$

$$= -10\alpha_{AB}\,\hat{i} + (-2.88k) \times (28.8i) \quad — (3)$$

Relating D to B

$$a_D = a_B + a_{D/B}$$

$$= a_B + (\alpha_{BB} \times r_{D/A}) + \cancel{\omega_{BD}}^{0} \times (\cancel{\omega_{BD}}^{0} \times r_{D/B})$$

$$= a_B + [\alpha_{DB} \times (10\cos 30^\circ\,\hat{i} + 10\sin 30\,\hat{j})] \quad — (4)$$

Using a_B from (3) in (4), and using

$a_D = 0$ as it moves with constant velocity

we get an expression.

Equating i & j components gives

$$\alpha_{BD} = -9.577k \text{ rad/s}^2$$

For point G $\quad a_G = a_D + a_{G/D}$

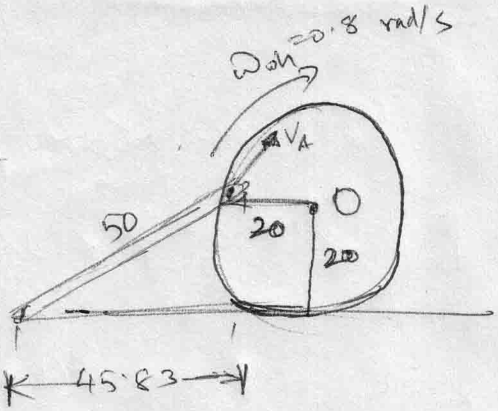
$$= a_D + (\vec{\alpha}_{BD} \times r_{G/D})$$

$$= 0 + (-9.577\hat{k}) \times (5\cos 30^\circ\,\hat{i} + 5\sin 30\,\hat{j})$$

$$= 23.94\hat{i} - 41.47\hat{j} \text{ m/s}^2$$

a_G = magnitude of $\vec{a}_G$ = 47.9 m/s^2.

16.66



Using the velocity relations we can relate

$$V_A = V_C + V_{A/C}$$

$V_C = 0$ (no slipping)

$$\Rightarrow V_A = (\omega_{AC} \times r_{AC}) = 22.63 \text{ in/s}$$

Using instant centre method

$$\omega_{AB} = \frac{V_A}{\overline{EA}} = 0.3492 \text{ rad/s}$$ where E is the instant center

Now $a_A = a_O + a_{A/O}$

$$= 0 + \vec{\omega}_{wh} \times (\vec{\omega}_{wh} \times r_{A/O}) \quad \text{— (1)}$$

[$\alpha_{AO} = 0$ - constant angular velocity]

$$a_B = a_A + a_{B/A}$$

$$= \left[\vec{\omega}_{wh} \times (\vec{\omega}_{wh} \times r_{A/O})\right] + (\alpha_{AB} \times r_{B/A})$$
$$+ \left[\vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times r_{B/A})\right]$$

$$a_B i = (-0.8k) \times (-0.8k \times -20i)$$
$$+ \alpha_{AB} k \times (-45.83\hat{i} - 20j) + 0.3492k \times (0.3492k \times (45.83i - 20j))$$

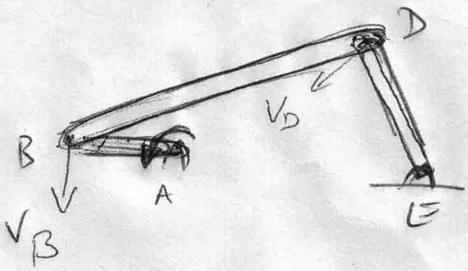
$$a_B i = 12.80\hat{i} - 45.83\alpha_{AB}\hat{j} + 20\alpha_{AB}\hat{i} + 5.587i + 2.439j$$

Equating $\hat{i}$ & $\hat{j}$ components we can solve for a_B & α_{AB}

Hence $a_B = 19.45\hat{i}$ in/s^2

16.68

Using velocity analysis



$V_B = V_A + (\omega_{AB} \times r_{AB})$

$V_A = 0$

$\therefore V_B = (4\hat{k} \times -6i) = -24\hat{j}$ in/s²

Using instant center method (which is point E) for both BD & DE

$\omega_{BD} = \omega_{DE} = 1.143$ rad/s (CCW)

Now relating accelerations

$a_B = a_A + a_{B/A}$

$a_B = 0 + (\alpha_{AB} \times r_{B/A}) + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times r_{B/A})$

$\alpha_{AB} = 0$ — moves with constant angular velocity

$\therefore a_B = \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times r_{B/A})$ — (1)

Now relating point D to E

$a_D = a_E + a_{D/E}$ $(a_E = 0)$

$= 0 + (\alpha_{DE} \times r_{D/E}) + \vec{\omega}_{DE} \times (\vec{\omega}_{DE} \times r_{D/E})$ — (2)

Now relating B & D

$a_B = a_D + a_{B/D}$

$a_B = a_D + (\alpha_{BD} \times r_{B/D}) + \vec{\omega}_{BD} \times (\vec{\omega}_{BD} \times r_{B/D})$ — (3)

Plugging in ① & ② into ③ and applying numerical values.

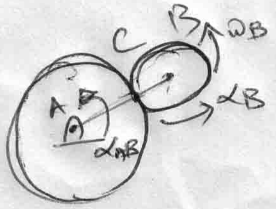
$$0 + 4k \times (4k \times (-6i)) = \alpha_{DE} k \times (-6i + 8j) + \cancel{1.143k \times (1.143}$$
$$+ 1.143k \times [1.143k \times (-6i + 8j)]$$
$$+ \alpha_{BD} k \times (-15i + 8j)$$
$$+ 1.143k \times [1.143k \times (-15i - 8j)]$$

Equating $\hat{i}$ & $\hat{j}$ components we can solve for the two unknowns α_{BD} & α_{DE}

$\vec{\alpha}_{BD} = 2.45k \text{ rad/s}^2$ $\quad\quad$ $\vec{\alpha}_{DE} = -6.12\hat{k} \text{ rad/s}^2$

<u>16.72</u>

$V_C = 0$ as point of contact doesn't slip.



$\therefore V_B = V_C + V_{B/C}$

$= V_{B/C} = (\omega_B \times r_B)$ — ①

Relating A to B

$V_B = V_A + V_{B/A}$

$= 0 + \omega_{AB} \times (r_{B/A})$

$= \omega_{AB} \times (\overrightarrow{r_A + r_B})$ — ②

From ① & ②

$(r_A + r_B)\omega_{AB} = r_B \cdot \omega_B$

$\Rightarrow \omega_{AB} = \dfrac{(r_A + r_B)\omega_{AB}}{r_B} = \left(\dfrac{60+25}{25}\right)(5) = 17.0 \text{ rad/s}$

Time derivative of the above provides

$$\alpha_B = \frac{r_A + r_B}{r_B} \alpha_{AB} = \frac{60+25}{25}(12.5) = 42.5 \text{ rad/s}^2$$

ccw.

Relating accelerations

$a_B = a_A + a_{B/A}$

$= 0 + (\alpha_{AB} \times r_{B/A}) + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times r_{B/A})$ — (3)

Relating C to B

$a_C = a_B + a_{C/B}$

$= a_B + (\alpha_{BC} \times r_{C/B}) + \omega_{BC} \times (\vec{\omega}_{BC} \times r_{C/B})$

Using (3) & plugging in values

$a_C = (12.5k \times 85i) + 5k \times (5k \times 85i)$

$+ 42.5k \times (-25i) + 17k \times [17k \times (-25i)]$

$\therefore a_C = 5.10 \, i \text{ m/s}^2$

16.74 Relating velocities

$V_B = V_A + V_{B/A}$

$= 0 + (\omega_{AB} \times r_{B/A}) = 0.96 \text{ m/s}$

Using "instant center" method $\vec{\omega}_{BD} = 1.3577 \text{ rad/s}$

$V_D = 0.67885 \text{ m/s}$ $\quad \omega_{DE} = V_D / \overline{DE} = 1.131 \text{ rad/s (cw)}$

Relating acceleration

$$a_B = a_A + a_{B/A}$$
$$= 0 + (\alpha_{AB} \times r_{B/A}) + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times r_{B/A}) \quad -①$$

$$a_D = a_E + a_{D/E}$$
$$= 0 + (\vec{\alpha}_{DE} \times r_{D/E}) + \vec{\omega}_{DE} \times (\vec{\omega}_{DE} \times r_{D/E}) \quad -②$$

Now relating B & D

$$a_D = a_B + a_{D/B} \quad -③$$
$$= a_B + (\alpha_{DB} \times r_{D/B}) + \vec{\omega}_{BD} \times (\vec{\omega}_{BD} \times r_{D/B})$$

Using ① and ② ~~&~~ in ③ and plugging in values

$$\alpha_{DE} k \times 0.6j + (-1.131k) \times [(-1.131k) \times 0.6j)]$$
$$= 0 + (-1.2k) \times [(-1.2k) \times (0.5657\hat{i} + 0.5657j)]$$
$$+ \alpha_{BD} k \times 0.5\hat{i} + 1.358k \times (1.358k \times 0.5\hat{i})$$

Equating $\hat{i}$ components we can solve for

α_{DE}

$$\alpha_{DE} = 2.894 \text{ rad/s}^2 \quad (\text{ccw})$$

$$\therefore a_D = -0.6(2.894)\hat{i} - 0.7675j$$
$$= -1.736\hat{i} - 0.7675\hat{j} \text{ m/s}^2$$