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A simplified homogenisation method for composite soils

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Abstract

Composite soil may consist of composite ground made up of soft soil reinforced by stiff soil columns or mixed soil made up of thoroughly mixed soft and stiff soils. In general, the behavior of composite soil is not well understood due to non-homogeneous structure of soil matrices. In this paper, a simplified homogenisation method is proposed to model the behavior of mixed soil and composite ground. It assumes that micro-stress/micro-strain is homogeneous in the matrix and the reinforcement of a composite soil and a localization tensor is developed. A microstructure knowledge-based model is proposed to distribute the micro-stress/micro-strain in each phase. This assumption largely simplifies the numerical procedure at the unit cell of composite soils. The validity of the proposed method is verified by comparing the predicted parameters with existing experimental data on mixed soils and composite grounds. Examples show that the proposed method can be applicable to well-randomly mixed soil and oriented composite ground. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Homogenisation method; Stress ratio; Localization tensor; Mixed soil; Composite ground; Deformation modulus; Strength

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Nomenclature

$\hat{\mathbf{A}}$	homogenized compliance matrix
B	bulk modulus
b_p	stress ratio for mean stress
b_s	homogenized stress ratio
$\mathbf{C}$	stress localization tensor.
C	electrical conductivity
C_c	compression index of the soil
$\mathbf{D}$	strain localization tensor.
$\tilde{\mathbf{E}}$	homogenized stiffness matrix
E	Young's modulus
E_i	initial modulus
E_{ijkl}	stiffness of the matrix material.
E_{50}	deformation modulus
e	void ratio
e_0	initial void ratio
F	clay fraction by dry weight
f	volume fraction of inclusion
f'	volume factor
f_s	volume factor
FR	critical content ratio of sand by dry weight
$\mathbf{I}$	fourth order unit tensor.
K	modulus number
K_b	bulk modulus number
M	critical state parameter
m	bulk modulus exponent
m_v	coefficient of compressibility
n	modulus exponent
n_j	outer normal unit vector of domain V
P_a	atmospheric pressure
p^0	mean stress
q^0	generalized shear stress
t_i	uniform traction.
u_i	component of a displacement vector
u_i^*	fluctuation part of a displacement vector.
V	domain for a unit cell
$\|V\|$	volume of domain V
∂V	boundary of domain V
V_{per}	domain with periodic condition
$\mathbf{W}$	characteristic function when $\boldsymbol{\varepsilon}^0$ is applied
$\hat{Y}$	$\left\{y_i \in V \mid \sigma_{ij}^0 n_j = t_i\right\}$,
	$\tilde{\mathbf{Y}} = \left\{y \in V \mid \varepsilon_{ij}^0 y_j = u_i^0\right\}$
y_j	jth component of local co-ordinate $\mathbf{y}$
σ	compressive stress
σ_3'	effective confining pressure
σ_{ij}	components of stress tensor
$\langle\sigma_{ij}\rangle$	volume average stress in a unit cell or macro-stress
$\boldsymbol{\sigma}^0$	macro-stress
$\boldsymbol{\sigma}^*$	fluctuation stress
ε_{kl}	components of strain tensor
$\langle\varepsilon_{kl}\rangle$	volume average strain in a unit cell
$\boldsymbol{\varepsilon}^0$	macro-strain
$\boldsymbol{\varepsilon}^*$	fluctuation strain
$\boldsymbol{\chi}$	characteristic function when $\boldsymbol{\sigma}^0$ is applied
ϕ	internal friction angle

Subscripts

f	reinforcement phase
m	matrix phase

Superscripts

d	series model
s	parallel model
h	homogenized model

1. Introduction

Composite soil is a commonly used soil improvement technique in which the in-situ soft soil is reinforced by the insertion of stiff soil. In some cases, soft soil is reinforced by deep mixing, stone columns or sand compaction piles. Such composite soil matrix would typically consist of soft clay reinforced by stiff soil columns installed at a regular spacing as shown in Fig. 1(a). This improved ground is commonly termed as composite ground. In other cases, the soil matrix consists of thoroughly mixed soft and stiff soils as shown in Fig. 1(b) and this is commonly termed as mixed soil. The study on the behaviour of composite soils has been an interesting topic in geotechnical engineering. For example, Refs. [1–5] investigated the mechanical properties of mixed soil using conventional uniaxial or triaxial compression tests. On the other hand, numerical or analytical approaches [6–9] have been proposed to predict non-linear behaviour of stone-column reinforced foundation by considering micro-equilibrium and deformation compatibility. Up to date, the

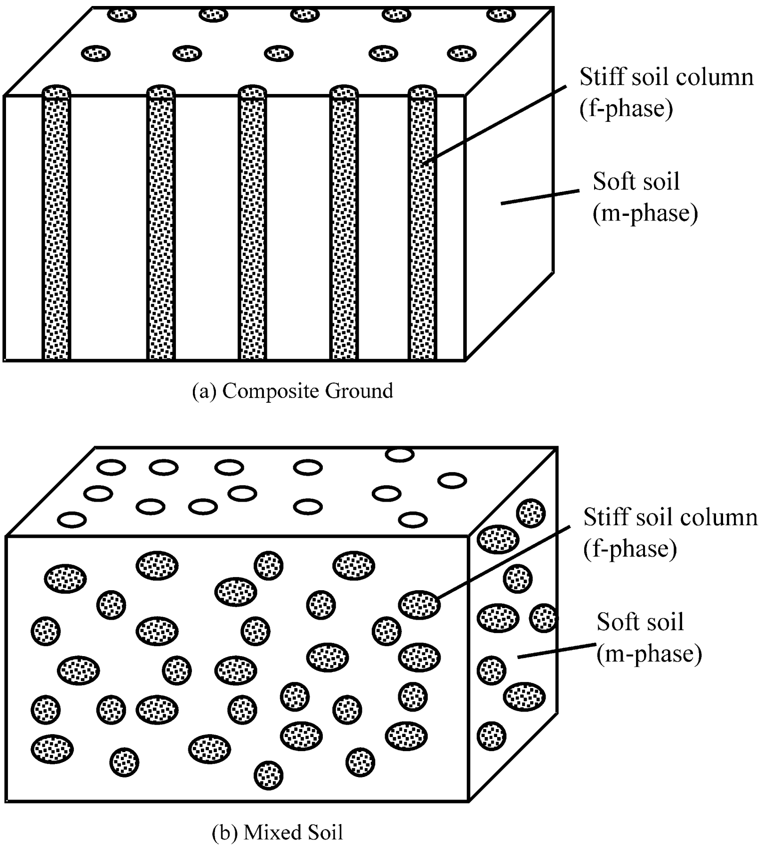

Fig. 1. Composite soils.

mechanical behavior of mixed soils is still not well understood due to non-homogeneous microstructures of the soil matrices.

Mathematicians developed a homogenisation method to understand the macro-behaviors of composite materials with microstructures. Sanchez-Palencia [10] and Bakhvalov and Panasenko [11] summarized the early applications of homogenisation method in science and engineering fields. Significant progress has been made recently in both theoretical and application aspects of the homogenisation method. These include the theoretical convergence of asymptotic (see for example [12]), applications in solid mechanics (see for example [13]), seepage problems (see for example [14]) and ground engineering [15]. However, relatively few studies had been carried out using the homogenisation method for the nonlinear or elastoplastic deformation of composite soils. Up to date, the majority of theoretical developments on the stress–strain behavior of composite ground and mixed soils have been either empirical or very complex and tedious.

In this paper, a simplified homogenisation method is proposed to predict the behavior of mixed soil and composite ground. A microstructure knowledge-based model is proposed to determine the model parameters. The proposed method is verified using experimental data on composite alloys, randomly mixed soils and composite ground.

2. Homogenisation method and homogenized moduli

In order to understand the key point of the homogenization procedure, let us consider a unit cell of composite soils consisting of the matrix material (termed *m*-phase) and the reinforcement material (termed *f*-phase) as shown in Fig. 1. This unit cell is a representative microstructure of the soil mass. It is a small boundary-value problem if boundary conditions are given. The *micro-equilibrium equation* that governs the unit cell is given as

$$\frac{\partial \sigma_{ij}}{\partial y_j} = 0 \tag{1}$$

where σ_{ij} are the components of the stress tensor and y_j is the *j*th component of local co-ordinate $\mathbf{y}$. As a unit cell comprises of different materials, the *micro-constitutive law* that governs each material or phase in a unit cell is given by

$$\sigma_{ij} = E_{ijkl}\varepsilon_{kl} \tag{2}$$

where σ_{ij}, ε_{kl} are the components of stress and strain tensor respectively and E_{ijkl} is the stiffness of the material. On the other hand, the macro-stress and macro-strain on the macro-level are directly associated with the global analysis of a geotechnical problem. On the macro-level, a unit cell is just regarded as a point with a homogenized constitutive law. The macro-stress, σ_{ij}^0, is usually defined as the volume average stress in a unit cell, $\langle \sigma_{ij} \rangle$, as follow:

$$\sigma_{ij}^0 = \langle \sigma_{ij} \rangle = \frac{1}{|V|} \int_V \sigma_{ij} \mathrm{d}V \tag{3}$$

where V is the domain for a unit cell, and $|V|$ is its volume.

The volume average strain in a unit cell, $\langle \varepsilon_{ij} \rangle$, can also be defined as

$$\langle \varepsilon_{ij} \rangle = \frac{1}{|V|} \int_V \varepsilon_{ij} \mathrm{d}V \tag{4}$$

However, the above strain is equal to macro-strain $\left(\langle \varepsilon_{ij} \rangle = \varepsilon_{ij}^0 \right)$ only if all phases of materials in a unit cell are free of discontinuity. Such a definition can satisfy the macro-homogenisation condition and the micro-energy is equivalent to the macro-energy for the following three types of boundary conditions [15]:

a. Uniform traction, t_i, on boundary ∂V(outer normal unit vector is denoted by n_j) of unit cell V, whose space is denoted as $\hat{Y}$ (that is, $\hat{Y} = \left\{ y_i \in V | \sigma_{ij}^0 n_j = t_i \right\}$)

$$\sigma_{ij} n_j = \sigma_{ij}^0 n_j \tag{5}$$

b. Uniform displacement on boundary ∂V. The component of a displacement vector in a unit cell, u_i, is given as

$$u_i = \varepsilon_{ij}^0 y_j \tag{6}$$

The space is denoted by $\tilde{Y} = \left\{ y \in V | \varepsilon_{ij}^0 y_j = u_i^0 \right\}$

c. Periodic condition on boundary ∂V whose space is denoted as V_{per}

$$u_i = \varepsilon_{ij}^0 y_j + u_i^* \tag{7}$$

where u_i^*, being periodic, denotes the fluctuation part of the displacement. $\sigma_{ij} n_j$ is anti-periodic, that is, $\sigma_{ij}(\mathbf{x}, \mathbf{y}) n_j = -\sigma_{ij}(\mathbf{x}, \mathbf{y} + \mathbf{Y}) n_j$.

After solving the boundary-value problem of a unit cell under one of above three boundary conditions, the micro-distribution of stress and strain within a unit cell is determined, thus determining the stress/strain localization tensor as follow.

Let $\mathbf{D}$ denote the strain localization tensor. The micro-strain, $\boldsymbol{\varepsilon}$, is given as

$$\boldsymbol{\varepsilon} = \mathbf{D} \boldsymbol{\varepsilon}^0 = \boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}^* \tag{8}$$

where $\boldsymbol{\varepsilon}^0$, $\boldsymbol{\varepsilon}^*$ are the macro-strain and micro fluctuation strain, respectively.

The same procedure defines $\mathbf{C}$ as the stress localization tensor. The micro-stress, $\boldsymbol{\sigma}$, is given as

$$\boldsymbol{\sigma} = \mathbf{C} \boldsymbol{\sigma}^0 = \boldsymbol{\sigma}^0 + \boldsymbol{\sigma}^* \tag{9}$$

where $\boldsymbol{\sigma}^0$, $\boldsymbol{\sigma}^*$ are the macro-stress and its micro fluctuation stress, respectively. Note that $\boldsymbol{\varepsilon}^*$ and $\boldsymbol{\sigma}^*$are higher order variables than $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$, respectively.

The homogenized moduli, which are represented by homogenized stiffness matrix $\tilde{\mathbf{E}}$ and homogenized compliance matrix $\hat{\mathbf{A}}$, are defined as

$$\tilde{\mathbf{E}} = \langle \mathbf{D}^T \mathbf{E} \rangle \tag{10}$$

and

$$\hat{\mathbf{A}} = \langle \mathbf{A}\mathbf{C} \rangle \tag{11}$$

$\mathbf{A}$ is the compliance coefficient of each phase material. Eqs. (10) and (11) reveal that the homogenized stiffness/compliance matrix is the weighted average of those of each component in a unit cell. The weights are stress/strain localization tensors, $\mathbf{C}$ and $\mathbf{D}$. The localization tensors have following properties (where $\mathbf{I}$ is unit tensor):

$$\langle \boldsymbol{\varepsilon} \rangle = \langle \mathbf{D}\boldsymbol{\varepsilon}^0 \rangle = \langle \mathbf{D} \rangle \boldsymbol{\varepsilon}^0 = \boldsymbol{\varepsilon}^0 \;\Rightarrow\; \langle \mathbf{D} \rangle = \mathbf{I} \tag{12}$$

$$\langle \boldsymbol{\sigma} \rangle = \langle \mathbf{C}\boldsymbol{\sigma}^0 \rangle = \langle \mathbf{C} \rangle \boldsymbol{\sigma}^0 = \boldsymbol{\sigma}^0 \;\Rightarrow\; \langle \mathbf{C} \rangle = \mathbf{I} \tag{13}$$

3. Solutions for stress/strain localization tensor

Suquet [16] proposed a simplified method for a periodic structure by assuming that the heterogeneity undergoes a uniform (homogeneous) strain. Wang [15] presented a numerical algorithm of non-linear homogenisation theory based on a rigorous mathematical approach. The expressions for the respective localization tensors can be obtained as (see [15] for details)

$$D_{ijkl} = \delta_{ik}\delta_{jl} - \frac{\partial W_i^{kl}}{\partial y_j} \tag{14}$$

or

$$C_{ijkl} = \delta_{ik}\delta_{jl} - \frac{\partial \chi_i^{kl}}{\partial y_j} \tag{15}$$

where $\mathbf{W}$ is the characteristic function when $\boldsymbol{\varepsilon}^0$ is applied to the unit cell, $\boldsymbol{\chi}$ is the characteristic function when $\boldsymbol{\sigma}^0$ is applied, and $\delta_{ik} = \{^{1\ \ i=k}_{0\ \ i\neq k}$.

However, it is difficult to apply above rigorous mathematical approach to composite soils because soil microstructures are usually unknown except for some micro-parameter such as volume fraction for each material phase. A simplified homogenisation method is proposed in the present study to determine the stress/

strain localization tensor. That is, the distribution of micro-stress and micro-strain in a unit cell is assumed to be homogeneous. This assumption would lead to a closed-form solution of stress/strain localization tensor.

Let the volume fraction of the reinforcement (f-phase) and matrix (m-phase) of a two-phase material of unit volume shown in Fig. 1 be f and $(1-f)$, respectively. The symbol f refers to the ratio of f-phase volume to the whole volume in a unit cell. In the subsequent discussion, the subscript f refers to the f-phase and the subscript m refers to the m-phase. For example, E_f refers to the stiffness of the f-phase and σ_m refers to the stress of the m-phase. Eq. (10) can be rewritten as

$$\tilde{\mathbf{E}} = \mathbf{E}_m + f\left\langle \mathbf{D}^T : \left(\mathbf{E}_f - \mathbf{E}_m\right)\right\rangle \tag{16}$$

If the distribution of the micro-stress/strain in a unit cell is known, the localization tensor can be determined. For example, Eshelby [17] studied a special case whereby a single anisotropic ellipsoidal inhomogeneity was imbedded into an infinite matrix. He obtained the strain locatization tensor as

$$\mathbf{D} = \left[\mathbf{I} + \mathbf{E}_0(\mathbf{A}_m)^{-1}\left(\mathbf{A}_f - \mathbf{A}_m\right)\right]^{-1} \tag{17}$$

where $\mathbf{E}_0$ is related to $\boldsymbol{\varepsilon}^f$ by

$$\boldsymbol{\varepsilon}^f = \mathbf{E}_0\boldsymbol{\varepsilon}^* + \boldsymbol{\varepsilon}^0 \tag{18}$$

where ε^* is the uniform eigenstrain and ε^0 is the strain in infinite matrix.

The present study assumes that the stress in each phase of a unit cell is homogeneous and the stresses in different phases are distributed according to some proportion. For two-phase materials, one can assume that the incremental distribution of the micro-stress satisfies the following:

$$\mathrm{d}\sigma_f = b_s \mathrm{d}\sigma_m \tag{19}$$

where b_s is termed the stress ratio which is a function of stress state or history that varies with deformation and loading paths. Eqs. (3), (9) and (19) imply that the stress localization tensor $\mathbf{C}$ is taken as

$$\mathbf{C} = \begin{cases} \dfrac{b_s}{(b_s-1)f+1}\mathbf{I} & \text{in } f-\text{phase} \\ \dfrac{1}{(b_s-1)f+1}\mathbf{I} & \text{in } m-\text{phase} \end{cases} \tag{20}$$

in which b_s is determined from the microstructure of a unit cell. It is noted that this assumption may make the boundary of phases not satisfying micro-equilibrium.

However, this assumption is a good approximation for micro-stress distributions. Furthermore, the above assumption and equation did not require periodicity conditions, thus the stress localization tensor may be applicable for any microstructure.

Determination of the parameter b_s depends on complex microstructures of mixed soils. For truly random materials, the strain in a unit cell can be simulated by a series model whose parameters are denoted by the superscript d, while stress distribution in the unit cell may be described by a parallel model whose parameters are denoted by the superscript s. These models are schematically shown in Fig. 2. The two models express different aspects of composite soils as follows:

a. Phase stress is homogeneous, i.e. Eq. (19) is true. The corresponding moduli are E_f^s and E_m^s for the two phases. From the parallel conceptual model, the two moduli have following relationship:

$$E_f^s = b_s E_m^s \tag{21}$$

b. Phase strain is also homogeneous. That is

$$\mathrm{d}\varepsilon_m = b_d\,\mathrm{d}\varepsilon_f \tag{22}$$

The corresponding moduli are E_f^d and E_m^d. The following can be derived from the series conceptual model:

$$E_f^d = b_d E_m^d \tag{23}$$

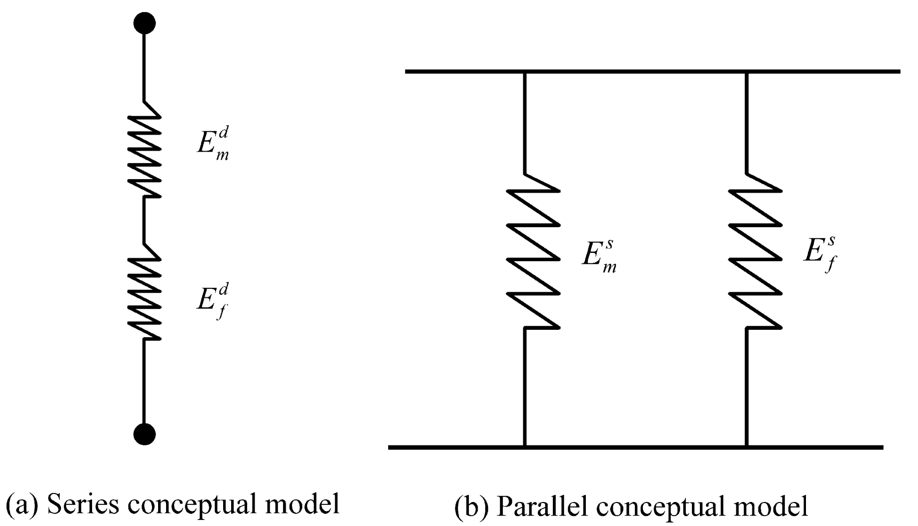

Fig. 2. Two conceptual models to simulate network structure.

In a truly random material, the two mechanisms exist at the same time and at the same space. From Eqs. (21) and (23), one has

$$E_f^d E_f^s = b_s b_d E_m^d E_m^s \tag{24}$$

As comparing with Eq. (19), phase moduli should have $E_f = b_s E_m$. This implies that Eq. (24) corresponds to $E_f^d E_f^s = E_f$ and $E_m^d E_m^s = E_m$. For a composite soil without obvious orientation, it is reasonable to assume that $E_f^d = E_f^s$ and $E_m^d = E_m^s$. That is

$$E_f^d = E_f^s = \sqrt{E_f} \tag{25}$$

and

$$E_m^d = E_m^s = \sqrt{E_m} \tag{26}$$

Finally, one gets the stress ratio as

$$b_s = b_d = \sqrt{\frac{E_f}{E_m}} \tag{27}$$

4. Comparison with experimental data on alloys

A linearly elastic composite metal with well-random microstructures is examined first. The experimental data was taken from Nishimatsu and Gurland [18] who conducted uniaxial compression tests on polycrystalline alloy made of carbonized tungsten and cobalt. The carbonized tungsten as inclusion of reinforcement material has a Young's modulus $E_f = 7.03 \times 10^5$MPa and the cobalt as matrix material has a Young's modulus $E_m = 2.07 \times 10^5$MPa. The parameter b_s is hence determined to be 1.843 using Eq. (27). By taking the stress localization tensor as that given in Eq. (20), the homogenized Young's modulus, E^h, at any intermediate volume fraction, f, can be determined as

$$\frac{1}{E^h} = \frac{\dfrac{b_s f}{E_f} + \dfrac{1-f}{E_m}}{(b_s - 1)f + 1} \tag{28}$$

For two extremes, Eq. (28) reduces to $E^h = E_m$ when $f = 0$ and $E^h = E_f$ when $f = 1$. Fig. 3 gives a comparison between experimental data and theoretical predictions obtained by Eq. (28) . For the full range of f, the predictions agree reasonably well with the experimental data. They also lie within the lower and upper bounds proposed by Hashin and Shtrikman [6]. The difference between the predictions and the experimental data is relatively large for $f = 0.6$ and 0.8. This may be attributed to the interaction among inclusions.

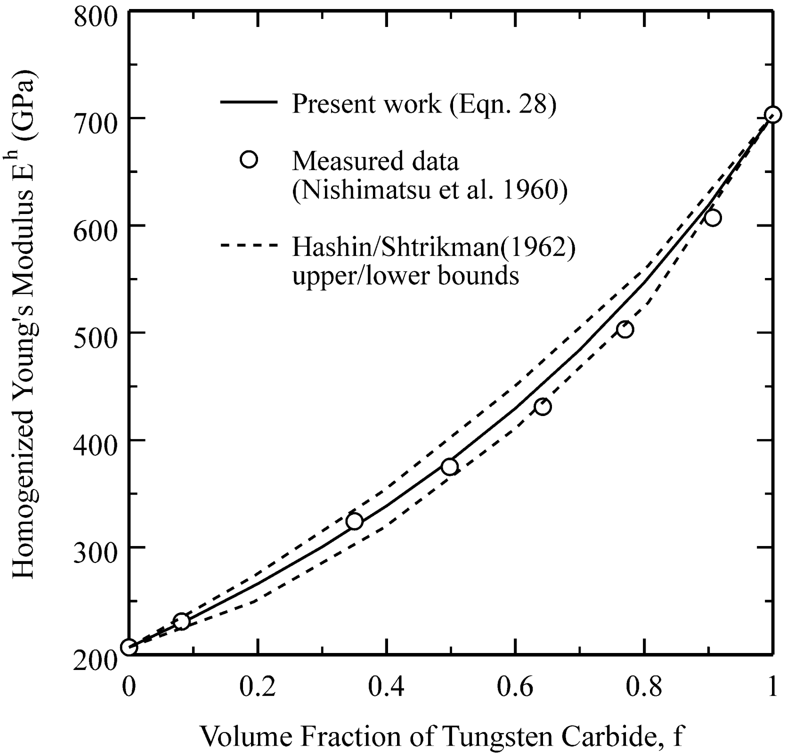

Fig. 3. Comparison of homogenized Young's modulus of a two-phase alloy.

Besides predicting the stiffness parameters of a metal alloy, the current method can be also applied to other properties of an alloy. Landauer [19] presented a set of experimental data on electrical conductivity on a mixture of lead magnesium ($M_{g2}Pb$) and lead (Pb). Electrical conductivity parameters, C_m and C_f, are given as 55.186×10^5 and 473.404×10^5 mho/m, respectively. An expression similar to Eq. (28) can be derived for the homogenized electrical conductivity, C^h, in terms of C_m, C_f and f. The value of b_s is determined to be 2.929 using Eq. (27). A comparison between the measured and predictions of C^h is shown in Fig. 4. The predictions again agree reasonably well with experimental results for the entire range of f. This verifies that the current method can be used to determine non-mechanical parameters of a composite alloy.

5. Applications to mixed soils

The simplified homogenisation method is applied to mixed soils in this section. First of all, it is used to determine the material parameters of Duncan–Chang [20] model. It is then used to predict the strength parameter and the coefficient of volume change.

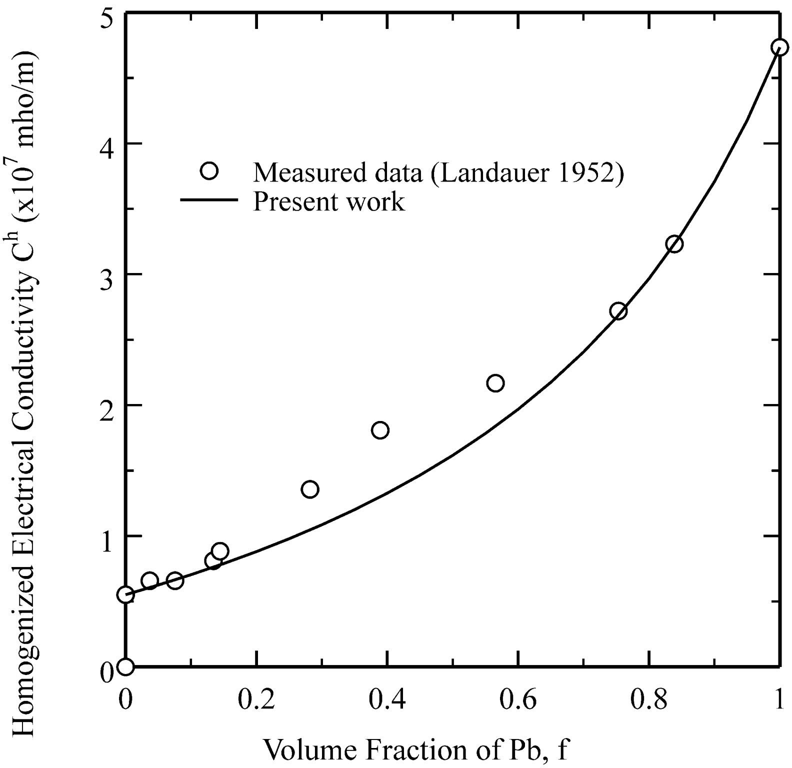

Fig. 4. Comparison of theoretical prediction and experimental data for electrical conductivity of $M_{g2}Pb - Pb$ mixture.

5.1. Prediction of Duncan–Chang model parameters

Stark et al. [3] carried out triaxial compression tests to determine the material parameters of mixed soils for the Duncan–Chang hyperbolic model. The silt mixtures were fabricated using a dry mixing technique. The samples were composed of processed silt and clay mineral contents between 0 to 50% by dry weight. The samples were compacted at standard Proctor relative compactions (PRC) between 85 and 100%. They investigated the effect of clay mineral content on the stress–strain behavior and the Mohr–Coulomb strength parameters for mixed soils.

Two important parameters for the Duncan–Chang hyperbolic model are summarized. The first one is the initial modulus, E_i, which is defined by

$$E_i = KP_a\left(\frac{\sigma_3'}{P_a}\right)^n \tag{29}$$

where P_a is the atmospheric pressure (101.3 kPa), K is the modulus number, σ'_3 is the effective confining pressure and n is the modulus exponent. The second one is the bulk modulus, B, which describes the nonlinear and stress-dependent volume change characteristics of soil:

$$B = K_b P_a \left(\frac{\sigma'_3}{P_a} \right)^m \tag{30}$$

where K_b is the bulk modulus number and m is bulk modulus exponent.

In the previous example, the simplified homogenisation method was applied to predict mechanical and non-mechanical properties of alloy based on the volume fraction of the f-phase, f. However, the proportion of sand and clay for mixed soils is usually measured by percentage by dry weight. A relationship between f and the clay fraction by dry weight, F, is therefore necessary in order that the proposed method can be employed to predict the behavior of mixed soils. In the experiments, F varies from 0 to 50%. Taking $K_f = K_0$ for $F = 0\%$ and $K_m = K_{50}$ for $F = 50\%$ for the two

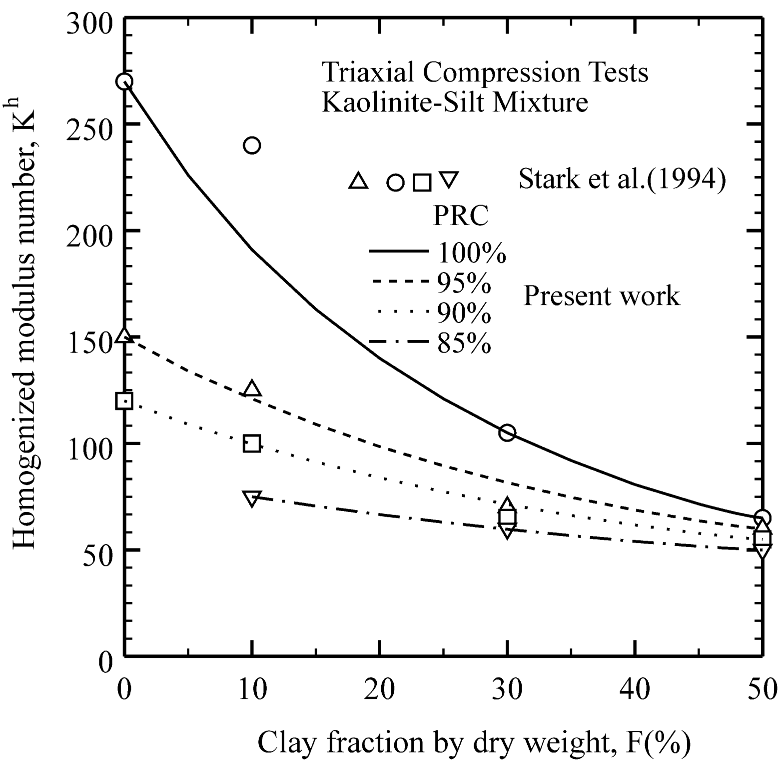

Fig. 5. Young's modulus number at different Proctor relative compaction (PRC).

extremes of F. A parameter termed as volume factor, f', is employed as an alternative for f where

$$f' = (1 - 2F) \tag{31}$$

For two extremes of F, Eq. (31) reduces to $f' = 0$ when F = 50% and $f' = 1$ when $F = 0\%$.

Two important parameters, K and K_b, can be approximated using the simplified homogenisation method. From Eqs. (11) and (20), the homogenized modulus number, K^h, can be determined by the following unified equation:

$$K^h = \frac{(b_s - 1)f' + 1}{\dfrac{b_s f'}{K_f} + \dfrac{1 - f'}{K_m}} \tag{32}$$

Homogenized bulk modulus number, K_b^h, can be also determined in a similar manner. Figs. 5 and 6 show the measured K^h and K_b^h values for kaolinite–silt

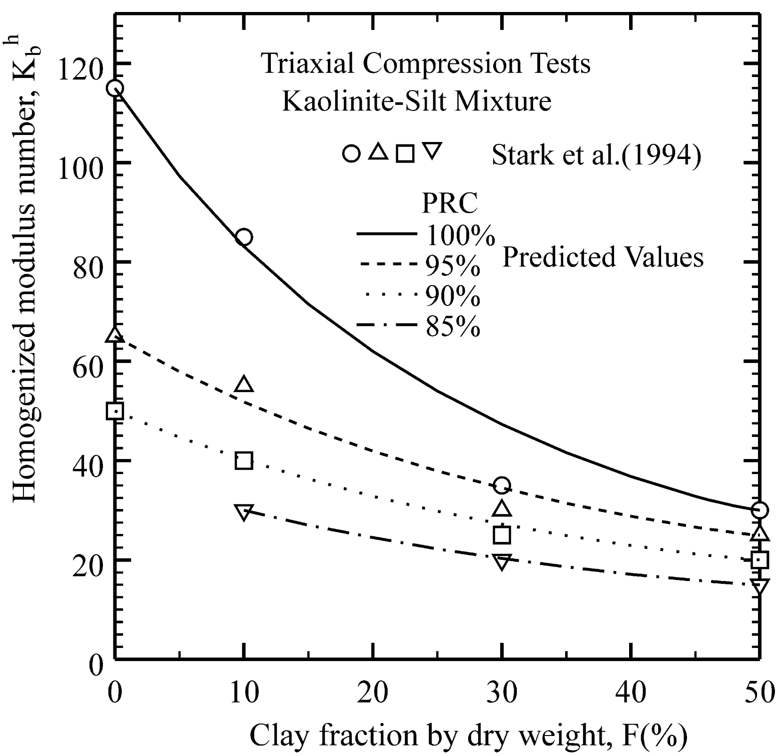

Fig. 6. Bulk modulus number at different Proctor relative compaction (PRC).

mixtures of various F. It is evident that reasonable agreements are obtained between experimental data and predictions.

5.2. Strength criterion for mixed soils

The criterion of stress limit (or simply strength) refers to the stress states under an infinite deformation. Wang [15] showed that the homogenized constitutive law could be generalized as

$$\{\Delta \boldsymbol{\sigma}^0\} = [\mathbf{E}^h]\{\Delta \boldsymbol{\varepsilon}^0\} \tag{33}$$

The stress limit implies that the determinant of the homogenized Young's modulus of the mixed soil, $\mathbf{E}^h$, is equal to zero when the soil reaches its failure status. In this section, only the critical stress state is examined. Each constituent of the unit cell is assumed to reach its critical state at the same time when the unit cell is at its critical

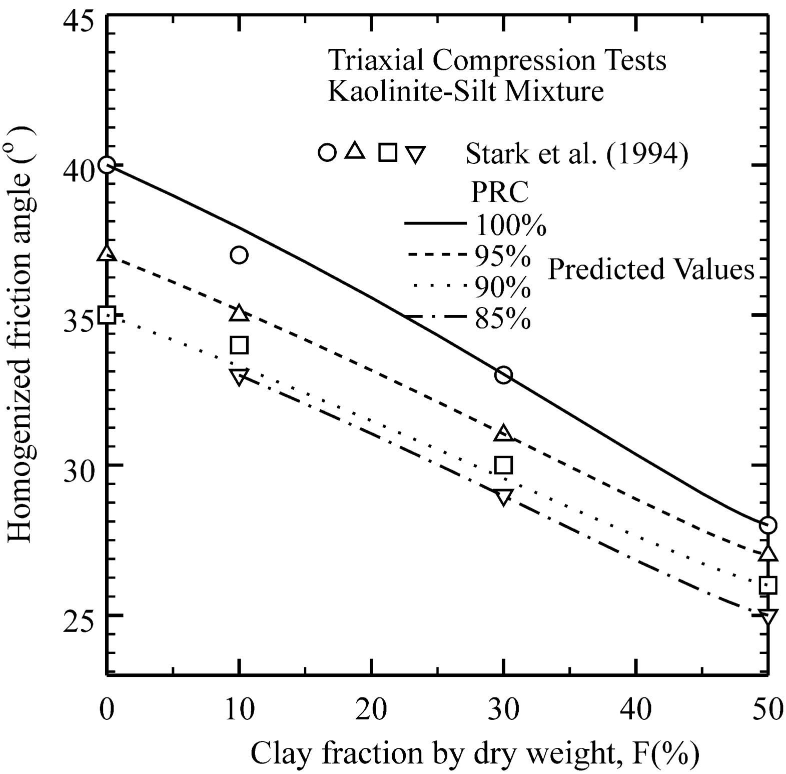

Fig. 7. Friction angle at different Proctor relative compaction.

Table 1
Properties of mixed soil A [2]

Sample	Clay (%)	Silt (%)	Sand (%)	Liquid limit (%)	Plastic limit (%)	Plasticity index (%)
A-1[a]	80	20	0	51.6	28	23.6
A-2	64	16	20	41.9	24.3	17.6
A-3	48	12	40	33.2	18.3	14.9
A-4	32	8	60	24.0	12.7	11.3
A-5	24	6	70			
A-6	16	4	80			
A-7	8	2	90			
A-8[b]	0	0	100			

[a] Kaolin ($G_s = 2.70$).
[b] Toyoura sand (G_s) = 2.65.

Table 2
Properties of mixed soil B [2]

Sample	Clay (%)	Silt (%)	Sand (%)	Liquid limit (%)	Plastic limit (%)	Plasticity index (%)
B-1[a]	72	28	0	163.8	20.9	142.9
B-2	57	23	20	123.4	16.5	106.9
B-3	43	17	40	89.6	12.7	76.9
B-4	29	11	60	60.0	11.5	48.5
B-5	22	8	70	45.6	12.0	33.6
B-6	15	5	80			
B-7	8	2	90			
B-8[b]	0	0	100			

[a] Kaolin and bentonite ($G_s = 2.50$).
[b] Toyoura sand ($G_s = 2.65$).

Table 3
Material parameters

Mixed soil	Initial void ratio e_0	Compression index of sand C_{cf}	Compression index of clay C_{cm}	Critical content ratio *FR*
A	0.5	0.0367	0.345	0.18
B	0.689	0.0351	1.219	0.12

stress state. Its limit stress state is expressed by the generalized shear stress, q^0, and the mean stress, p^0. They are the volume average of their partners given as

$$q^0 = f'q_f + (1 - f')q_m \tag{34}$$

$$p^0 = f'p_f + (1 - f')p_m \tag{35}$$

The homogenized critical state parameter for a whole unit cell, M^h, is introduced as

$$M^h = \frac{q^0}{p^0} \tag{36}$$

For the reinforcement component,

$$M_f = \frac{q_f}{p_f} \tag{37}$$

and for the matrix component,

$$M_m = \frac{q_m}{p_m} \tag{38}$$

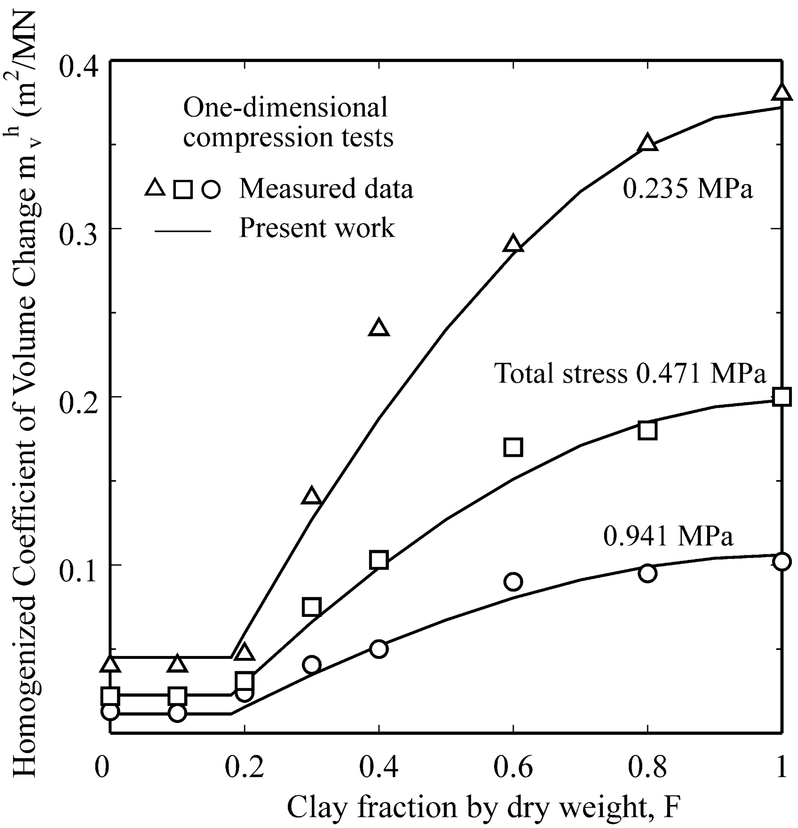

Fig. 8. Homogenized coefficient of volume change for mixed soil A.

As compression and shear properties for soils are different, the stress ratios for both properties should be different. If $p_f = b_p p_m$ is assumed for mean stress, following equation can be obtained using Eq. (35).

$$p^0 = \left[f'\left(b_p - 1\right) + 1\right]p_m \tag{39}$$

or

$$p^0 = \frac{\left[f'\left(b_p - 1\right) + 1\right]}{b_p}p_f \tag{40}$$

where b_p is the stress ratio for the mean stress of a unit cell. The generalized shear stress should be the volume average of each phase. That is,

$$q^0 = f'q_f + (1 - f')q_m \tag{41}$$

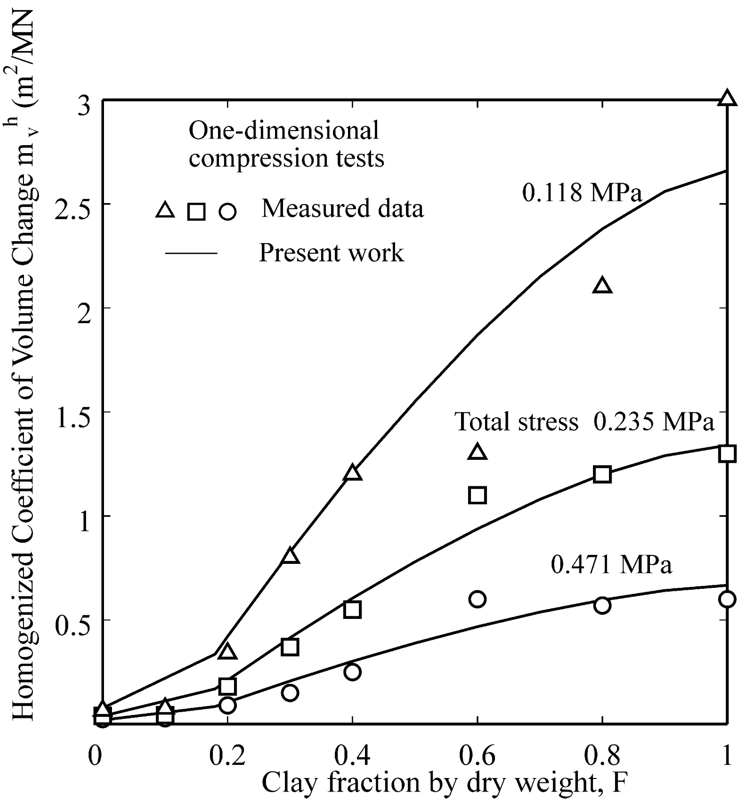

Fig. 9. Homogenized coefficient of volume change for mixed soil B.

Therefore, the critical state parameter of this unit cell is obtained as

$$M^h = \frac{b_p f'}{f'(b_p - 1) + 1} M_f + \frac{1 - f'}{f'(b_p - 1) + 1} M_m \tag{42}$$

For simplicity, it is assumed that $b_p = b_s$ and the following form for the homogenized stress ratio is obtained as

$$b_s = \sqrt{\frac{M_f}{M_m}} \tag{43}$$

Fig. 7 shows a comparison between the measured friction angle and the prediction obtained using Eq. (42) for the kaolinite-silt mixture. Note that the homogenized friction angle $\phi^h = \tan(M^h)$. The predictions are again in good agreement with experimental data.

5.3. Uniaxial compression experimental data from Omine and Ochiai [2]

Omine and Ochiai [2] carried out uniaxial compression tests on two types of mixed soils, mixed soils A and B. The properties of mixed soils A and B are shown in

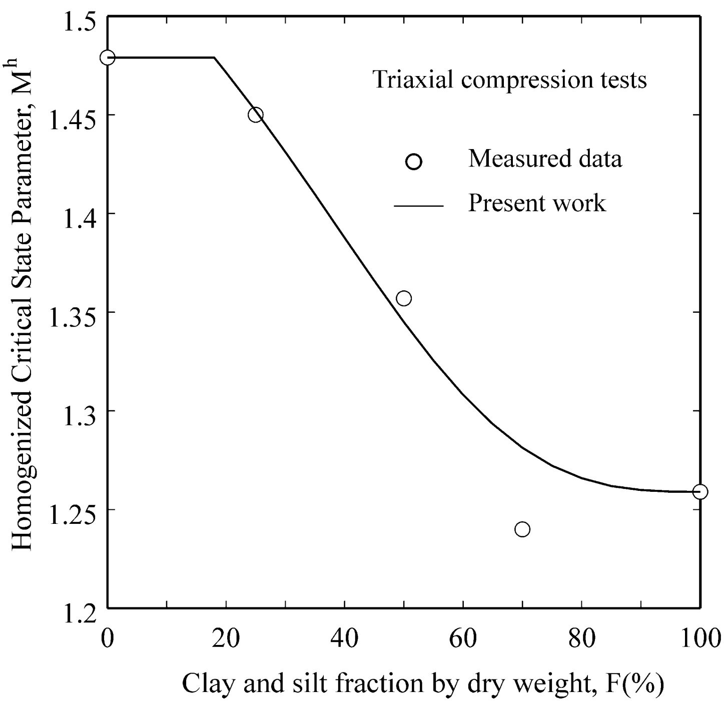

Fig. 10. Critical state parameter for mixed soil A.

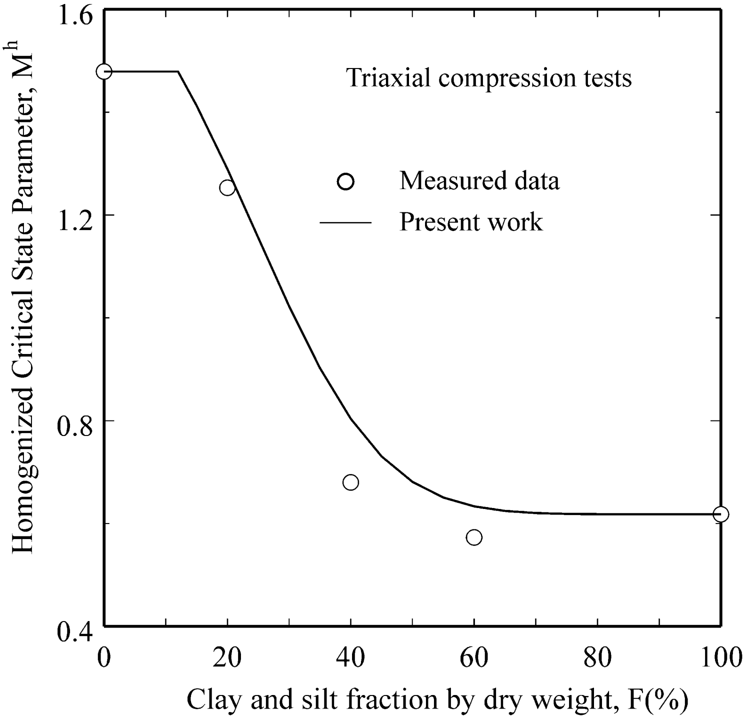

Fig. 11. Critical state parameter for mixed soil B.

Tables 1 and 2, respectively. The proportion of clay, silt and sand refers to percentage by dry weight. They established through experiments that there was a critical content ratio of sand (denoted by *FR*) that was found to be 18% for mixed soil A and 12% for mixed soil B. When the percentage by weight of sand is below the *FR*, the reinforcement effect of sands is so small that it can be ignored. The mechanical properties of mixed soils are almost the same as those of matrix. From Eqs. (11) and (20), the homogenized coefficient of volume change, m_v^h at any f', is obtained as follow:

$$m_v^h = \frac{b_s f' \ m_{vf} + (1 - f')m_{vm}}{(b_s - 1)f' + 1} \tag{44}$$

where m_{vf} and m_{vm} are the coefficient of volume change of sand and clay, respectively.

Experimental data obtained for mixed soil A reveal that the change of void ratio e with confining pressure σ is relatively small. Taking the compression index C_c as the gradient of the straight line on the $e - \log \sigma$ plot and using initial void ratio e_0 as reference, the coefficient of volume change m_v can be derived as

$$m_v = \frac{0.435 C_c}{\sigma(1 + e_0)} \tag{45}$$

The change in e is relatively large for mixed soil B. It would be more appropriate to use the void ratio e at a given σ rather than e_0 as the reference. In such case,

$$m_v = \frac{0.435C_c}{\sigma(1 + e_0 - C_c \log(\sigma))} \tag{46}$$

As the effect of the soil mixture is only dominant from $F=FR$ to $F=1$, these two values will be taken as the two extremes for the homogenization method. The same principle of deriving Eq. (31) is used to obtain a relationship between volume factor f' and F as

$$f' = \left(\frac{1-F}{1-FR}\right) \tag{47}$$

Eqs. (44) and (47) are then used to determine the m_v^h values for the soil under different loading pressures. The relevant m_{vf} and m_{vm} values are obtained using Eq. (45) for mixed soil A and Eq. (46) for mixed soil B, respectively. The material properties for the two soils are summarized in Table 3. The predicted and measured m_v^h values are given in Figs. 8 and 9. These figures show good agreement with the measured data.

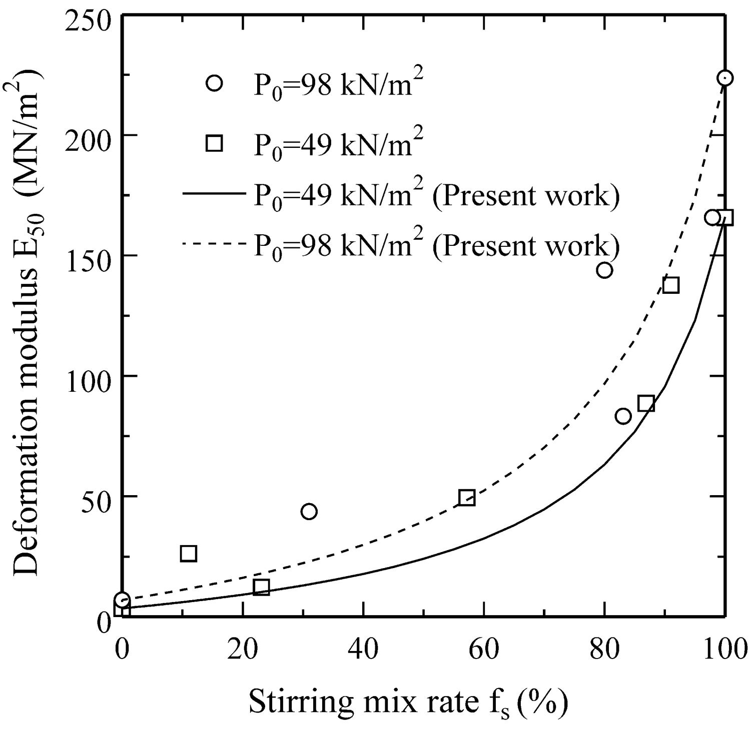

Fig. 12. Relationship between E_{50} and f_s.

Omine et al. [4] also carried out triaxial compression tests on the soils and the measured critical state parameters M^h for mixed soil A and B are shown in Figs. 10 and 11, respectively. Homogenized M^h are also plotted in Figs. 10 and 11 using the relationship between f' and F given in Eq. (47). Again good agreement is obtained between the measured and predictions.

6. Applications to composite ground with definite reinforcement orientation

Unlike mixed soils that are a thorough mixture of soft and stiff soils, composite ground [4,7–9] consists of improved and unimproved parts because stiff soil is installed as columns and does not mix with the in-situ original soil outside the stiff soil columns. Furthermore, reinforcements have definite reinforcement orientation in space. For example, stone-columns are usually installed vertically. For such a composite ground, two important parameters, deformation modulus E_{50} and unconfined compressive strength q_u in vertical direction, were usually important to boundary-value problems. Omine et al. [4] measured these two parameters for a laboratory prepared cement treated soil. The volume content of improved part in treated soils is defined as a stirring mix ratio f_s. It was measured through the ratio of areas of improved to unimproved parts in a cross-section of sampling specimen.

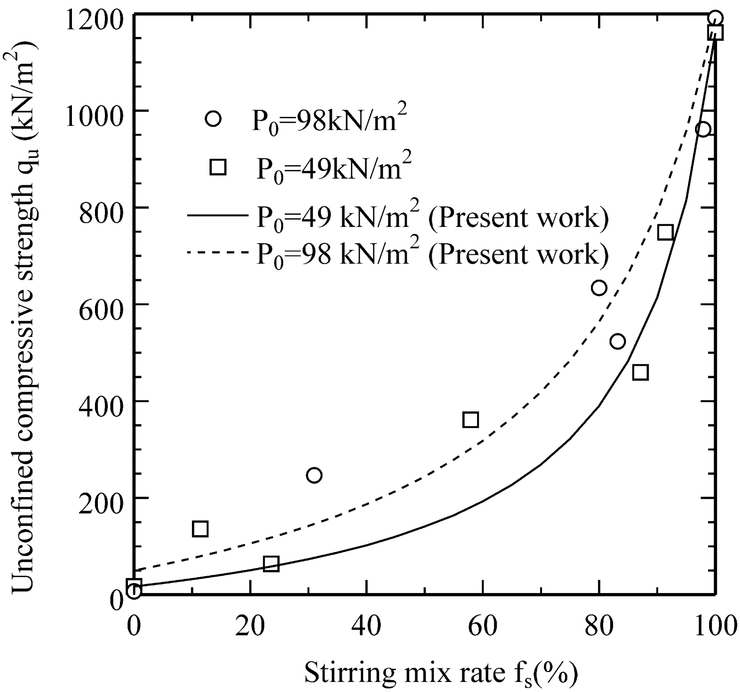

Fig. 13. Relationship between q_u and f_s.

Micro-stress is again assumed to be homogeneous in improved and unimproved parts. Because micro-stress is distributed according to stress localization tensor given by Eq. (20), the micro-stress at the failure status should be

$$q'_{uf} = \frac{b_s}{(b_s - 1)f_s + 1} q_{uf} \qquad q'_{um} = \frac{1}{(b_s - 1)f_s + 1} q_{um} \tag{48}$$

Therefore, the homogenized strength is established as follows:

$$q_u^h = f_s q'_{uf} + (1 - f_s) q'_{um} = \frac{b_s f_s q_{uf} + (1 - f_s) q_{um}}{(b_s - 1)f_s + 1} \tag{49}$$

For such a case, micro-strain in vertical direction should be the same for each phase (equal to macro-strain). If Eq. (49) is divided by macro-strain, the homogenized deformation modulus is obtained as follows:

$$E_{50}^h = \frac{b_s f_s E_{50f} + (1 - f_s) E_{50m}}{(b_s - 1)f_s + 1} \tag{50}$$

The parameter b_s is again determined using Eq. (27). The predictions of E_{50} and q_u in Figs. 12 and 13 show reasonably good agreement with measured data. This shows that the current method is also applicable to composite ground. Eq. (50) is a little different from Eq. (28) of homogenized Young's modulus. This is because for composite ground, microstructures have definite reinforcement orientation while the microstructures of alloy or mixed soils are randomly distributed.

7. Conclusion

A simplified homogenisation method is proposed based on the assumption that micro-stress/micro-strain is homogeneous in the matrix and the reinforcement of a composite soil. A conceptual model consisting of a series model and a parallel model is proposed to determine the stress ratio in each phase. Thus, a localization tensor for a two-phase material is obtained in Eq. (20). This localization tensor is applied to predict some characteristic parameters such as Young's modulus or modulus number, strength parameter for mixed soils and composite ground. The following conclusions can be drawn:

1. The simplified homogenisation method directly assumes the micro-stress distribution in each phase, while rigorous procedure of homogenisation theory should solve a boundary-value problem for a unit cell under periodicity condition or truly random condition. A closed-form stress localization tensor is then developed to distribute the macro-stress in each phase. The stress concentration tensor has two important parameters: stress ratio b_s and volume factor f'. For mixed soils, f' is related to the clay fraction by dry weight F. For composite ground, f' is simply the volume fraction of inclusion/reinforcement. This assumption may not satisfy the

micro-equilibrium especially at the boundaries of each phase. However, case study shows that this micro-non-equilibrium is not critical to the parameter prediction.

2. Although homogenisation procedures require periodicity conditions for a unit cell, the simplified homogenisation method does not limit itself to that condition. In fact, this method can be applied to both randomly mixed soils and composite ground with definite reinforcement orientation. This is because the equivalent parameter includes the main characteristics of microstructures. Thus, the current method provides a simple way to predict the material parameter of a composite material at any intermediate f' if material parameters of matrix and reinforcement are known.

3. The stress ratio b_s is directly associated with microstructure. Eq. (27) is applicable not only for Young's modulus, but also for the coefficient of volume change and the strength parameter. This makes the current method applicable to a wide range of material properties such as electrical conductivity, strength, and the parameters for the Duncan–Chang's hyperbolic model. The simplified homogenisation method can also effectively predict the mechanical properties of randomly mixed soil and composite ground with definite reinforcement orientation. However, if the material has more than two phases and its microstructure has definite orientations, the current method has its limitation and rigorous procedure of homogenisation method is recommended for such case.

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