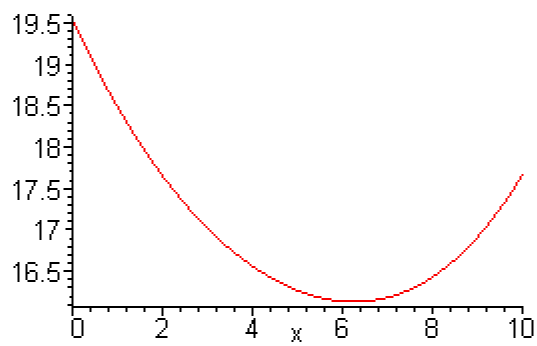
It is without question that Quantum Mechanics has posed as many questions as answers since it was established a century ago. With the introduction of the Schrödinger Wave Equation, questions began to surface regarding the interpretation of the solution Ψ and how it related to the mechanics of a particle in an atom. Born interpreted the function as a measurement of probability of localizing a particle in a region of space. $|\Psi^2|$ gives a probability distribution, indicating regions where the particle most likely exists.

Consider for the moment Fermat's Principle of least time. When light is transmitted from a medium with refractive index n_1 to one with index n_2, there are an infinite number of possibilities for the trajectory of the beam. Choosing the initial point as $(x_i, 0, z_i)$ and the final point as $(x_f, 0, z_f)$, one develops an equation describing the time required for light to travel through the medium:

$$t = \frac{n_1\left(\sqrt{(x - x_i)^2 + z_i^2}\right) + n_2\left(\sqrt{(x_f - x)^2 + z_f^2}\right)}{c}$$

which yields the following graph:



Fermat's Principle states that the path light travels in the medium is that which minimizes the time required to transmit through the medium. So, from the graph, we see the minimum of the parabola corresponds to the path taken by the light.

Let us analyze how we first generated this graph. We wished to see how light would travel from point A to point B. We didn't know explicitly which direction it would take, so we had to generate a list of all possible paths, and then judge from the outcome how light would propagate through the medium. Further analysis of the graph reveals that at regions close to the minimum, the states of the photons are similar. This is due to the constructive interference of the similar disturbances. In regions which require a longer propagation time, destructive interference due to dissimilar disturbances dominates.

This is precisely the purpose of the wave function Ψ. It provides a list (or set) of all possible states of a particle, and when plotted, reveals regions where a particle is more likely to be found. Ψ is an indication of the instantaneous state configuration of a particle.

Let us consider some key points from statistical mechanics:

1) A group of particles can be organized to be distributed among different states.
2) A system's particles exist in all different states at once.
3) Certain states contain a greater number of particles than others.
4) The particles in all states are constantly moving and changing.

These main ideas from statistical mechanics can be transformed into quantum mechanics rather successfully, if one considers a single particle as a system, and the different physical properties (observables) to be the individual parts of the system. Then, the main results for quantum mechanics will be

1) The values of observables can be distributed among different states.
2) The possible values of the observables occupy many different states at once.
3) Certain states contain a greater number of similar-valued observables.
4) The values of the observables in each state are constantly changing.

The "state space" for the particle must span both the temporal and spatial planes, since the distribution will change over time, and space. Thus, for a single particle at any given place and time, there is a unique distribution of observables which change over time.

Postulate 1

The locations where the observables are most similar in value indicate regions where the particle occupies much of it's time.

Postulate2

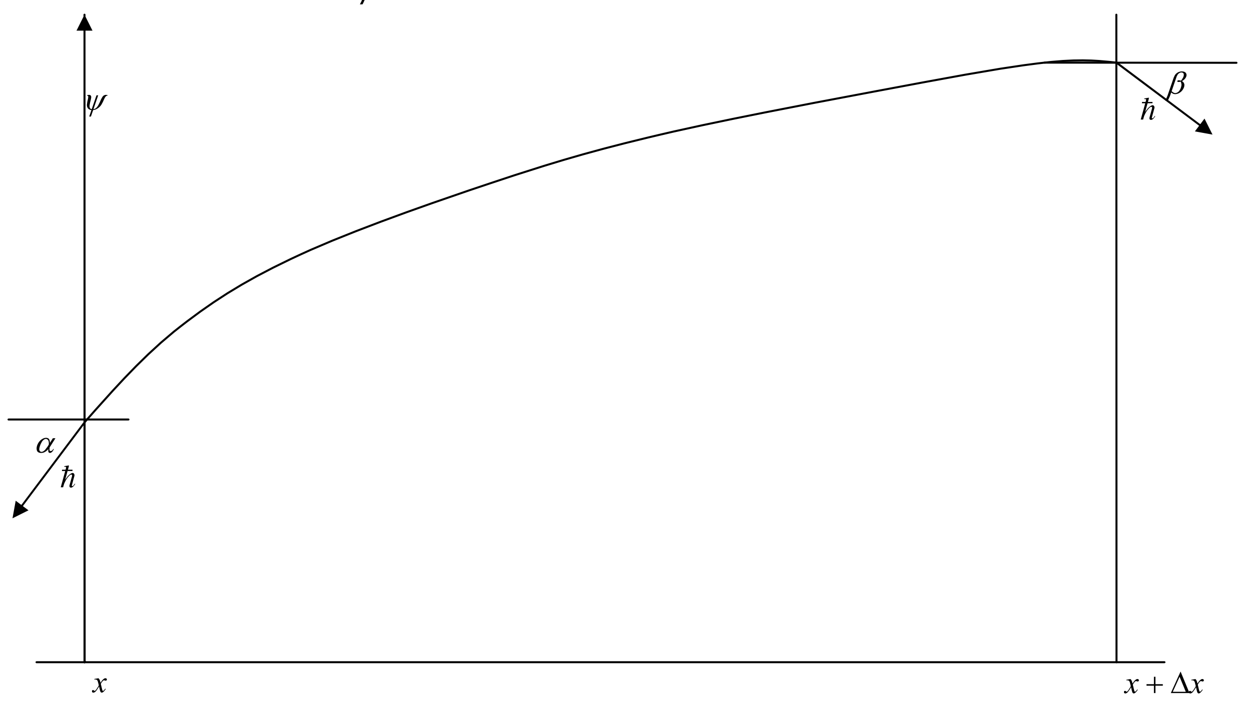
The wave function describes the changes in observables of a particle throughout time and space. The wave nature of the function suggests a periodic, repeating pattern of such changes, according to the energy of the particle and any external interactions. Thus, there is a pattern to the changes in observables which form a particle's distinct quantum signature. In other words, the quantum signature describes the natural progression in the values of certain observable quantities of a particle. The particle behaves in such a way that the action involved in the evolution of these changes is constant and quantized. This action $\hbar$ ensures that the signature will be formed in precisely the right way. In other words, a particle behaves in such a way that the action involved in changing its observable quantities is $\hbar$.

We start by analyzing the case of no external potential interactions. Let S represent the action along each stage of the signature. Here we will make an assumption:

Postulate 3

The action along the signature is a constant and uniform along the entire length. This constant action is the quanta of action $\hbar$.

Consider the portion of the quantum signature between two points A and B, located at x and $x+\Delta x$. Only the vertical components of the action are considered, since there is no evolution of the signature in the horizontal direction. The vertical components of the action are $-\hbar \sin\alpha$ and $\hbar \sin\beta$.



Postulate 4

The equation describing the evolution of the quantum signature $\psi(x,t)$ is given by

$$\left(\rho_1 \Delta x\right)\frac{d\psi(x,t)}{dt} = -\hbar\sin\alpha + \hbar\sin\beta + \left(\rho_2 \Delta x\right)V\left(x,t\right)\psi\left(x,t\right)$$

Where $V(x,t)$ is an external potential.

Thus, the potential $V(x,t)$ serves to "drive" the progression of the quantum signature, just as the classical example of a forced vibration in a string.

Using $\tan\alpha = \dfrac{\partial\psi(x,t)}{\partial x}$ and $\tan\beta = \dfrac{\partial\psi(x+\Delta x,t)}{\partial x}$,

Which yields

$$\rho_1 \frac{\partial\psi(x,t)}{\partial t} = \hbar \frac{\partial^2\psi(x,t)}{\partial x^2} + \rho_2 V(x,t)\psi(x,t)$$

or

$$\frac{\rho_1}{\rho_2}\frac{\partial\psi(x,t)}{\partial t} = \frac{\hbar}{\rho_2}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t)$$

With $\rho_2 = -\dfrac{2m}{\hbar}$ and $\rho_1 = -2mi$, the familiar one-dimensional Schrödinger Wave Equation emerges:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t)$$