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EVAPORATION FROM SHALLOW WATER TABLE THROUGH LAYERED SOIL PROFILES

by
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ABSTRACT

The purpose of this study is to estimate the steady state evaporation rates from layered soils in the presence of high water table under isothermal conditions. A finite difference numerical scheme based upon the one-dimensional Richards equation has been employed to estimate the evaporation rates from a two-layered soil profile overlying a shallow water for appropriate initial and boundary conditions. The method takes into account the relevant atmospheric factors and soil moisture characteristics of the two layers. The effects of sequence and thickness of the soil layers and water table depth on the evaporation rates have been examined.

KEY WORDS : Evaporation, Water table, Soil moisture, Hydraulic conductivity, Soil layers.

INTRODUCTION

It is desirable to estimate the water loss from soils by evaporation and the amount of ground water available to plants due to the upward movement of water from a water table. Soils may also become saline due to the upward movement of saline ground water and its subsequent evaporation at the soil surface. In order to determine what depth to water table should be maintained, the relation between depth to water table, soil properties and evaporation rate must be known.

Evaporation from shallow water table through a homogeneous soil profile has been studied theoretically and experimentally by many workers. However, uniform soil profiles rarely occur in nature. It is more common to find the soils having well-defined layers differing from each other either in texture or in structure. Therefore, it becomes necessary to determine the effect of layered soils on evaporation from a shallow water table.

The actual evaporation rate is governed by the atmospheric conditions, thickness and transmitting properties of the soil layers and the water table depth. While the maximum possible (potential) rate of evaporation from a given soil depends only on atmospheric conditions, the actual flux across the soil surface is limited by the ability of the porous medium water from below.

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The steady state upward flow of water from a water table through the soil profile to an evaporation zone at the soil surface was first studied by Moore (1939). Theoretical solutions of the flow equation for this process were given by several workers including Gardner (1958) and Anat et al. (1965). A theoretical analysis of steady evaporation from a two layered soil profile was carried out by Willis (1960). A more flexible treatment of steady state evaporation from mulitlayer profiles was developed by Ripple et al. (1972).

In the present study, an attempt has been made to examine the effect of layered soil profiles on steady state evaporation rates from a shallow water table under isothermal conditions by using a finite difference numerical scheme for solution of the one-dimensional Richards equation. The evaporation rates are shown to be related to the water table depth and sequence and thickness of the soil layers.

METHODOLOGY

General Equation of Unsaturated Flow

Using Darcy's law and continuity principle, the partial differential equation for one-dimensional vertical water movement in nonswelling soil can be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z} - 1\right)\right] \qquad (1)$$

where θ is the volumetric water content (cm^3/cm^3), K(h) the hydraulic conductivity of the soil (cm/h), h the soil water pressure (relative to the atmosphere) expressed in cm of water and z the gravitational head (cm) considered positive in downward direction.

With the specific water capacity, C defined as

$$C(h) = \frac{d\theta}{\partial h} \qquad (1/cm) \qquad (2)$$

Equation (1) can also be written as

$$C(h)\frac{\partial h}{\partial x} = \frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z} - 1\right)\right] \qquad (3)$$

Written in this form, Eq. (3) provides the basis for predicting soil water movement in non-hysteretic layered soils of which each layer may have different physical properties. It is a non-linear partial differential equation (known as Richards equation) because the parameters K(h) and C(h) depend on the actual solution of h(z, t). The non-linearity of the equation causes problems in its solution. Analytical solutions are known for special cases only. The majority of practical field problems can therefore only be solved by numerical methods.

Initial and Boundary Conditions

To obtain a solution for the one-dimensional vertical flow, Eq. (3) must be supplemented by appropriate initial and boundary conditions.

For the present study, the initial and boundary conditions have been defined as follows.

I. **Initial Condition :**

$$h(z,0) = h_0 \quad \text{for } z \geq 0, \qquad t = 0 \tag{4}$$

(approximate equilibrium moisture profile)

II. **Upper Boundary Condition :**

If the relative humidity (f) and the temperature of the air (T) as a function of time are known, and if it may be assumed that the pressure head at the soil surface is at equilibrium with the atmosphere, then h(0, t) can be derived from the thermodynamic relation (Edlefson and Anderson, 1943) :

$$h(0,t) = \frac{RT(t)}{Mg} \; ln\,[f(t)] \tag{5}$$

where R is the universal gas constant (8.314 x 10^7 erg/mole/K), T is the absolute temperature (K), g is acceleration due to gravity (980.665 cm/s^2), M is the molecular weight of water (18 gm/mole), f is the relative humidity of the air (fraction) and h is in bars. Knowing h(0, t), $\theta(0, t)$ can be derived from the soil water retention curve.

III. **Lower Boundary Condition :**

The phreatic surface acts as lower boundary of the system in case of evaporation from shallow water table. The lower boundary condition has therefore set as

$$h(z = L,\; t) = 0 \tag{6}$$

where L is the depth of the ground water table.

Soil Moisture Characteristics

For the present study, functional relations, as reported by Haverkamp et al. (1977), for characterising the hydraulic properties of two soils, sand and yolo light clay, were used. The following analytical expressions were used for characterising the sand :

$$K = K_s \frac{A}{A + |h|^{\beta_1}} \; ; \tag{7}$$

K_s = 34 cm/h

A = 1.175 x 10^6

β_1 = 4.74

and

$$\theta = \frac{\alpha(\theta_s - \theta_r)}{\alpha + |h|^{\beta_2}} + \theta_r \tag{8}$$

θ_s = 0.287

θ_r = 0.075

α = 1.611 x 10^6

β_2 = 3.96

where subscript s refers to saturation, i.e. the value of θ for which h = 0, and the subscript r to residual water content.

The soil characteristics of Yolo light clay are given in Eqs. (9) and (10), using the same representations as in the previous case. The data points for (h) were taken from Philip (1969, pp. 221). Values for K(h) were presented by Philip (1957, pp. 353). Points for K(h) were determined from (h) and K_s, as reported by Haverkamp et al. (1977).

$$K = K_s \frac{A}{A + |h|^{\beta_1}} \tag{9}$$

K_s = 4.428 x 10^{-2} cm/h

A = 124.6

β_1 = 1.77

and $$\theta = \frac{\alpha(\theta_s - \theta_r)}{\alpha + (\ln|h|)^{\beta_2}} + \theta_r \quad \text{for } h < -1 \text{ cm} \tag{10}$$

θ_s = 0.495

θ_r = 0.124

α = 739

β_2 = 4

$\theta = \theta_s$ for h ≥ -1 cm

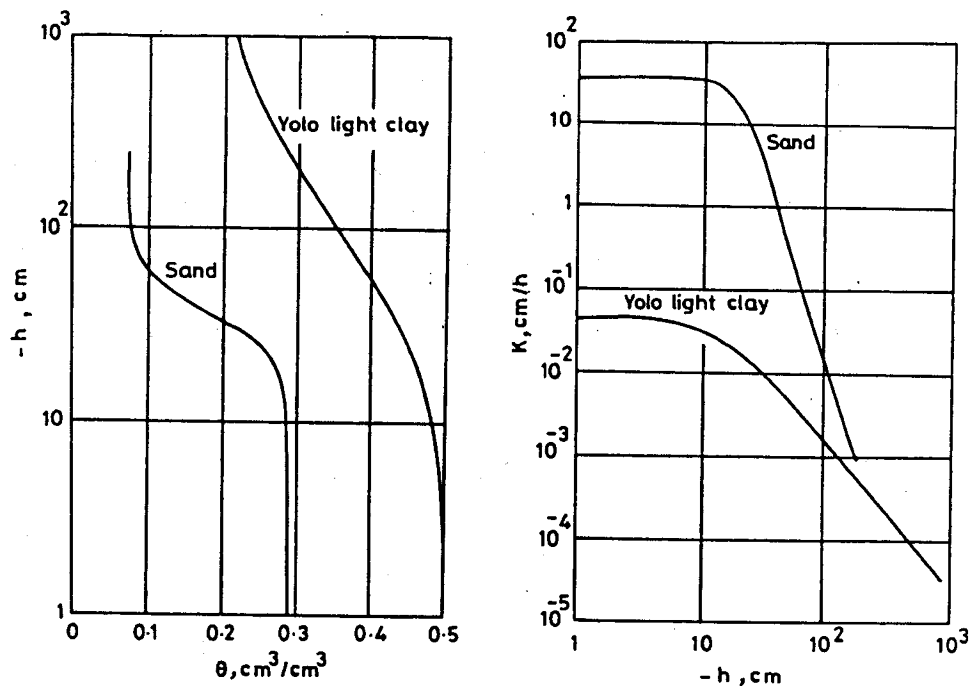

FIG. 1 : RELATIONSHIPS BETWEEN THE SOIL WATER PRESSURE h, THE WATER CONTENT θ AND THE HYDRAULIC CONDUCTIVITY K FOR THE TWO SOILS USED IN THE STUDY

The functional relationships represented by the above equations describe fairly well the data, as tabulated by Philip (1957, 1969), except near h = -100 cm for the K(h) curve.

Figure 1 presents the relationships between the soil water pressure h, the water content and the hydraulic conductivity K for the two soils used in this study. These relations were used to estimate the evaporation rates from various combinations of layered soil profiles.

Finite Difference Approximation

Equation (3) can be solved by a finite difference technique and appropriate initial and boundary conditions. The values of the coefficients C(h) and K(h) can be expressed from a prediction at time (t + 1/2 t) using a method described by Douglas and Jones, 1963 (implicit lineariszation). From Eq. (3), we

$$C\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}[K(\frac{\partial h}{\partial z}-1)]$$

or

$$C\frac{\partial h}{\partial t} = \frac{\partial K}{\partial z}(\frac{\partial h}{\partial z}-1)+K\frac{\partial^2 h}{\partial z^2}$$

or

$$\frac{C}{K}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial z^2}+\frac{1}{K}\frac{\partial K}{\partial z}(\frac{\partial h}{\partial z}-1) \qquad (11)$$

Prediction (estimation of C_i^j and K_i^j)

From Eq. (11), by taking time step as t/2, we have

$$\frac{2C_i^j}{K_i^j}\cdot\frac{h_i^{j+1/2}-h_i^j}{\Delta t} = \frac{h_{i+1}^{j+1/2}-2h_i^{j+1/2}+h_{i-1}^{j+1/2}}{(\Delta z)^2}+\frac{1}{K_i^j}\cdot\frac{K_{i+1}^j-K_{i-1}^j}{2\Delta z}\left[\frac{h_{i+1}^j-h_{i-1}^j}{2\Delta z}-1\right]$$

where i refers to depth and j refers to time. Rearranging the terms, we get

$$-\frac{\Delta t}{(\Delta z)^2}h_{i-1}^{j+1/2}+\left[\frac{2C_i^j}{K_i^j}+\frac{2\Delta t}{(\Delta z)^2}\right]h_i^{j+1/2}-\frac{\Delta t}{(\Delta z)^2}h_{i+1}^{j+1/2}$$

$$=\frac{2C_i^j}{K_i^j}h_i^j+\frac{1}{2}\frac{K_{i+1}^j-K_{i-1}^j}{K_i^j}\frac{\Delta t}{\Delta z}\left[\frac{h_{i+1}^j-h_{i-1}^j}{2\Delta z}-1\right] \qquad (12)$$

Correction (estimation of h_i^j)

From Eq. (11), by taking time step as t, we have

$$\frac{C_i^{j+1/2}}{K_i^{j+1/2}}\cdot\frac{h_i^{j+1}-h_i^j}{\Delta t} = \frac{1}{2}\left[\frac{h_{i+1}^{j+1}-2h_i^{j+1}+h_{i-1}^{j+1}}{(\Delta z)^2}+\frac{h_{i+1}^j-2h_i^j+h_{i-1}^j}{(\Delta z)^2}\right]$$

$$+\frac{1}{K_i^{j+1/2}}\cdot\frac{K_{i+1}^{j+1/2}-K_{i-1}^{j+1/2}}{2\Delta z}\left[\frac{h_{i+1}^{j+1/2}-h_{i-1}^{j+1/2}}{2\Delta z}-1\right]$$

rearranging the terms, we get

$$-\frac{1}{2}\frac{\Delta t}{(\Delta z)^2}h_{i-1}^{j+1}+\left[\frac{C_i^{j+1/2}}{K_i^{j+1/2}}+\frac{\Delta t}{(\Delta z)^2}\right]h_i^{j+1}-\frac{1}{2}\frac{\Delta t}{(\Delta z)^2}h_{i+1\,c}^{j+1}$$

$$=\frac{C_i^{j+1/2}}{K_i^{j+1/2}}h_i^j+\frac{1}{2}\frac{\Delta t}{(\Delta z)^2}\left[h_{i+1}^j-2h_i^j+h_{i-1}^j\right]$$

$$+\frac{1}{2}\frac{K_{i+1}^{j+1/2}-K_{i-1}^{j+1/2}}{K_i^{j+1/2}}\frac{\Delta t}{\Delta z}\left[\frac{h_{i+1}^{j+1/2}-h_{i-1}^{j+1/2}}{2\Delta z}-1\right] \quad (13)$$

When Eqs. (12) and (13) are applied at all nodes, the result is a system of simultaneous linear algebraic equations with a tridiagonal coefficient matrix with zero elements outside the diagonals and unknown values of h. In solving this system of equations, the so-called direct method was used by applying a tridiagonal algorithm of the kind discussed by Remson et al. (1971).

Estimation of Evaporation Rates

Steady state evaporation rates from the layered profiles were estimated by applying Darcy's equation for two vertically adjacent nodal points after obtaining the equilibrium moisture profile for the given set of water table depth and thickness of the top soil layer.

$$q = K_{i+1/2}^j\left(\frac{h_{i+1}^j-h_i^j}{\Delta z}-1\right) \quad (14)$$

where, $K_{i+1/2}^j=\sqrt{K_i^j\ K_{i+1}^j}$

Geometric mean of K was taken following suggestions of Haverkamp and Vauclin (1979). Various sets of water table depths and thickness of the two soil layers were considered for the study.

ANALYSIS AND RESULTS

The numerical model described earlier was tested by comparing water content profiles calculated at given times with results obtained from quasi-analytical solution of Philip subject to condition of a constant pressure at the soil surface (θ = 0.267 cm^3/cm^3). The infiltration profiles at various times for infiltration in the sand (one of the two soils under consideration) obtained by quasi-analytical solution of Philip, were reported by Haverkamp et al. (1977). The model yielded good agreement with water content profiles at various times (Kumar and Mishra, 1991).

The present study was carried out for bare surface (i.e. no vegetation) and therefore transpiration by plants was not taken into account. The sub-surface profile was divided into uniform layers of thickness 4 cm each (depth interval, Δz) down to the water table position which was varied from 60 cm to 140 cm below the soil surface.

Keeping in view the stability of the numerical scheme, the time step, Δt was taken as 40 seconds during the entire study period. Uniform evaporative conditions (temperature = 25°C, relative humidity = 0.75) were assumed for the study. The value of potential evaporation was obtained through Meyer's equation (for temperature = 25°C, relative humidity = 0.75 and wind speed = 10 miles/hour) as 5.99 mm/day. Therefore, the maximum limit of evaporation from soil surface was imposed as the equivalent 0.025 cm/hour.

For the given external evaporative conditions, water table depth and the layered soil profiles, the equilibrium moisture profile was obtained by using the numerical scheme presented in the section 'Finite Difference Approximation', and assuming the pressure head at the soil surface to be in equilibrium with the surrounding atmosphere ($h(0, t) = -396.14$ cm). The initial and boundary conditions were defined by the Eqs. (4), (5) and (6) respectively. The rate of loss of water (Darcian flux q) served as a measure of the evaporation rate, once the steady state was attained. Ample time was allowed for steady state to be attained. The steady values of soil water pressures at different nodes during consecutive time steps gave assurance that steady state had been attained. At steady state, the rate of loss of water (the flux q), which is approximately the same at every depth, equals the evaporation rate. The evaporation rates under each set of water table depth and soil layerings were evaluated by using computer program written in FORTRAN.

In order to examine the possible behaviour of layered profiles consisting of sand and Yolo light clay, the dependence of steady evaporation rate upon top layer thickness was determined for various ground water depths. The computed results are given in Tables 1 and 2. The variations of evaporation rates from layered profiles as a function of top layer depth and three water table depths are also presented in Figs. 2 and 3 for the two soil layering sequences. The evaporation rates for homogeneous soil profiles (i.e., top layer thickness = 0) are also indicated for comparison.

For the case of Yolo light clay over sand (Table 1 and Fig. 2), the top layer was found to have little effect on the evaporation rate when ground water is at a depth of 140 cm. However, for ground water depth of 120 cm, the evaporation rate increases markedly as the top layer thickness increases. For ground water depth of 100 cm, the evaporation occurs at potential rate (6 mm/day) for the top layer thickness greater than 20 cm.

TABLE - 1

STEADY STATE EVAPORATION RATES FROM LAYERED PROFILES OF YOLO LIGHT CLAY OVER SAND

S. No.	Thickness of Top Layer (cm)	Evaporation Rates (mm/day)		
		L = 100 cm	L = 120 cm	L = 140 cm
1	0	4.030484	1.699340	0.814898
2	6	4.462968	1.875703	0.899408
3	10	4.840748	2.010576	0.958741

S. No.	Thickness of Top Layer (cm)	Evaporation Rates (mm/day)		
		L = 100 cm	L = 120 cm	L = 140 cm
4	14	5.284893	2.161296	1.019669
5	18	5.809570	2.331051	1.086894
6	22	6.000000	2.522819	1.160336
7	26	6.000000	2.740535	1.240665
8	30	6.000000	2.985332	1.329966
9	34	6.000000	3.267940	1.429026
10	38	6.000000	3.590603	1.537989
11	42	6.000000	3.966448	1.660899
12	46	6.000000	4.400203	1.800100
13	50	6.000000	4.906572	1.955429

TABLE - 2

STEADY STATE EVAPORATION RATES FROM LAYERED PROFILES OF SAND OVER YOLO LIGHT CLAY

S. No.	Thickness of Top Layer (cm)	Evaporation Rates (mm/day)		
		L = 60 cm	L = 80 cm	L = 100 cm
1	0	1.833106	1.088800	0.705278
2	6	1.685597	0.992043	0.647764
3	10	2.307044	0.954659	0.590985
4	14	4.937681	1.200225	0.582285
5	18	6.000000	1.720937	0.644286
6	22	6.000000	2.418169	0.766910
7	26	6.000000	3.202290	0.931421
8	30	6.000000	4.005234	1.119970
9	34	6.000000	4.789237	1.321853
10	38	6.000000	5.540245	1.526910
11	42	6.000000	6.000000	1.730477
12	46	6.000000	6.000000	1.930086
13	50	6.000000	6.000000	2.122212

For the case of sand overlying Yolo light clay (Table 2 and Fig. 3), the evaporation rate is seen to fall marginally upto top layer thickness of around 14 cm, for ground water depth of 100 cm. With increase in the top layer thickness beyond 14 cm, the evaporation rate increases gradually. If the top layer depth is increased upto 100 cm, then it will correspond to zero top layer depth (Yolo light clay over sand) in Fig. 2. For ground water depth of 80 cm, the evaporation rate marginally falls upto top layer thickness of around 10 cm, increases steeply thereafter and attains potential rate at top layer thickness of around 42 cm. For ground water depth of 60 cm, the effect of top layer thickness becomes more pronounced, the decrease in rate being upto top layer thickness of around 5 cm and attaining potential rate at top layer thickness of only 18 cm.

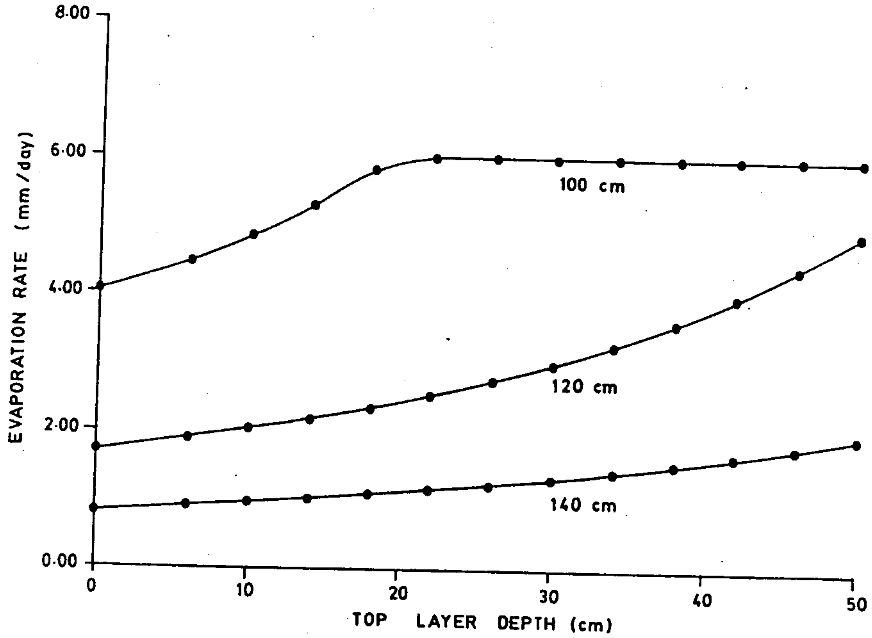

FIG. 2 : EVAPORATION RATES FROM LAYERED PROFILES OF YOLO LIGHT CLAY OVER SAND AS A FUNCTION OF TOP LYER DEPTH AND THREE WATER TABLE DEPTHS

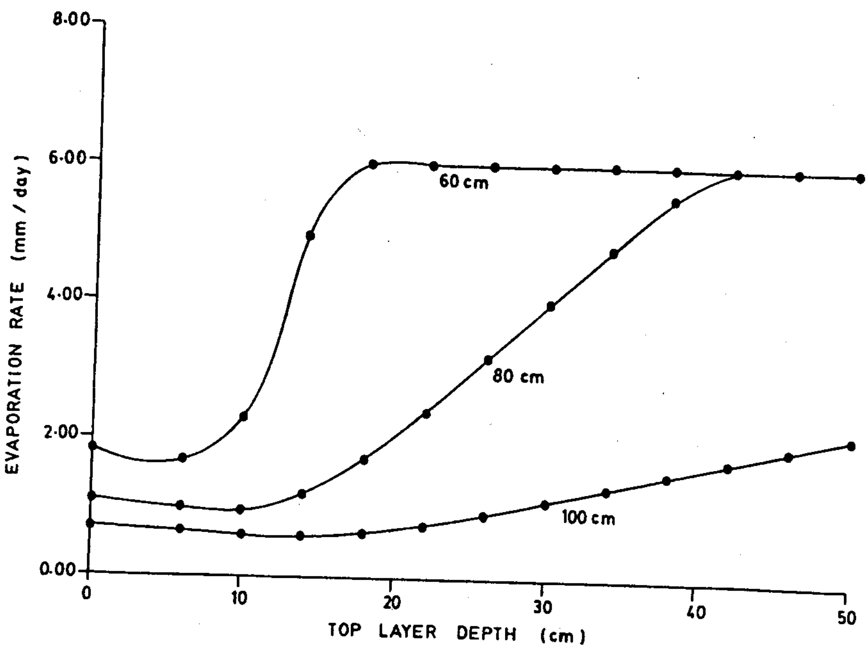

FIG. 3 : EVAPORATION RATES FROM LAYERED PROFILES OF SAND OVER YOLO LIGHT CLAY AS A FUNCTION OF TOP LAYER DEPTH AND THREE WATER TABLE DEPTHS

Figures 2 and 3 also indicate that for deeper water tables, the evaporation rates were independent of the external evaporativity (6 mm/day). For shallower water tables, the evaporation rates followed the external "potential" rate as the top layer thickness was increased. It should be emphasized that the above results have not been subjected to empirical testing in the laboratory and field.

The significance of these results is that the presence of a fine-textured soil overlying a coarse-textured one, may increase the evaporative water loss for a given ground water depth. This effect will become more pronounced as the ground water becomes shallower. It is due to the higher unsaturated hydraulic conductivity of the finer material at the high suction values which prevail near the surface of the soil profile during evaporation. If the sequence of layers is a coarse-textured soil overlying a fine-textured one, the evaporative loss can be slightly reduced in case of thin top layers for corresponding water table depths. The effect of a tilled zone on the evaporation loss, as compared to a homogeneous soil profile, can thus be anticipated. The tilled top layer acts as if the soil has a coarse-textured layer on top, even though its texture is homogeneous. This implies that tillage operations can reduce evaporation and salinization to some extent in regions having shallow water tables.

Steady state conditions were assumed in this study, though the numerical model permits variable climatic parameters i.e. temperature and relative humidity. In nature, however, the systems considered are seldom in a steady state condition, principally because of the variations in meteorological conditions, soil salt content and water table depth. The changes in soil salt content and water table depth are relatively slow and therefore their short-period effects might be negligible. Their long-range influences, however, could be of considerable importance and should be taken into account, with different experimentally determined soil parameters and measured or predicted water table depths.

CONCLUSIONS

The evaporation rates are shown to be related to the sequence and thickness of soil layers, their hydraulic properties and water table depths. It was found that soil layering reduces evaporation only marginally when a thin layer of coarse-textured soil overlies a fine-textured soil, as compared to a homogeneous profile. However, evaporation increases for thicker top layers of coarse-textured soil. The soil layering also increases the evaporation in case of fine-textured soil overlying a coarse-textured soil. The effect of layering was found to be more pronounced for shallow ground water depths.

The dependence of the actual steady state evaporation rate on water table depth and soil layerings, might be very useful in hydrologic practice. The extent, to which the above results can be applied quantitatively to the field, depends upon the correspondence between the input values of soil moisture characteristics for each layer and those existing in the field.

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NOTATIONS

C	=	Specific water capacity
f	=	Relative humidity of the air (fraction)
g	=	Acceleration due to gravity
h	=	Soil water pressure
i	=	Refers to depth
j	=	Refers to time
K	=	Hydraulic conductivity of the soil
K_s	=	Hydraulic conductivity at saturation
M	=	Molecular weight of water
q	=	Flux (evaporation rate)
R	=	Universal gas constant
t	=	Time
Δt	=	Time step
T	=	Absolute temperature (^{0}K)
z	=	Vertical distance from the soil surface downward (i.e., the depth)
Δz	=	Depth interval
θ	=	Volumetric water content
θ_r	=	Residual water content
θ_s	=	Saturated water content