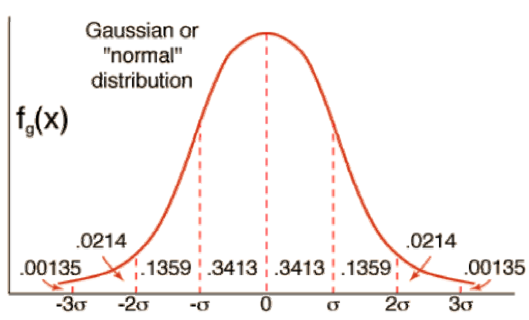
Before one can begin to understand quantum mechanics, it is beneficial to study the results from statistical mechanics. While quantum mechanics deals with the behaviour of isolated particles, statistical mechanics summarizes the behaviour of a collection of many particles (in the order of 10^{23} !). Basically, statistical mechanics is a form of "book-keeping". It considers all possible states that a system of particles can be in, then makes quantitative measurements of the tendency of the particles to be in one particular state rather than the other.

Quantum mechanics is simply a special case of statistical mechanics; actually, it is the limiting case: the absolute minimum extreme for which the results of statistical mechanics can be applied. Instead of results describing the possible state configurations of a **system of particles**, quantum mechanics describes all possible state configurations of **individual particles**, or smaller groups of particles.

So, can the successful theory of statistical mechanics (from which we get an understanding of semi-conductors, Bose-Einstein condensation and more), be married to the fundamental theory of quantum mechanics (from which we gain knowledge of atomic structure and tunnelling effects)? The short answer is: **they'd better!** For two theories to be considered successful in terms of physical accuracy there must be continuity between them; the one must give way to the other (and vice-versa) in a complimentary way. For instance, the theory of Special Relativity and Newtonian/Lagrangian mechanics are complimentary in the case of "slow" motion (i.e. at speeds much less than 2.99×10^8 m/s). So too, must the two theories of quantum and statistical mechanics be complimentary.

Let's delve into the world of statistical mechanics for just a moment. Imagine a large system of particles. Now, imagine describing the behaviour of every single particle at every single place and time. Unless you have a lot of free time (and energy), I doubt you'd be up to the challenge. However, you can take a step back and observe the overall system as a whole. We should all be familiar with the distribution pattern for a purely random system. Randomness means simply the absence of any predetermined behaviour or outcomes. As far as the particles are concerned, they have no consciousness, and so they simply exist and move about **randomly**. For any random system, the distribution of outcomes will follow the Gaussian curve which looks like:



The function which generates this curve is given by

$$f_g(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(x-a)^2}{2\sigma^2}}$$

Thus, we can safely conclude that there will be a similar exponential function describing a system of many particles in statistical mechanics. An attractive way of defining a system is in terms of its energy. If there is an overall energy τ of a particular configuration, then the distribution of particles with energy ε can be expressed as $f(x) = Ae^{-\frac{\varepsilon}{\tau}}$. This is known as the **Boltzmann Distribution** and the factor $e^{-\frac{\varepsilon}{\tau}}$ is termed the "**Boltzmann factor**". This factor gives us the relative frequency of finding a ***single particle*** with energy ε in a ***system of particles*** with thermal kinetic energy τ. It shows us that a system prefers to be in a lowest energy state, with the likelihood of finding highly energetic particles being extremely low (but not impossible!). This hints at the reason why "absolute zero", the phenomenon in which all thermal motion ceases, is unattainable: It is impossible to have all (fermion) particles of a system occupying the same low-energy state. ***There is always a distribution of particles in different states***.

Of course, it is rather restricting to consider only one particle with energy ε. It would be much better to have a way of considering all particles of a system, each with their own energies. This is where the **Partition Function** makes its debut. The partition function is a summation of all the Boltzmann Factors, and this stores all the "information" about all the different possible states of a system of N particles. Its expressed as $Z = \sum_{i=1}^{N} e^{-\frac{\varepsilon_i}{\tau}}$. Here, ε_i indicates the energy of an individual particle. Finally, the probability of finding a particle in an energy state ε_i in a system of thermal energy τ is

$$P(\varepsilon_n) = \frac{e^{-\frac{\varepsilon_i}{\tau}}}{\sum_{i=1}^{N}\left(e^{-\frac{\varepsilon_i}{\tau}}\right)}$$

Essentially, we are done with statistical considerations. All other applications arise from manipulations of the Boltzman Factors and the Partition Function. We are now ready to take these results and relate them to quantum mechanics.

While statistical mechanics can be thought of as the ***mechanics of the very many***, quantum mechanics can be seen as the ***mechanics of the very few***.