CHAPTER 16

Answer to Checkpoint Questions

1. (a) $-x_{\rm m}$; (b) $+x_{\rm m}$; (c) 0

2. a

3. (a) 5 J; (b) 2 J; (c) 5 J

4. all tie (in Eq. 16-32, m is included in I)

5. 1, 2, 3 (the ratio m/b matters; k does not)

Answer to Questions

1. c

2. (a) 2; (b) positive; (c) between 0 and $+x_{\rm m}$

3. (a) 0; (b) between 0 and $+x_{\rm m}$; (c) between $-x_{\rm m}$ and 0; (d) between $-x_{\rm m}$ and 0

4. a and b

5. (a) toward $-x_{\rm m}$; (b) toward $+x_{\rm m}$; (c) between $-x_{\rm m}$ and 0; (d) between $-x_{\rm m}$ and 0; (e) decreasing; (f) increasing

6. (a) π rad; (b) π rad; (c) $\pi/2$ rad

7. (a) 3, 2, 1; (b) all tie

8. all tie

9. 3, 2, 1

10. half spring

11. system with spring A

12. (a) same; (b) same; (c) increase; (d) increase; (e) increase

13. b (infinite period; does not oscillate), c, a

14. (a) same; (b) same; (c) smaller; (d) smaller; (e) and (f) larger ($T = \infty$)

15. (a) same; (b) same; (c) same; (d) smaller; (e) smaller; (f) and (g) larger ($T = \infty$)

16. one system: $k = 1500\,\mathrm{N/m}$, $m = 500\,\mathrm{kg}$ other system: $k = 1200\,\mathrm{N/m}$, $m = 400\,\mathrm{kg}$ the same ratio $k/m = 3$ gives resonance for both systems

Solutions to Exercises & Problems

1E

(a) The period is $T = 2(0.25\,\mathrm{s}) = 0.50\,\mathrm{s}$.
(b) The frequency is $f = 1/T = 1/0.50\,\mathrm{s} = 2.0\,\mathrm{Hz}$.
(c) The amplitude is $A = 36\,\mathrm{cm}/2 = 18\,\mathrm{cm}$.

2E

(a) By definition, $T = 0.75\,\mathrm{s}$.
(b) $f = 1/T = 1/0.75\,\mathrm{s} = 1.3\,\mathrm{Hz}$.
(c) $\omega = 2\pi f = 2\pi(1.3\,\mathrm{Hz}) = 8.4\,\mathrm{rad/s}$.

3E

(a) The spring constant is

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(4.00\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{0.160\,\mathrm{m}} = 245\,\mathrm{N/m}\,.$$

(b) The period is

$$T = 2\pi\sqrt{\frac{m'}{k}} = 2\pi\sqrt{\frac{0.500\,\mathrm{kg}}{245\,\mathrm{N/m}}} = 0.284\,\mathrm{s}\,.$$

4E

(a) The motion repeats every $0.500\,\mathrm{s}$ so the period must be $T = 0.500\,\mathrm{s}$.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500\,\mathrm{s}) = 2.00\,\mathrm{Hz}$.

(c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00\,\mathrm{Hz}) = 12.6\,\mathrm{rad/s}$.

(d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. Solve for k: $k = m\omega^2 = (0.500\,\mathrm{kg})(12.6\,\mathrm{rad/s})^2 = 79.0\,\mathrm{N/m}$.

(e) Let x_m be the amplitude. The maximum speed is $v_\mathrm{m} = \omega x_\mathrm{m} = (12.57\,\mathrm{rad/s})(0.350\,\mathrm{m}) = 4.40\,\mathrm{m/s}$.

(*f*) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0\,\mathrm{N/m})(0.350\,\mathrm{m}) = 27.6\,\mathrm{N}$.

5E

The effective spring constant k_{eff} satisfies

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\mathrm{eff}}}{m}}\,.$$

Solve for k_{eff}:

$$k_{\mathrm{eff}} = 4\pi^2 fm = 4\pi^2(10^{13}\,\mathrm{Hz})\left(\frac{0.108\,\mathrm{kg}}{6.02\times10^{23}}\right) = 708\,\mathrm{N/m}\,.$$

6E

The maximum acceleration is

$$a_{\max} = \omega^2 A = 4\pi^2 f^2 A = 4\pi^2(6.60\,\mathrm{Hz})^2(2.20\times10^{-2}\,\mathrm{m}) = 37.8\,\mathrm{m/s^2}\,.$$

7E

The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude. The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g/x_m}$ and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\mathrm{m}/s^2}{1.0\times10^{-6}\,\mathrm{m}}} = 500\,\mathrm{Hz}\,.$$

For frequencies greater than 500 Hz the acceleration exceeds g for some part of the motion.

8E

(*a*) The spring constant is $k = \Delta F/\Delta k = 32.0\,\mathrm{lb}/4.00\,\mathrm{in.} = 8.00\,\mathrm{lb/in.} = 96.0\,\mathrm{lb/ft}$.
(*b*) Use $f = (1/2\pi)\sqrt{k/m}$ and solve for the weight W:

$$W = mg = \frac{kg}{4\pi^2 f^2} = \frac{(8.00\,\mathrm{lb/in.})(32.2\,\mathrm{ft/s^2})(12\,\mathrm{in./ft})}{4\pi^2(2.00\,\mathrm{Hz})^2} = 19.6\,\mathrm{lb}\,.$$

9E

(*a*) The spring constant is

$$k = \frac{mg}{x} = \frac{20\,\mathrm{N}}{0.20\,\mathrm{m}} = 100\,\mathrm{N/m}\,.$$

(b) The period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5.0\,\mathrm{N}}{(100\,\mathrm{N/m})(9.80\,\mathrm{m/s^2})}} = 0.45\,\mathrm{s}\,.$$

10E

(a) The spring constant k satisfies $T = 2\pi\sqrt{m/k}$, which gives

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(50.0\,\mathrm{g})(10^{-3}\,\mathrm{kg/g})}{(0.500\,\mathrm{s})^2} = 7.90\,\mathrm{N/m}\,.$$

(b) Use $v_\mathrm{m} = \omega A = (2\pi/T)A$ to solve for A:

$$A = \frac{v_\mathrm{m}T}{2\pi} = \frac{(15.0\,\mathrm{cm/s})(0.500\,\mathrm{s})}{2\pi} = 1.19\,\mathrm{cm}\,.$$

(c) $f = 1/T = 1/0.500\,\mathrm{s} = 2.00\,\mathrm{Hz}$.

11E

(a) The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period. The relationship $f = 1/T$ was used to obtain the last form. Thus $\omega = 2\pi/(1.00 \times 10^{-5}\,\mathrm{s}) = 6.28 \times 10^5\,\mathrm{rad/s}$.

(b) The maximum speed v_m and maximum displacement x_m are related by $v_\mathrm{m} = \omega x_\mathrm{m}$, so

$$x_\mathrm{m} = \frac{v_\mathrm{m}}{\omega} = \frac{1.00 \times 10^3\,\mathrm{m/s}}{6.28 \times 10^5\,\mathrm{rad/s}} = 1.59 \times 10^{-3}\,\mathrm{m}\,.$$

12E

(a) and (b) The spring constant k satisfies $T = 2\pi\sqrt{m/k}$, or

$$k = m\left(\frac{2\pi}{T}\right)^2 = (0.12\,\mathrm{kg})\left(\frac{2\pi}{0.20\,\mathrm{s}}\right)^2 = 1.2 \times 10^2\,\mathrm{N/m}\,.$$

The maximum force is then $F_\mathrm{m} = kA = (1.2 \times 10^2\,\mathrm{N/m})(0.085\,\mathrm{m}) = 10\,\mathrm{N}$.

13E

(a) The amplitude is half the range of the displacement, or $x_\mathrm{m} = 1.0\,\mathrm{mm}$.

(b) The maximum speed v_m is related to the amplitude x_m by $v_\mathrm{m} = \omega x_\mathrm{m}$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency, $v_\mathrm{m} = 2\pi f x_\mathrm{m} = 2\pi(120\,\mathrm{Hz})(1.0 \times 10^{-3}\,\mathrm{m}) = 0.75\,\mathrm{m/s}$.

(c) The maximum acceleration is $a_{\mathrm{m}} = \omega^2 x_{\mathrm{m}} = (2\pi f)^2 x_{\mathrm{m}} = [2\pi(120\,\mathrm{Hz})]^2(1.0\times 10^{-3}\,\mathrm{m}) = 570\,\mathrm{m/s^2}$.

14E

(a) The angular frequency is $\omega = 2\pi f = 2\pi(440\,\mathrm{Hz}) = 2.8\times 10^3\,\mathrm{rad/s}$.
(b) The maximum speed is $v_{\mathrm{m}} = \omega A = (2.8\times 10^3\,\mathrm{rad/s})(0.75\times 10^{-3}\,\mathrm{m}) = 2.1\,\mathrm{m/s}$.
(c) The maximum acceleration is $a_{\mathrm{m}} = \omega^2 A = (2.76\times 10^3\,\mathrm{rad/s})^2(0.75\times 10^{-3}\,\mathrm{m}) = 5.7\times 10^3\,\mathrm{m/s^2}$.

15E

(a) The effcetive spring constant for the whole car of mass m is $K_{\mathrm{eff}} = 4k$, where k is the spring constant of each spring. Solve $f = (2\pi)^{-1}\sqrt{k_{\mathrm{eff}}/m}$ for k:

$$k = \frac{1}{4}k_{\mathrm{eff}} = \pi^2 f^2 m = \pi^2(3.00\,\mathrm{Hz})^2(1450\,\mathrm{kg}) = 1.29\times 10^5\,\mathrm{N/m}\,.$$

(b) The vibraton frequency will be

$$f = \frac{1}{2\pi}\sqrt{\frac{4(1.29\times 10^5\,\mathrm{N/m})}{1450\,\mathrm{kg} + 5(73.0\,\mathrm{kg})}} = 2.68\,\mathrm{Hz}\,.$$

16E

(a) The displacement at $t = 2.0\,\mathrm{s}$ is

$$x|_{t=2.0\,\mathrm{s}} = (6.0\,\mathrm{m})\cos\left[(3\pi\,\mathrm{rad/s})(2.0\,\mathrm{s}) + \frac{\pi}{3}\,\mathrm{rad}\right] = 3.0\,\mathrm{m}\,.$$

(b) The velocity at $t = 2.0\,\mathrm{s}$ is

$$v|_{t=2.0\,\mathrm{s}} = -(6.0\,\mathrm{m})(3\pi\,\mathrm{rad/s})\sin\left[(3\pi\,\mathrm{rad/s})(2.0\,\mathrm{s}) + \frac{\pi}{3}\,\mathrm{rad}\right] = -49\,\mathrm{m/s}\,.$$

(c) The acceleration at $t = 2.0\,\mathrm{s}$ is

$$a|_{t=2.0\,\mathrm{s}} = -\omega^2 x|_{t=2.0\,\mathrm{s}} = -(3\pi\,\mathrm{rad/s})^2(3.0\,\mathrm{m}) = -2.7\times 10^2\,\mathrm{m/s^2}.$$

(d) The phase of the motion at $t = 2.0\,\mathrm{s}$ is

$$\phi|_{t=2.0\,\mathrm{s}} = (3\pi\,\mathrm{rad/s})(2.0\,\mathrm{s}) + \frac{\pi}{3}\,\mathrm{rad} = \frac{19\pi}{3} = 20\,\mathrm{rad}\,.$$

(e) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{3\pi\,\mathrm{rad/s}}{2\pi} = 1.5\,\mathrm{Hz}\,.$$

(*f*) The period is $T = 1/f = 1/1.5\,\text{Hz} = 0.67\,\text{s}$.

17E

(*a*) The period is $T = 1/f = 1/0.25\,\text{Hz} = 4.0\,\text{s}$.
(*b*) The angular frequency is $\omega = 2\pi f = 2\pi(0.25\,\text{Hz}) = 1.57\,\text{rad/s}$.
(*c*) The amplitude is $A = 0.37\,\text{cm}$.
(*d*) The displacement at time t is

$$x(t) = A\cos(\omega t) = (0.37\,\text{cm})\cos\left(\frac{\pi t}{2}\right).$$

(*e*) The velocity at time t is

$$\begin{aligned} v(t) = \frac{dx(t)}{dt} &= -\omega A \sin\omega t \\ &= -(1.57\,\text{rad/s})(0.37\,\text{cm})\sin\left(\frac{\pi t}{2}\right) = (-0.58\,\text{cm/s})\sin\left(\frac{\pi t}{2}\right). \end{aligned}$$

(*f*) The maximum speed is $v_\text{m} = \omega A = (1.57\,\text{rad/s})(0.37\,\text{cm}) = 0.58\,\text{cm/s}$.
(*g*) The maximum acceleration is $a_\text{m} = \omega^2 A = (1.57\,\text{rad/s})^2(0.37\,\text{cm}) = 0.91\,\text{cm/s}^2$.
(*h*) The displacement at $t = 3.0\,\text{s}$ is

$$x|_{t=3.0\,\text{s}} = (0.37\,\text{cm})\cos\left[\frac{\pi}{2}(3.0)\right] = 0.$$

(*i*) The speed at $t = 3.0\,\text{s}$ is $v|_{t=3.0\,\text{s}} = v_\text{m} = 0.58\,\text{cm/s}$ (since $x|_{t=3.0\,\text{s}} = 0$).

18E

Use $v_\text{m} = \omega x_\text{m} = 2\pi f x_\text{m}$. The frequency is $180/(60\,\text{s}) = 3.0\,\text{Hz}$ and the amplitude is half the stroke, or $0.38\,\text{m}$. Thus $v_\text{m} = 2\pi(3.0\,\text{Hz})(0.38\,\text{m}) = 7.2\,\text{m/s}$.

19P

(*a*) From $T = 2\pi\sqrt{(m+M)/R}$, we solve for M to get $M = (k/4\pi^2)T^2 - m$.
(*b*) Since $M = 0$,

$$m = \frac{kT^2}{4\pi^2} = \frac{(605.6\,\text{N/m})(0.90149\,\text{s})}{4\pi^2} = 12.47\,\text{kg}.$$

(*c*) The mass of the astronaut is

$$M = (k/4\pi^2)T^2 - m = \frac{(605.6\,\text{N/m})(2.08832\,\text{s})}{4\pi^2} - 12.47\,\text{kg} = 54.43\,\text{kg}.$$

20P

(a) The spring constant is

$$k = \frac{\Delta F}{\Delta x} = \frac{(300 \times 10^{-3}\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{2.00 \times 10^{-2}\,\mathrm{m}} = 147\,\mathrm{N/m}\,.$$

(b) The period of motion is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2.00\,\mathrm{kg}}{147\,\mathrm{N/m}}} = 0.733\,\mathrm{s}\,.$$

21P

Since $T \propto \sqrt{M}$, we get

$$\sqrt{\frac{m}{m + 2.0\,\mathrm{kg}}} = \frac{2.0\,\mathrm{s}}{3.0\,\mathrm{s}}\,.$$

Solve for m to obtain $m = 1.6\,\mathrm{kg}$.

22P

(a) The maximum acceleration is

$$a_\mathrm{m} = \omega^2 A = (2\pi f)^2 A = 4\pi^2 (1000\,\mathrm{Hz})^2 (0.40 \times 10^{-3}\,\mathrm{m}) = 1.6 \times 10^4\,\mathrm{m/s^2}\,.$$

(b) The maximum speed is

$$v_\mathrm{m} = \omega A = 2\pi f A = 2\pi(1000\,\mathrm{Hz})(0.40 \times 10^{-3}\,\mathrm{m}) = 2.5\,\mathrm{m/s}\,.$$

(c) When the displacement of the prong is $x = 0.20\,\mathrm{mm}$, its acceleration is

$$a|_{x=0.20\,\mathrm{mm}} = -\omega^2 x = -4\pi^2 (1000\,\mathrm{Hz})(0.20 \times 10^{-3}\,\mathrm{m}) = 7.9 \times 10^3\,\mathrm{m/s}\,.$$

(d) The speed of the prong at this moment is

$$v|_{x=0.20\,\mathrm{mm}} = v_\mathrm{m}\sqrt{1 - \frac{x^2}{A^2}} = (2.5\,\mathrm{m/s})\sqrt{1 - \left(\frac{0.20\,\mathrm{mm}}{0.40\,\mathrm{mm}}\right)} = 2.2\,\mathrm{m/s}\,.$$

23P

(a)$f = (10\,\mathrm{rad/s})/(2\pi) = 1.6\,\mathrm{Hz}$.
(b) $v_\mathrm{m} = \omega A = (10\,\mathrm{rad/s})(10\,\mathrm{cm}) = 100\,\mathrm{cm/s}$.
(c) $a_\mathrm{m} = \omega^2 A = (10\,\mathrm{rad/s})^2 (10\,\mathrm{cm}) = 1.0 \times 10^3\,\mathrm{cm/s^2}$. It occurs at $x = \pm A = \pm 10\,\mathrm{cm}$.

(d) The force applied as a function of time is given by

$$F = ma = -m\omega^2 x = -(0.10\,\mathrm{kg})(10\,\mathrm{rad/s})^2 x = (-10\,\mathrm{N/m})x\,.$$

24P

The displacement x of the ocean surface from its equilibrium position can be written as $x(t) = \frac{1}{2}d\cos(2\pi t/T)$. As the wave falls a distance $d/4$ from its maximum height of $d/2$, the displacement of the wave is $x = d/2 - d/4 = d/4$. The corresponding time t satisfies $d/4 = (d/2)\cos(2\pi t/T)$. Thus

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{1}{2}\right) = \left(\frac{12.5\,\mathrm{h}}{2\pi}\right)\cos^{-1}\left(\frac{1}{2}\right) = 2.08\,\mathrm{h}\,.$$

25P

Since the maximum horizontal force exerted on the block of mass m is $f_{s,\max} = \mu mg$, the block is only capable of reaching a maximum acceleration $a_\mathrm{m} = f_{s,\max}/m = \mu g$. Since m is in SHM, its actual maximum acceleration is $a'_\mathrm{m} = \omega^2 A = kA/(m+M)$. Thus the condition for m and M to be in contact is

$$a'_\mathrm{m} = \frac{kA}{m+M} \leq a_\mathrm{m} = \mu g\,,$$

or

$$A \leq \frac{\mu g(m+M)}{k} = \frac{(0.40)(9.80\,\mathrm{m/s^2})(1.0\,\mathrm{kg} + 10\,\mathrm{kg})}{200\,\mathrm{N/m}} = 0.22\,\mathrm{m}\,.$$

26P

The maximum force that can be exerted by the surface must be less than $\mu_s N$ or else the block will not follow the surface in its motion. Here μ_s is the coefficient of static friction and N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically you know that $N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion the magnitude of the maximum force exerted on it is given by $F = ma_\mathrm{m} = m\omega^2 x_\mathrm{m} = m(2\pi f)^2 x_\mathrm{m}$, where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form.

Substitute $F = m(2\pi f)^2 x_\mathrm{m}$ and $N = mg$ into $F < \mu_s N$ to obtain $m(2\pi f)^2 x_\mathrm{m} < \mu_s mg$, or

$$x_\mathrm{m} < \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\,\mathrm{m/s^2})}{(2\pi \times 2.0\,\mathrm{Hz})^2} = 0.031\,\mathrm{m}\,.$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

27P

(a) For SHM of amplitude A, $a_{\rm m} = \omega^2 A = 4\pi^2 A/T^2$. But the downward acceleration cannot exceed g if the block remains in contact with the piston. Thus

$$a_{\rm m} = \frac{4\pi^2 A}{T^2} \le g\,,$$

or

$$A \le \frac{gT^2}{4\pi^2} = \frac{(9.80\,{\rm m/s^2})(1.0\,{\rm s})^2}{4\pi^2} = 0.25\,{\rm m}\,.$$

(b) Use $a_{\rm m} = 4\pi^2 f^2 A \le g$. Thus

$$f \le \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\pi}\sqrt{\frac{9.80\,{\rm m/s^2}}{0.050\,{\rm m}}} = 2.2\,{\rm Hz}\,.$$

28P

(a) Use $a = -\omega^2 x = -4\pi^2 f^2 x$ to find f:

$$f = \frac{1}{2\pi}\sqrt{\frac{-a}{x}} = \frac{1}{2\pi}\sqrt{\frac{123\,{\rm m/s^2}}{0.100\,{\rm m}}} = 5.58\,{\rm Hz}\,.$$

(b) The mass m of the block satisfies $f = (2\pi)^{-1}\sqrt{k/m}$, or

$$m = \frac{k}{4\pi^2 f^2} = \frac{400\,{\rm N/m}}{4\pi^2(5.58\,{\rm Hz})^2} = 0.325\,{\rm kg}\,.$$

(c) The amplitude A of the oscillation satisfies $\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$, so

$$A = \sqrt{x^2 + \frac{mv^2}{k}} = \sqrt{(0.100\,{\rm m})^2 + \frac{(0.325\,{\rm kg})(-13.6\,{\rm m/s})^2}{400\,{\rm N/m}}} = 0.400\,.$$

29P

(a) Use $E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ to solve for A:

$$A = \sqrt{x^2 + \frac{mv^2}{k}} = \sqrt{(0.129\,{\rm m})^2 + \frac{(2.00\,{\rm kg})(3.415\,{\rm m/s})^2}{100\,{\rm N/m}}} = 0.500\,{\rm m}\,.$$

(b) Write $x(t) = A\sin(\omega t + \phi)$ and $v(t) = \omega A\cos(\omega t + \phi)$. At $t = t_0 = 1.0\,{\rm s}$ we have $\omega x(t_0)/v(t_0) = \tan(\omega t_0 + \phi)$, which gives

$$\begin{aligned}\phi &= \tan^{-1}[\omega x(t_0)/v(t_0)] - \omega t_0 \\ &= \tan^{-1}\left[\sqrt{\frac{100\,{\rm N/m}}{2.00\,{\rm kg}}}\left(\frac{0.129\,{\rm m}}{3.415\,{\rm m/s}}\right)\right] - \sqrt{\frac{100\,{\rm N/m}}{2.00\,{\rm kg}}}(1.0\,{\rm s}) \\ &= -6.81\,.\end{aligned}$$

Thus

$$x|_{t=0} = A\sin\phi = (0.500\,\mathrm{m})\sin(-6.81) = -0.251\,\mathrm{m/s}$$

and

$$v|_{t=0} = \omega A\cos\phi = \sqrt{\frac{100\,\mathrm{N/m}}{2.00\,\mathrm{kg}}}(0.500\,\mathrm{m})\cos(-6.81) = 3.06\,\mathrm{m/s}\,.$$

30P

(*a*) The object oscillates about its equilibrium point, where the downward force of gravity is balanced by the upward force of the spring. If ℓ is the elongation of the spring at equilibrium, then $k\ell = mg$, where k is the spring constant and m is the mass of the object. Thus $k/m = g/\ell$ and $f = \omega/2\pi = (1/2\pi)\sqrt{k/m} = (1/2\pi)\sqrt{g/\ell}$. Now the equilibrium point is halfway between the points where the object is momentarily at rest. One of these points is where the spring is unstretched and the other is the lowest point, 10 cm below. Thus $\ell = 5.0\,cm = 0.050\,\mathrm{m}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8\,\mathrm{m/s^2}}{0.50\,\mathrm{m}}} = 2.2\,\mathrm{Hz}\,.$$

(*b*) Use conservation of energy. Take the zero of gravitational potential energy to be at the initial position of the object, where the spring is unstretched. Then both the initial potential and kinetic energies are zero. Take the y axis to be positive in the downward direction and let $y = 0.080\,\mathrm{m}$. The potential energy when the object is at this point is $U = \frac{1}{2}ky^2 - mgy$. The energy equation becomes $0 = \frac{1}{2}ky^2 - mgy + \frac{1}{2}mv^2$. Solve for v:

$$\begin{aligned} v &= \sqrt{2gy - \frac{k}{m}y^2} = \sqrt{2gy - \frac{g}{\ell}y^2} \\ &= \sqrt{2(9.8\,\mathrm{m/s^2})(0.080\,\mathrm{m}) - \left(\frac{9.8\,\mathrm{m/s^2}}{0.050\,\mathrm{m}}\right)(0.080\,\mathrm{m})^2} = 0.56\,\mathrm{m/s}\,. \end{aligned}$$

(*c*) Let m be the original mass and Δm be the additional mass. The new angular frequency is $\omega' = \sqrt{k/(m+\Delta m)}$. This should be half the original angular frequency, or $\frac{1}{2}\sqrt{k/m}$. Solve $\sqrt{k/(m+\Delta m)} = \frac{1}{2}\sqrt{k/m}$ for m. Square both sides of the equation, then take the reciprocal to obtain $m + \Delta m = 4m$. This gives $m = \Delta m/3 = (300\,\mathrm{g})/3 = 100\,\mathrm{g}$.

(*d*) The equilibrium position is determined by the balancing of the gravitational and spring forces: $ky = (m+\Delta m)g$. Thus $y = (m+\Delta m)g/k$. You will need to find the value of the spring constant k. Use $k = m\omega^2 = m(2\pi f)^2$. Then

$$y = \frac{(m+\Delta m)g}{m(2\pi f)^2} = \frac{(0.10\,\mathrm{kg} + 0.30\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{4\pi^2(0.10\,\mathrm{kg})(2.24\,\mathrm{Hz})^2} = 0.20\,\mathrm{m}\,.$$

This is measured from the initial position.

31P

(a) Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A the amplitudes are both $A/2$. The arguments of the cosine functions are in radians.

Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50\,\mathrm{s}$:

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}}\right) = -0.250A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}} + \frac{\pi}{6}\right) = -0.433A\,.$$

Their separation at that time is $x_1 - x_2 = -0.250A + 0.433A = 0.183A$.

(b) The velocities of the particles are given by

$$v_1 = \frac{dx_1}{dt} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{dx_2}{dt}$$

The solutions to this equation within $(0, 2\pi)$ are $\phi = 0$ or $\phi = 120° = 2\pi/3$. Since v_1 and v_2 have opposite signs at t_0, we need to choose $\phi = 2\pi/3$.

33P

Consider a displacement x (say, to the right) of the mass m. The force f_1 exerted by the left spring on the mass is $f_1 = -kx$, while the force f_2 exerted by the right one is also $f_2 = -kx$. Thus the net force on m is $f_{\text{net}} = f_1 + f_2 = -2kx = -k_{\text{eff}}x$. So

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}\,.$$

34P

The effective spring constant of the system is $k_{\text{eff}} = k_1 + k_2$. Thus the corresponding frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1}{m} + \frac{k_2}{m}} = \sqrt{f_1^2 + f_2^2}\,,$$

where we have used $f_{1,2} = (1/2\pi)\sqrt{k_{1,2}/m}$.

35P

We wish to find the effective spring constant for the combination of springs shown in Fig. 28. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$.

Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\,\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\,\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the mass and its magnitude is $F = k\,\Delta x_\ell$. Thus $k_{\text{eff}} = k\,\Delta x_\ell/2\Delta x_r = k/2$. The mass behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}\,.$$

36P

(a) As it reaches its equilibrium position, the displacement x_0 of the block of mass m from the position when the spring is unstretched is given by $mg\sin\theta = kx_0$, or

$$x_0 = \frac{mg\sin\theta}{k} = \frac{(14.0\,\text{N})(\sin 40.0°)}{120\,\text{N/m}} = 7.50 \times 10^{-2}\,\text{m}\,.$$

The position of the block when it stops is therefore a distance $0.450+0.075 = 0.525\,\mathrm{m}$ from the top of the incline.
(b) The net restoring force F as a function of the block's displacement x from its equilibrium position is $F = -kx$. The period of the oscillation is then

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{14.0\,\mathrm{N}}{(9.80\,\mathrm{m/s^2})(120\,\mathrm{N/m})}} = 0.686\,\mathrm{s}\,.$$

37P

(a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If it is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1.

The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is $\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n+1)$, where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n+1)/n$. Substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n+1)/n$ into $k\,\Delta x = k_1\,\Delta x_1$ and solve for k_1. The result is $k_1 = k(n+1)/n$.

Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a fore on it. Now $k\,\Delta x = k_2\,\Delta x_2$. Substitute $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n+1)$, then solve for k_2. The result is $k_2 = k(n+1)$.

(b) To find the frequency when spring 1 is attached to mass m, replace k in $(1/2\pi)\sqrt{k/m}$ with $k(n+1)/n$ to obtain

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}} = \sqrt{\frac{n+1}{n}}\,f\,,$$

where the substitution $f = (1/2\pi)\sqrt{k/m}$ was made. To find the frequency when spring 2 is attached to the mass, replace k with $k(n+1)$ to obtain

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{n+1}\,f\,,$$

where the same substitution was made.

38P

(a) Let the mass of each of the cars be m. The restoring force of the cable before it breaks is given by $f = 3mg\sin 30° = 3(10,000\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(\sin 30°) = 1.47\times 10^5\,\mathrm{N}$. The effective spring constant of the cable is $k_{\text{eff}} = 1.47 \times 10^5\,\text{N}/0.15\,\text{m} = 9.8 \times 10^5\,\text{N/m}$. The frequency of oscillation is then

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{2m}} = \frac{1}{2\pi}\sqrt{\frac{9.8 \times 10^5\,\text{N/m}}{2(10,000\,\text{kg})}} = 1.1\,\text{Hz}\,.$$

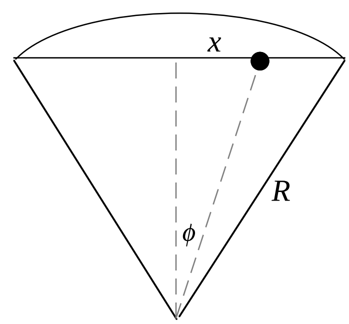
(*b*) The difference in the equilibrium length of the cable with two and three cars attached is

$$\begin{aligned}\Delta x &= \frac{\Delta F}{k_{\text{eff}}} = \frac{mg\sin 30^\circ}{k_{\text{eff}}} \\ &= \frac{(10,000\,\text{kg})(9.80\,\text{m/s}^2)(\sin 30^\circ)}{9.8 \times 10^5\,\text{N/m}} = 0.050\,\text{m} = 5.0\,\text{cm}\,,\end{aligned}$$

which is also the amplitude of the oscillation.

39P

(*a*) Measure the position of the train with the variable x shown on the diagram to the right. If the train is to the left of the center of its path x it negative; if the train is to the right x is positive. When the train is at the point shown the gravitational force due to all of the Earth's mass outside a sphere of radius r is 0. The mass within the sphere is given by $M = M_e(r/R)^3$, where M_e is the total mass of the Earth and R is the radius of the Earth. The magnitude of the gravitation force on the train is $F = GMm/r^2 = GM_emr/R^3$, where m is the mass of the train.



We are interested in the component of the gravitational force along the train's path. This is the component that produces the acceleration of the train. The component perpendicular to the path is balanced by the normal force of the track on the train. The component along the path is $F_x = -F\sin\phi = -(GM_em/R^3)r\sin\phi$. now $x = r\sin\phi$, so $F_x = -(GM_em/R^3)x$. Note that this force is toward the center of the train's path. That is, it is to the right (positive) if x is negative and to the left (negative) if x is positive. It is a restoring force that is proportional to the displacement. We conclude that the train is in simple harmonic motion. Since the train is at rest at the beginning and end of the path, a one-way trip takes half a period.

(*b*) Newton's second law yields

$$-\frac{GM_em}{R^3}\,x = m\,\frac{dx^2}{dt^2}$$

or, when m is canceled,

$$-\left(\frac{GM_e}{R^3}\right)x = \frac{dx^2}{dt^2}\,.$$

By comparison with the analogous equation for a spring-mass system, $-\omega^2 x = dx^2/dt^2$, we can identify the angular frequency: $\omega = \sqrt{GM_e/R^3}$. The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^3}{GM_e}} = 2\pi\sqrt{\frac{(6.37\times10^6\,\mathrm{m})^3}{(6.67\times10^{-11}\,\mathrm{m^3/kg\cdot s^2})(5.98\times10^{24}\,\mathrm{kg})}} = 5.06\times10^3\,\mathrm{s}\,.$$

The time for a trip is $T/2 = 2.53\times10^3\,\mathrm{s} = 42\,\mathrm{min}$.

40E

When the block is at the end of its path and is momentarily at rest, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E = \frac{1}{2}kx_\mathrm{m}^2 = \frac{1}{2}(1.3\times10^2\,\mathrm{N/m})(0.024\,\mathrm{m})^2 = 3.7\times10^{-2}\,\mathrm{J}\,.$$

41E

Use $E = \frac{1}{2}mv_\mathrm{m}^2 = \frac{1}{2}kA^2$ and $v_\mathrm{m} = 2\pi fA$.
(*a*) $k = 2E/A^2 = 2(1.00\,\mathrm{J})/(0.100\,\mathrm{m})^2 = 200\,\mathrm{N/m}$.
(*b*) $m = 2E/v_\mathrm{m}^2 = 2(1.00\,\mathrm{J})/(1.2\,\mathrm{m/s})^2 = 1.39\,\mathrm{kg}$.
(*c*) $f = v_\mathrm{m}/2\pi A = (1.20\,\mathrm{m/s})/[2\pi(0.100\,\mathrm{m})] = 1.91\,\mathrm{Hz}$.

42E

(*a*) The frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000\,\mathrm{N/m}}{5.00\,\mathrm{kg}}} = 2.25\,\mathrm{Hz}\,.$$

(*b*) The initial potential energy is

$$U_i = \frac{1}{2}kx_i^2 = \frac{1}{2}(1000\,\mathrm{N/m})(0.500\,\mathrm{m})^2 = 125\,\mathrm{J}\,.$$

(*c*) The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(5.00\,\mathrm{kg})(10.0\,\mathrm{m/s})^2 = 250\,\mathrm{J}\,.$$

(*d*) Use $\frac{1}{2}kA^2 = U_i + K_i$ to find the amplitude of oscillation:

$$A = \sqrt{\frac{2(U_i+K_i)}{k}} = \sqrt{\frac{2(125\,\mathrm{J}+250\,\mathrm{J})}{1000\,\mathrm{N/m}}} = 0.866\,\mathrm{m} = 86.6\,\mathrm{cm}\,.$$

43E

(a) The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky = mg$, where y is the elongation of the spring at equilibrium, k is the spring constant, and m is the mass of the block. Thus $k = mg/y = (1.3\,\mathrm{kg})(9.8\,\mathrm{m/s^2})/(0.096\,\mathrm{m}) = 130\,\mathrm{N/m}$.

(b) The period is given by $T = 1/f = 2\pi/\omega = 2\pi\sqrt{m/k} = 2\pi\sqrt{(1.3\,\mathrm{kg})/(133\,\mathrm{N/m})} = 0.62\,\mathrm{s}$.

(c) The frequency is $f = 1/T = 1/0.62\,\mathrm{s} = 1.6\,\mathrm{Hz}$.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) The block has maximum speed as it passes the equilibrium point. Then the spring is elongated by $y = 9.6\,\mathrm{cm}$ and the spring potential energy is $\frac{1}{2}ky^2 = \frac{1}{2}(133\,\mathrm{N/m})(0.096\,\mathrm{m})^2 = 0.612\,\mathrm{J}$ and the gravitational potential energy is $-mgy = -(1.3\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.096\,\mathrm{m}) = -1.22\,\mathrm{J}$. The total potential energy is $0.612\,\mathrm{J} - 1.22\,\mathrm{J} = -0.61\,\mathrm{J}$. At the initial position the block is not moving but it has potential energy $U_i = -0.44\,\mathrm{J}$. Write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44\,\mathrm{J} + 0.61\,\mathrm{J})}{1.3\,\mathrm{kg}}} = 0.51\,\mathrm{m/s}\,.$$

44E

(a) Use $\frac{1}{2}kA^2 = \frac{1}{2}mv^2$ to find k:

$$k = m\left(\frac{v}{A}\right)^2 = (0.130\,\mathrm{kg})\left(\frac{11.2\times 10^3\,\mathrm{m/s}}{1.50\,\mathrm{m}}\right)^2 = 7.25\times 10^6\,\mathrm{N/m}\,.$$

(b) The number of people required is

$$n = \frac{kA}{220\,\mathrm{N}} = \frac{(7.25\times 10^6\,\mathrm{N/m})(1.50\,\mathrm{m})}{220\,\mathrm{N}} = 49,400\,.$$

45E

(a) and (b) The total energy is given by $E = \frac{1}{2}kx_\mathrm{m}^2$, where k is the spring constant and x_m is the amplitude. When $x = \frac{1}{2}x_\mathrm{m}$ the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{8}kx_\mathrm{m}^2$. The ratio is

$$\frac{U}{E} = \frac{kx_\mathrm{m}^2/8}{kx_\mathrm{m}^2/2} = \frac{1}{4}\,.$$

The fraction of the energy that is kinetic is

$$\frac{K}{E} = \frac{E-U}{E} = 1 - \frac{U}{E} = 1 - \frac{1}{4} = \frac{3}{4}\,.$$

(c) Since $E = \frac{1}{2}kx_{\rm m}^2$ and $U = \frac{1}{2}kx^2$, $U/E = x^2/x_{\rm m}^2$. Solve $x^2/x_{\rm m}^2 = 1/2$ for x. You should get $x = x_{\rm m}/\sqrt{2}$.

46E

(a) Use $mv = MV + m$ for the velocity V of the block: $V = mv/(M+m)$.
(b) Use $\frac{1}{2}(M+m)V^2 = \frac{1}{2}kA^2$ to solve for A:

$$A = V\sqrt{\frac{M+m}{k}} = \frac{mv}{\sqrt{k(M+m)}}\,.$$

47P

(a) Let the value fo x be x_0 when $U(x) = \frac{1}{2}E$. Thus $U(x) = \frac{1}{2}kx_0^2 = \frac{1}{2}E = \frac{1}{4}kA^2$, or $x_0 = A/\sqrt{2} = 5.0\,\text{m}/\sqrt{2} = 3.5\,\text{m}$.
(b) The phase difference between the two positions (the equilibrium position and the position $x = x_0$) is

$$\Delta\phi = \frac{\pi}{2} - \cos^{-1}\left(\frac{3.5\,\text{m}}{5.0\,\text{m}}\right) = 0.775\,.$$

The time Δt it takes is then

$$\Delta t = \left(\frac{\Delta\phi}{2\pi}\right)\frac{1}{f} = \frac{0.775}{(2\pi)(\pi/3)(1/2\pi)} = 0.75\,\text{s}\,.$$

48P

(a) Let the displacement of the particle be $x(t) = x_{\rm m}\cos(\omega t - \phi)$, then the correpsonding force $F(t)$ is given by $F(t) = ma(t) = -m\omega^2 x(t) = -F_{\rm m}(\cos\omega t - \phi)$, where $F_{\rm m} = ma_{\rm m} = (10\times10^{-3}\,\text{kg})(8.0\times10^3\,\text{m/s}^2) = 80\,\text{N}$, $\omega = \sqrt{a_{\rm m}/A} = \sqrt{(8.0\times10^3\,\text{m/s}^2)/(2.0\times10^{-3}\,\text{m})} = 2.0\times10^3\,\text{rad/s}$, and $\phi = -\pi/3\,\text{rad}$. Thus $F(t) = -(80\,\text{N})\cos[(2.0\times10^3\,\text{rad/s})t - \pi/3]$.
(b) $T = 2\pi/\omega = 2\pi/(2.0\times10^3\,\text{rad/s}) = 3.1\times10^{-3}\,\text{s}$.
(c) $v_{\rm m} = \omega A = (2.0\times10^3\,\text{rad/s})(2.0\times10^{-3}\,\text{m}) = 4.0\,\text{m/s}$.
(d) $E = \frac{1}{2}mv_{\rm m}^2 = \frac{1}{2}(10\times10^{-3}\,\text{kg})(4.0\,\text{m/s})^2 = 8.0\times10^{-2}\,\text{J}$.

49P

(a) Suppose that the spring stretches by an amount ℓ before the body stops momentarily. According to the work-energy theorem, the total work done on the body is $W = -\frac{1}{2}k\ell^2 + mgl = \Delta K = 0$, or

$$l = \frac{2mg}{k} = \frac{2(0.20\,\text{kg})(9.80\,\text{m/s}^2)}{19\,\text{N/m}} = 0.21\,\text{m}\,.$$

(b) The frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{19\,\mathrm{N/m}}{0.20\,\mathrm{kg}}} = 1.6\,\mathrm{Hz}\,.$$

(c) The amplitude A is equal to half of the distance between the two turning points of the oscillator, i.e., $A = l/2 = 0.21\,\mathrm{m}/2 = 0.10\,\mathrm{m}$.

50P

(a) Assume that the bullet becomes imbedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet-block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m + M)v$, so

$$v = \frac{mv_0}{m+M} = \frac{(0.050\,\mathrm{kg})(150\,\mathrm{m/s})}{0.050\,\mathrm{kg} + 4.0\,\mathrm{kg}} = 1.85\,\mathrm{m/s}\,.$$

After the collision the block with the bullet embedded oscillates with simple harmonic motion. The energy is all kinetic and is given by $E = \frac{1}{2}(m + M)v^2$. When the block is at the end of its path the energy is all potential and is given by $E = \frac{1}{2}ky_\mathrm{m}^2$, where k is the spring constant and y_m is the amplitude. Since energy is conserved, $\frac{1}{2}(m + M)v^2 = \frac{1}{2}ky_\mathrm{m}^2$ and

$$y_\mathrm{m} = \sqrt{\frac{(m+M)v^2}{k}} = \sqrt{\frac{(0.050\,\mathrm{kg} + 4.0\,\mathrm{kg})(1.85\,\mathrm{m/s})^2}{500\,\mathrm{N/m}}} = 0.167\,\mathrm{m}\,.$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050\,\mathrm{k}g)(150\,\mathrm{m/s})^2 = 563\,\mathrm{J}$. The energy after the collision is $E = \frac{1}{2}(m + M)v^2 = \frac{1}{2}(0.050\,\mathrm{kg} + 4.0\,\mathrm{kg})(1.85\,\mathrm{m/s})^2 = 6.94\,\mathrm{J}$. The ratio is $E/E_0 = (6.94\,\mathrm{J})/(563\,\mathrm{J}) = 0.0123$ or 1.23%.

51P

(a) Use conservation of energy. Initially the energy is all potential in nature and is given by $E = \frac{1}{2}kx_\mathrm{m}^2$, where k is the spring constant and x_m is the initial elongation of the spring. When the disk is at the equilibrium point (and the kinetic energy is maximum) the energy is all kinetic in nature and is given by $E = \frac{1}{2}Mv_\mathrm{m}^2 + \frac{1}{2}I\omega_\mathrm{m}^2$, where M is the mass of the disk, v_m is the speed of its center of mass, I is its rotational inertia, and ω_m is its angular speed (about the center of mass). The first term in the energy is the translational kinetic energy and the second is the rotational kinetic energy. Since the wheel rolls without slipping $\omega_\mathrm{m} = v_\mathrm{m}/R$, where R is its radius. Furthermore the rotational inertia of a disk is given by $I = \frac{1}{2}MR^2$. When these substitutions are made the expression for the energy at the equilibrium point becomes $E = \frac{1}{2}Mv_\mathrm{m}^2 + \frac{1}{4}Mv_\mathrm{m}^2 = \frac{3}{4}Mv_\mathrm{m}^2$. Conservation of energy yields $\frac{1}{2}kx_\mathrm{m}^2 = \frac{3}{4}Mv_\mathrm{m}^2$. Thus $Mv_\mathrm{m}^2 = \frac{2}{3}kx_\mathrm{m}^2$ and the translational kinetic energy is $\frac{1}{2}Mv_\mathrm{m}^2 = \frac{1}{3}kx_\mathrm{m}^2 = \frac{1}{3}(3.0\,\mathrm{N/m})(0.25\,\mathrm{m})^2 = 0.0625\,\mathrm{J}$.

(b) The rotational kinetic energy at the equilibrium point is $\frac{1}{2}I\omega_{\rm m}^2$. Replace I with $\frac{1}{2}MR^2$ and $\omega_{\rm m}$ with $v_{\rm m}/R$ to obtain $\frac{1}{4}Mv_{\rm m}^2$. Now replace $Mv_{\rm m}^2$ with $\frac{2}{3}kx_{\rm m}^2$ to obtain $\frac{1}{6}kx_{\rm m}^2 = \frac{1}{6}(3.0\,{\rm N/m})(0.25\,{\rm m/s})^2 = 0.031\,{\rm J}$.

(c) Suppose the center of mass executes simple harmonic motion with angular frequency ω (not to be confused with the angular speed of rotation). Then $v_{\rm m} = \omega x_{\rm m}$. Use $\frac{1}{2}Mv_{\rm m}^2 = \frac{1}{3}kx_{\rm m}^2$ to find that $v_{\rm m} = \sqrt{2k/3M}x_{\rm m}$. Thus $\sqrt{2k/3M}x_{\rm m} = \omega x_{\rm m}$ and $\omega = \sqrt{2k/3M}$. The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3M}{2k}}\,.$$

52E

(a) $I = \frac{1}{2}mR^2 = \frac{1}{2}(3.00\,{\rm kg})(70.0\times10^{-2}\,{\rm m})^2 = 0.735\,{\rm kg\cdot m^2}$.
(b) $\kappa = \tau/\theta = (0.0600\,{\rm N\cdot m})/2.50\,{\rm rad} = 0.0240\,{\rm N\cdot m}$.
(c) $\omega = \sqrt{\kappa/I} = \sqrt{(0.0240\,{\rm N\cdot m})/(0.735\,{\rm kg\cdot m^2})} = 0.181\,{\rm rad/s}$.

53P

The torque τ applied to the wire is related to the turning angle θ by $\tau = -\kappa\theta$, where $\kappa = |\tau/\theta| = 0.20\,{\rm N{\cdot}m}/0.85\,{\rm rad} = 0.235\,{\rm N{\cdot}m/rad}$. The period of oscillation of sphere of mass m and radius R is then

$$T = 2\pi\sqrt{\frac{I}{\kappa}} = 2\pi\sqrt{\frac{2mR^2}{5\kappa}} = 2\pi(0.15\,{\rm m})\sqrt{\frac{2(95\,{\rm kg})}{5(0.235\,{\rm N\cdot m/rad})}} = 12\,{\rm s}\,.$$

54P

Use $T = 2\pi\sqrt{I/\kappa}$ to solve for I:

$$I = \frac{T^2\kappa}{4\pi^2} = \frac{(50\,{\rm s}/20)^2(0.50\,{\rm N\cdot m})}{4\pi^2} = 7.9\times10^{-2}\,{\rm kg\cdot m^2}.$$

55P

(a) Take the angular displacement of the wheel to be $\theta = \theta_{\rm m}\cos(2\pi t/T)$, where $\theta_{\rm m}$ is the amplitude and T is the period. Differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_{\rm m}\sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_{\rm m} = \frac{2\pi\theta_{\rm m}}{T} = \frac{(2\pi)(\pi\,{\rm rad})}{0.500\ {\rm s}} = 39.5\,{\rm rad/s}\,.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1 - \cos^2(2\pi t/T)} = \sqrt{1 - (1/2)^2} = \sqrt{3}/2\,,$$

where the trigonometric identity $\cos^2 A + \sin^2 A = 1$ was used. Thus

$$\Omega = -\frac{2\pi}{T}\,\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500\,\text{s}}\right)(\pi\,\text{rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2\,\text{rad/s}\,.$$

The minus sign is not significant. During another portion of the cycle its angular speed is $+34.2\,\text{rad/s}$ when its angular displacement is $\pi/2\,\text{rad}$.

(c) The angular acceleration is

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta\,.$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500\,\text{s}}\right)^2\left(\frac{\pi}{4}\right) = -124\,\text{rad/s}^2\,.$$

Again the minus sign is not significant.

56E

Use $T = 2\pi\sqrt{L/g}$ to solve for L:

$$L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\,\text{s})^2(32.2\,\text{ft/s}^2)}{4\pi^2} = 9.79\,\text{in.}\,.$$

57E

(a) The period of the swing of length L is given by

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{17\,\text{m}}{9.80\,\text{m/s}^2}} = 8.3\,\text{s}\,.$$

(b) No. T depends only on L and g.

58E

The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$, where L is its length. Thus $L = T^2 g/4\pi^2 = (2.0\,\text{s})^2(9.8\,\text{m/s}^2)/(4\pi^2) = 0.99\,\text{m}$.

59E

Use $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{1.50\,\mathrm{m}/g} = 180\,\mathrm{s}/72.0$ and solve for g:

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1.50\,\mathrm{m})}{(180\,\mathrm{s}/72.0)^2} = 9.47\,\mathrm{m/s^2}\,.$$

60E

Since the spring constant k is given by $k = mg/h$,

$$T_{\mathrm{spring}} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}} = T_{\mathrm{pendulum}}\,.$$

61E

Let the effective length of the simple pendulum be L_1 when the performer is seated on the trapeze. Then $T_1 = 8.85\,\mathrm{s} = 2\pi\sqrt{L_1/g}$, or

$$L_1 = \left(\frac{T_1}{2\pi}\right)^2 g = \left(\frac{8.85\,\mathrm{s}}{2\pi}\right)^2 (9.80\,\mathrm{m/s^2}) = 19.4\,\mathrm{m}\,.$$

Thus as the performer stands up, the new period T_2 is

$$T_2 = 2\pi\sqrt{\frac{L_2}{g}} = 2\pi\sqrt{\frac{19.4\,\mathrm{m} - 0.350\,\mathrm{m}}{9.80\,\mathrm{m/s^2}}} = 8.77\,\mathrm{s}\,.$$

62E

Since $T \propto \sqrt{L}$, the new period T' is

$$T' = T\sqrt{\frac{L/2}{L}} = \frac{T}{\sqrt{2}}\,.$$

63E

Use

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{mL^2/12 + mx^2}{mgx}}$$

to solve for x:

$$x = \frac{T^2 g}{8\pi^2} - \frac{1}{2}\sqrt{\left(\frac{T^2 g}{4\pi^2}\right)^2 - \frac{L^2}{3}}$$
$$= \frac{(2.5\,\mathrm{s})^2(9.80\,\mathrm{m/s^2})}{8\pi^2} - \frac{1}{2}\sqrt{\left[\frac{(2.5\,\mathrm{s})^2(9.80\,\mathrm{m/s^2})}{4\pi^2}\right]^2 - \frac{(1.00\,\mathrm{m})^2}{3}}$$
$$= 0.056\,\mathrm{m}\,.$$

64E

(a) The period of the pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{mgd}}\,,$$

where I is its rotational inertia, m is its mass, and d is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ and, according to the parallel-axis theorem, its rotational inertia when it is pivoted a distance d from the center is $I = mL^2/12 + md^2$. Thus

$$T = 2\pi\sqrt{\frac{m(L^2/12 + d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}\,.$$

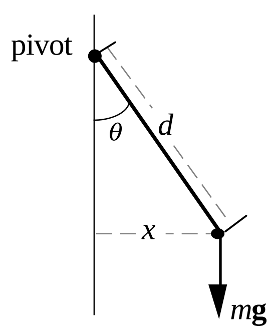
(b) $(L^2 + 12d^2)/12gd$, considered as a function of d, has a minimum at $d = L/\sqrt{12}$, so the period increases as d decreases if $d < L/\sqrt{12}$ and decreases as d decreases if $d > L/\sqrt{12}$.

(c) L occurs only in the numerator of the expression for the period, so T increases as L increases.

(d) The period does not depend on the mass of the pendulum, so T does not change when m increases.

65E

As the pendulum makes a small angular displacement θ as shown, the torque τ applied to it about its pivot is



$$\tau = -mgx = -mgd\sin\theta \approx -mgd\theta = -\kappa\theta\,.$$

Thus its period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{\kappa}} = 2\pi\sqrt{\frac{\frac{1}{2}mR^2 + md^2}{mgd}} = 2\pi\sqrt{\frac{R^2 + 2d^2}{2gd}}\,,$$

where we have made use of the parallel axis theorem to find I.

66E

(a) The period is

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}}$$
$$= 2\pi\sqrt{\frac{(12.5\,\mathrm{cm})^2/2 + (12.5\,\mathrm{cm})^2}{(980\,\mathrm{cm/s^2})(12.5\,\mathrm{cm})}} = 0.869\,\mathrm{s}\,.$$

(b) Let

$$T = 2\pi\sqrt{\frac{mR^2/2 + mr^2}{mgr}} = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}}$$

and solve for r: $r = R/2 = 12.5\,\mathrm{cm}/2 = 6.25\,\mathrm{cm}$.

67E

(a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}MR^2$, where M is its mass and R is its radius. The disk of this problem rotates about a point that is displaced from its center by $R+L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}MR^2+M(L+R)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is $I = \frac{1}{2}MR^2+M(L+R)^2+\frac{1}{3}mL^2 = \frac{1}{2}(0.500\,\mathrm{kg})(0.100\,\mathrm{m})^2+(0.500\,\mathrm{kg})(0.500\,\mathrm{m}+0.100\,\mathrm{m})^2+\frac{1}{3}(0.270\,\mathrm{kg})(0.500\,\mathrm{m})^2 = 0.205\,\mathrm{kg}\cdot\mathrm{m}^2$.

(b) Put the origin at the pivot. The center of mass of the disk is $\ell_d = L + R = 0.500\,\mathrm{m} + 0.100\,\mathrm{m} = 0.600\,\mathrm{m}$ away and the center of mass of the rod is $\ell_r = L/2 = (0.500\,\mathrm{m})/2 = 0.250\,\mathrm{m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M+m} = \frac{(0.500\,\mathrm{kg})(0.600\,\mathrm{m}) + (0.270\,\mathrm{kg})(0.250\,\mathrm{m})}{0.500\,\mathrm{kg} + 0.270\,\mathrm{kg}} = 0.477\,\mathrm{m}\,.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\,\mathrm{kg}\cdot\mathrm{m}^2}{(0.500\,\mathrm{kg}+0.270\,\mathrm{kg})(9.8\,\mathrm{m/s}^2)(0.447\,\mathrm{m})}} = 1.50\,\mathrm{s}\,.$$

68E

(a) and (b) If the pendulum is suspended at point P then in Eq. 16-32 we must change h to $h' = L_0 - h = \frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$ and I to $I' = I_0 + mh'^2 = \frac{1}{12}mL^2 + m(\frac{1}{6}L)^2 = \frac{1}{9}mL_0^2$.

Here we made use of the parallel axis theorem to calculate the moment of inertia I' about point P. Thus the new period of oscillation is

$$T' = 2\pi\sqrt{\frac{I'}{mgh'}} = 2\pi\sqrt{\frac{mL^2/9}{mg(L/6)}} = 2\pi\sqrt{\frac{2L}{3g}}\,.$$

This is the same as the period we obtained before in Sample Problem 16-6. The numerical value is once again 1.64 s.

69E

Solve for L_0 from

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L_0}{g}}$$

to obtain $L_0 = I/mh$.

70E

Let the frequency of oscillation of the meter stick of length L be $f_0 = c\sqrt{g/L}$, where c is a constant which you don't need to calculate for this problem. If L is reduced to $L/2$, then the new frequency f must be $f = c\sqrt{g/(L/2)} = \sqrt{2}c\sqrt{g/L} = \sqrt{2}f_0$.

71P

(a) Since $I_0 = mL^2/12 + mx^2$ the period of the pendulum is

$$T = 2\pi\sqrt{\frac{I_0}{mgx}} = 2\pi\sqrt{\frac{mL^2/12 + mx^2}{mgx}} = 2\pi\sqrt{\frac{L^2 + 12x^2}{12gx}}\,.$$

(b) Set

$$\frac{d}{dx}\left(\frac{L^2 + 12x^2}{12gx}\right)\Bigg|_{x=x_0} = \left(-\frac{L^2}{12gx^2} + \frac{1}{g}\right)\Bigg|_{x=x_0} = 0$$

and solve for x: $x_0 = L/\sqrt{12} = 1.00\,\text{m}/\sqrt{12} = 0.289\,\text{m}$.

(c)

$$T_{\min} = T|_{x=x_0} = 2\pi\sqrt{\frac{(1.00\,\text{m})^2 + 12(0.289\,\text{m})^2}{12(9.80\,\text{m/s}^2)(0.289\,\text{m})}} = 1.53\,\text{s}\,.$$

72P

(a) The linear acceleration a due to the force F exerted at point O is given by $F = ma$, or $a = F/m$.

(b) The angular acceleration about point C is

$$\alpha = \frac{\tau}{I} = \frac{F(L_0/6)}{mL_0^2/12} = \frac{2F}{mL_0}\,.$$

(c) The actual linear acceleration at point O is given by

$$a_0 = a - \frac{\alpha L_0}{2} = \frac{F}{m} - \frac{2F}{mL_0}\cdot\frac{L_0}{2} = 0\,.$$

(d) Since $a_0 = 0$ no force is felt at point O, so point P is indeed the "sweet spot."

73P

Denote Paris with subscript p and Cayenne with c. Then

$$\frac{T_c}{T_p} = \sqrt{\frac{g_p}{g_c}} = \frac{(1\,\mathrm{da})(86400\,\mathrm{s/da})}{(1\,\mathrm{da})(86400\,\mathrm{s/da}) - (2.5\,\mathrm{min})(60\,\mathrm{s/min})} = 1.0017\,.$$

Thus

$$g_c = \frac{g_p}{(1.0017)^2} = \frac{9.81\,\mathrm{m/s^2}}{(1.0017)^2} = 9.78\,\mathrm{m/s^2}\,.$$

74P

The difference in the measured values of the acceleration of gravity is due to the different apparent angular speed of the Earth's rotation as viewed from the submarine. Let the angular speed of the Earth viewed from a stationary observer on Earth be ω_e, then its apparent angular speed as viewed from the submarine is given by $\omega_\pm = \omega_e \pm V_s/R_e$, where V_s is the speed of the submarine and R_e the radius of the Earth. Since the Earth rotates from west to east, the plus sign corresponds to the case when the submarine moves westward (i.e., in the opposite direction to the spin of the Earth). If we denote the acceleration of gravity on the eauator to be a_g if the Earth were to stop rotating, then the measured acceleration of gravity satisfies $g_\pm = a_g - \omega_\pm^2 R_e$, i.e.,

$$|\Delta g_\pm| = |g_+ - g_-| = |(a_g - \omega_+^2 R) - (a_g - \omega_-^2 R_e)| = 4\omega_e V_s\,.$$

Thus

$$\frac{\Delta g_\pm}{g} = \frac{4\omega_e V_s}{g} = \frac{4(2\pi/86,400\,\mathrm{s})(16\times 10^3\,\mathrm{m}/3600\,\mathrm{s})}{9.8\,\mathrm{m/s^2}} = 1.3\times 10^{-4}\,.$$

The fractional error $\Delta g/g$ for either travel direction is therefore $\Delta g/g = \frac{1}{2}\Delta g_\pm/g = 1.3\times 10^{-4}/2 = 6.6\times 10^{-5}$.

75P

If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is $T = 2\pi/\omega = 2\pi\sqrt{I/C}$, where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities.

Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved $(L/2)(1-\cos\theta)$ to the right. The length of the spring is now $\sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}$. If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\,\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$.

The rotational inertia for a rod pivoted at it center is $I = mL^2/12$, where m is its mass. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{4mL^2}{12kL^2}} = 2\pi\sqrt{\frac{m}{3k}}\,.$$

76P

(a) The frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. This gives $f = (1/2\pi)\sqrt{(9.80\,\mathrm{m/s^2})/(2.0\,\mathrm{m})} = 0.35\,\mathrm{Hz}$.

(b) The forces acing on the pendulum are the tension $\mathbf{T}$ in the rod and the force of gravity $m\mathbf{g}$. Newton's second law yields $\mathbf{T}+m\mathbf{g} = m\mathbf{a}$, where m is the mass and $\mathbf{a}$ is the acceleration of the pendulum. Let $\mathbf{a} = \mathbf{a}_e + \mathbf{a}'$, where $\mathbf{a}_e$ is the acceleration of the elevator and $\mathbf{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\mathbf{g} - \mathbf{a}_e) + \mathbf{T} = m\mathbf{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\mathbf{g} - \mathbf{a}_e$. Since $\mathbf{g}$ and $\mathbf{a}_e$ are along the same line and in opposite directions we can find the frequency for small amplitude oscillations by replacing g with $g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\mathrm{m/s^2} + 2.0\,\mathrm{m/s^2}}{2.0\,\mathrm{m}}} = 0.39\,\mathrm{Hz}\,.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is $\mathbf{g} - \mathbf{a}_e = 0$. To find the frequency for small amplitude oscillations, replace g with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

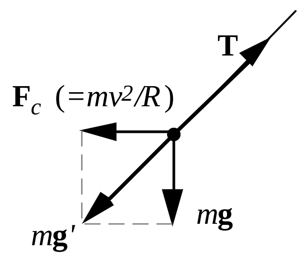
77P

Consider the motion of the pendulum in the frame of reference which is fixed on the moving car. A fictitious centrifugal force (pseudoforce) $F_c = mv^2/R$ must be intruduced to account for the fact that the frame of reference we are using is non-inertial in nature. Equivalently, as depicted in the figure to the right, we can regard this situation as if the pendulum were subject to an effective gravitational field $\mathbf{g}_{\text{eff}}$ which satisfies

$$m\mathbf{g}_{\text{eff}} = m\mathbf{g} + \mathbf{F}_c\,,$$

where

$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}\,.$$



Thus the frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{g'}} = \frac{1}{2\pi}\sqrt{\frac{L}{\sqrt{g^2 + (v^2/R)^2}}}\,.$$

78P

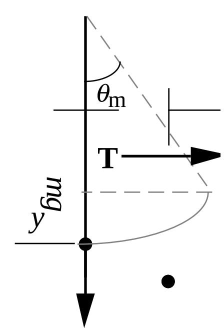
The approximation we made in deriving $T = 2\pi\sqrt{L/g}$ was to replace $\sin\theta$ by θ for small $|\theta|$. Thus the angular amplitude θ_{m} must satisfy

$$\left|\frac{\sin\theta_{\text{m}} - \theta_{\text{m}}}{\sin\theta_{\text{m}}}\right| = 0.01\,.$$

Substitute $\sin\theta_{\text{m}} \simeq \theta_{\text{m}} - \theta_{\text{m}}^3/3$ into the equation above and solve for θ_{m}. You should get $\theta_{\text{m}} \simeq 14.0°$.

79P

(a) The loss of gravitational potential energy of the bob in the Earth's gravitational field as it swings down is $mgh = mgR(1 - \cos\theta_{\text{m}})$. Thus its speed v at the bottom of the swing satisfies $\frac{1}{2}mv^2 = mgR(1 - \cos\theta_{\text{m}})$, or $mv^2 = 2mgR(1 - \cos\theta_{\text{m}}) \approx mgR\theta_{\text{m}}^2$, where we have used the expansion $\cos\theta_{\text{m}} \simeq 1 - \theta_{\text{m}}^2/2$ for small θ_{m}. Thus the tension T at that position satisfies $T - mg = F_c = mv^2/R \simeq mg\theta_{\text{m}}^2$, or



$$T \simeq mg(1 + \theta_{\text{m}}^2)\,.$$

(b) The tension T_θ at any other position (where θ_{m} is replaced by a smaller value θ) satisfies $T_\theta - mg\cos\theta = mv_\theta^2/R$ or $T_\theta = mg\cos\theta + mv_\theta^2/R$. Note that $0 < \cos\theta < 1$ and $v_\theta < v$, so $T_\theta < T$.

80P

(a) Suppose that the wheel is rotated through a small angle θ. The corresopnding change in the length of the spring is $x \simeq r\theta$. The torque applied by the spring to the wheel about its center is then $\tau = -Fr = -kxr \simeq -kr^2\theta = -\kappa\theta$. Thus the angular frequency of the small oscillation is

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{kr^2}{mR^2}} = \frac{r}{R}\sqrt{\frac{k}{m}}\,.$$

(b) If $r = R$ then ω reaches its maximum value of $\sqrt{k/m}$.
(c) If $r = 0$ then $\omega = 0$ (i.e., no oscillation).

81P

Denote the length of the rod as L and the diameter of the disk of mass m as d.
(a)

$$\begin{aligned} T &= 2\pi\sqrt{\frac{I}{mgx}} = 2\pi\sqrt{\frac{md^2/8 + m(L+d/2)^2}{mg(L+d/2)}} \\ &= 2\pi\sqrt{\frac{(42\,\mathrm{cm})^2/8 + (76\,\mathrm{cm} + 42\,\mathrm{cm}/2)^2}{(980\,\mathrm{cm/s^2})(76\,\mathrm{cm} + 42\,\mathrm{cm}/2)}} = 2.0\,\mathrm{s}\,. \end{aligned}$$

(b) Let $T' = 2\pi\sqrt{I/(mgh+\kappa)}$ and solve for κ:

$$\begin{aligned} \kappa =& I\left(\frac{2\pi}{T'}\right) - mgh \\ =& [(42\,\mathrm{cm})^2(2.5\,\mathrm{kg})/8 + (2.5\,\mathrm{kg})(76\,\mathrm{cm} + 42\,\mathrm{cm}/2)^2]\left(\frac{2\pi}{2.0\,\mathrm{s} - 0.50\,\mathrm{s}}\right)^2 \\ & - (2.5\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(76\,\mathrm{cm} + 42\,\mathrm{cm}/2) \\ =& 18.5\,\mathrm{N\cdot m/rad}\,. \end{aligned}$$

82P

Let the distances from points A and B to the center of mass be L_A and L_B, respectively. Then $T = 2\pi\sqrt{I_A/(mgL_A)}$ and $T = 2\pi\sqrt{I_B/(mgL_B)}$, where I_A and I_B are the moments of inertia of the pendulum about points A and B, respectively. Solve for L_A and L_B: $L_A = 4\pi^2 I_A/(mgT^2)$, $L_B = 4\pi^2 I_B/(mgT^2)$.

83P*

Let the distance from the pivot to the center of mass be x. Then

$$T(x) = 2\pi\sqrt{\frac{I}{mgx}} = 2\pi\sqrt{\frac{I_0 + mx^2}{mgx}}\,.$$

Set

$$\frac{d}{dx}\left(\frac{I_0 + mx^2}{mgx}\right)\bigg|_{x=x_0} = -\frac{I_0}{mgx_0^2} + \frac{1}{g} = 0$$

and solve for x_0:

$$x_0 = \sqrt{\frac{I_0}{m}} = \sqrt{\frac{mL^2}{12m}} = \frac{L}{\sqrt{12}} \approx 0.29L\,.$$

84E

Since $E = \frac{1}{2}kA^2 \propto A^2$, the fraction of the energy lost in each full oscillation is

$$\text{frac} = \frac{A^2 - [(1 - 3.0\%)A]^2}{A^2} = 5.9\%\,.$$

85E

Let $A = x_\mathrm{m}e^{-bt/2m}$. You want to evaluate $A/x_\mathrm{m} = e^{-bt/2m}$ for $t = 20T$, where T is the period. Values for b (70 g/s), and m (250 g) are given in Sample Problem 10 and T is found in that problem to be 0.34 s. Thus $bt/2m = (0.070\,\mathrm{kg/s})(6.82\,\mathrm{s})/2(0.250\,\mathrm{kg}) = 0.955$ and

$$\frac{A}{x_\mathrm{m}} = e^{-0.955} = 0.385\,.$$

Note the conversions to SI units.

86E

(a) You want to solve $e^{-bt/2m} = 1/3$ for t. Take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Now solve for t: $t = -(2m/b)\ln(1/3) = (2m/b)\ln 3$, where the sign was reversed when the argument of the logarithm was replaced by its reciprocal. Thus

$$t = \frac{2(1.50\,\mathrm{kg})}{0.230\,\mathrm{kg/s}}\ln 3 = 14.3\,\mathrm{s}\,.$$

(b) The angular frequency is

$$\omega_d = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\,\mathrm{N/m}}{1.50\,\mathrm{kg}} - \frac{(0.230\,\mathrm{k}g/s)^2}{4(1.50\,\mathrm{kg})^2}} = 2.31\,\mathrm{rad/s}\,.$$

The period is $T = 2\pi/\omega_d = (2\pi)/(2.31\,\text{rad/s}) = 2.72\,\text{s}$ and the number of oscillations is $t/T = (14.3\,\text{s})/(2.72\,\text{s}) = 5.27$.

87P

(a) Use $A(t) = x_\text{m} e^{-bt/2m}$. At $t_0 = 4T = 8\pi\sqrt{m/k} = 8\pi\sqrt{2.00\,\text{kg}/(10.0\,\text{N/m})} = 11.24\,\text{s}$ we have $A(t_0) = 3x_\text{m}/4$. Thus

$$e^{-bt_0/2m} = \frac{3}{4}\,.$$

Solve for b :

$$b = \frac{2m}{t_0}\ln\frac{4}{3} = \frac{2(2.00\,\text{kg})}{11.24\,\text{s}}\ln\frac{4}{3} = 0.102\,\text{kg/s}\,.$$

(b) The amount of energy lost is

$$|\Delta E| = \frac{1}{2}k|x_\text{m}^2 - A^2(t_0)| = \frac{1}{2}(10.0\,\text{N/m})\left(1 - \frac{9}{16}\right)(0.250\,\text{m})^2 = 0.137\,\text{J}\,.$$

88P

(a) Use $x(t) = x_\text{m} e^{-bt/2m}\cos(\omega_0 t + \phi)$. The maximum value of $(-b dx/dt)$ is attained when $dx(t)/dt$ reaches its maximum. Since $0 < e^{-bt/2m} = e^{-(70\,\text{g/s})(0.34\,\text{s})/[2(250\,\text{g})]} = 0.95 \simeq 1$, we have $dx(t)/dt \simeq -\omega_d x_\text{m} e^{-bt/2m}\sin(\omega_d t + \phi)$. Thus $[dx(t)/dt]_\text{max} \simeq -\omega_d x_\text{m}$. The maximum value of the damping force is then

$$f_\text{m} = -b\left(\frac{dx}{dt}\right)_\text{max} \simeq b\,\omega_d x_\text{m}\,.$$

The ratio in question is thus

$$\begin{aligned}\eta &\simeq \frac{f_\text{m}}{kx_\text{m}} \simeq \frac{b\omega_d x_\text{m}}{kx_\text{m}} = \frac{b\omega_d}{k} \simeq \frac{b\omega}{k} = \frac{b}{k}\sqrt{\frac{k}{m}} \\ &= \frac{70\times10^{-3}\,\text{kg/s}}{\sqrt{(0.25\,\text{kg})(85\,\text{N/m})}} = 0.015\,.\end{aligned}$$

(b) Since $\eta \simeq b/\sqrt{km}$, it should not change appreciably.

89P

The value of k is

$$k = \frac{m'g}{x_\text{m}} = \frac{(2000\,\text{kg})(980\,\text{cm/s}^2)}{4(10\,\text{m})} = 490\,\text{N/cm}\,,$$

where $m' = m/4$ (since there are four wheels). From $A(T) = x_{\rm m}e^{-bT/2m'} = 0.50x_{\rm m}$, we get $e^{-bT/2m'} = 0.50$. Solve for b:

$$\begin{aligned} b &= \frac{2m}{T}\ln 2.0 = \frac{2m}{2\pi}\sqrt{\frac{k}{m'}}\ln 2.0 \\ &= \frac{2000\,\text{kg}}{\pi}\sqrt{(49000\,\text{N/m})(500\,\text{kg})}\ln 2.0 = 1100\,\text{kg/s}\,. \end{aligned}$$

90E

(a) Since $\omega = \omega_0$ at resonance, the amplitude is $x_{\rm m}|_{\omega=\omega_0} = F_{\rm m}/(b\omega_0)$.
(b) The velocity amplitude is $v_{\rm m}|_{\omega=\omega_0} = (\omega x_{\rm m})|_{\omega=\omega_0} = \omega_0[F_{\rm m}/(b\omega_0)] = F_{\rm m}/b$.

91P

The natural frequency of the car is

$$\omega_0 = \frac{2\pi v}{\Delta x} = \frac{2\pi(10\,\text{mi/h})(1\,\text{h}/3600\,\text{s})(5279\,\text{ft/mi})}{13\,\text{ft}} = 7.1\,\text{rad/s}\,.$$

Let the mass of the car be m_1 and that of the passengers be m_2. Then $\omega_0 = \sqrt{k/(m_1+m_2)}$, which gives

$$k = (m_1+m_2)\omega_0^2 = \left[\frac{2200\,\text{lb} + 4(180\,\text{lb})}{32.2\,\text{ft/s}^2}\right](7.1\,\text{rad/s})^2 = 4.6\times 10^3\,\text{lb/ft}\,.$$

Thus the car rises by

$$\Delta x = \frac{\Delta F}{k} = \frac{m_2 g}{k} = \frac{4(180\,\text{lb})(12\,\text{in./ft})}{4.6\times 10^3\,\text{lb/ft}} = 1.9\,\text{in.}\,.$$

92

(a) $m_1 = 288\,\text{g}$, $m_2 = 120\,\text{g}$;
(b) $477\,\text{g}$;
(c) $8.0\,\text{cm}$

93

(a) $y_{\rm m} = 8.8\times 10^{-4}\,\text{m}$, $T = 0.18\,\text{s}$, $\omega = 35\,\text{rad/s}$;
(b) $y_{\rm m} = 5.6\times 10^{-2}\,\text{m}$, $T = 0.48\,\text{s}$, $\omega = 13\,\text{rad/s}$;
(c) $y_{\rm m} = 3.3\times 10^{-2}\,\text{m}$, $T = 0.31\,\text{s}$, $\omega = 20\,\text{rad/s}$