An Elastoplastic Homogenization Theory for Geomaterials

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ABSTRACT

Geomaterials are composed of discrete elements of microstructures. The studies of these materials are of increasing interest in geophysics and geotechnical engineering. We here treat the behavior of microscopically heterogeneous geomaterials with elastoplastic components, and show numerical results applied for a mixed soil involving hard gravel inclusions.

INTRODUCTION

If a material is composed by some elastoplastic constituents, what is the macro-mechanical properties of this composite? In the present work, we show an elastoplastic homogenization theory.

Let us show a material body with a microscale periodic structure in Figure 1. The repeated microstructure is called a unit cell. Then, the elastoplastic problem for this microscopically heterogeneous material are defined as follows:

Governing equations

Equilibrium equation

$$\frac{\partial \sigma_{ij}^{\varepsilon}}{\partial x_j} + f_i^{\varepsilon} = \frac{\partial \Delta \sigma_{ij}^{\varepsilon}}{\partial x_j} + \Delta f_i^{\varepsilon} + \left(\frac{\partial \sigma_{ij}^{*\varepsilon}}{\partial x_j} + f_i^{*\varepsilon} \right) = 0, \qquad \text{or} \qquad \frac{\partial \Delta \sigma_{ij}^{\varepsilon}}{\partial x_j} + \Delta f_i^{\varepsilon} = 0 \tag{1}$$

Geometrical relation

$$\varepsilon_{ij}^{\varepsilon} = \varepsilon_{ij}(\boldsymbol{u}^{\varepsilon}) = \frac{1}{2}\left(\frac{\partial u_i^{\varepsilon}}{\partial x_j} + \frac{\partial u_j^{\varepsilon}}{\partial x_i} \right), \qquad \text{or} \qquad \Delta\varepsilon_{ij}(\boldsymbol{u}^{\varepsilon}) = \frac{1}{2}\left(\frac{\partial \Delta u_i^{\varepsilon}}{\partial x_j} + \frac{\partial \Delta u_j^{\varepsilon}}{\partial x_i} \right) \tag{2}$$

Here we introduced current state variables $\sigma_{ij}^{\varepsilon}, \varepsilon_{ij}^{\varepsilon}, \cdots$ at the time $t + \Delta t$, reference ones $\sigma_{ij}^{*}, \varepsilon_{ij}^{*}, \cdots$ at the time t, and the increments $\Delta\sigma_{ij}^{\varepsilon}, \Delta\varepsilon_{ij}^{\varepsilon}, \cdots$ during the time step $[t, t + \Delta t]$:

1) Associate Professor
2) Graduate Student

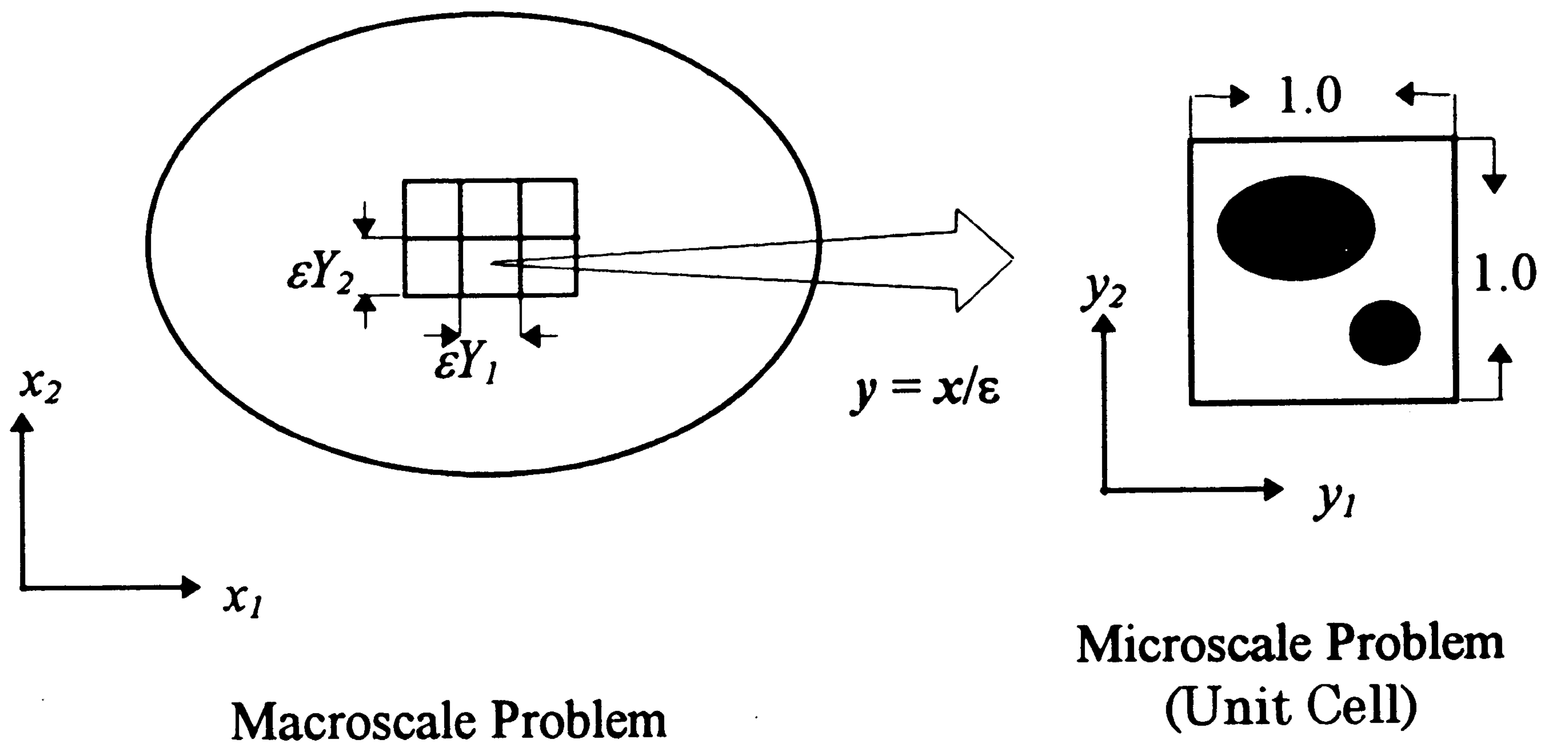


Figure 1. Material body with periodic microstructure.

$$\sigma^{\varepsilon}_{ij} = \sigma^{*}_{ij} + \Delta\sigma^{\varepsilon}_{ij}, \qquad \varepsilon^{\varepsilon}_{ij} = \varepsilon^{*}_{ij} + \Delta\varepsilon^{\varepsilon}_{ij}, \qquad f^{\varepsilon}_{i} = f^{*\varepsilon}_{i} + \Delta f^{\varepsilon}_{i}.$$

Note that $(\bullet)^{\varepsilon}$ implies the variable changes drastically in each microscopic constituent.

Constitutive equation: The flow theory is widely used in soil and rock mechanics (Roscoe and Burland 1968; Chen and Saleeb 1981, 1982). Its basic idea is stated as follows:

•Elastic and plastic strain resolution: $\Delta\varepsilon^{\varepsilon}_{ij} = \Delta\varepsilon^{e}_{ij} + \Delta\varepsilon^{p}_{ij}$ (3)

•Hooke's law: $\Delta\sigma^{\varepsilon}_{ij} = E^{\varepsilon}_{ijkl}\Delta\varepsilon^{e}_{kl}$, or $\Delta\varepsilon^{e}_{ij} = A^{\varepsilon}_{ijkl}\Delta\sigma^{\varepsilon}_{kl}$. (4)

•Flow rule: $d\varepsilon^{p}_{ij} = d\lambda^{\varepsilon}\dfrac{\partial g}{\partial\sigma_{ij}} \approx d\lambda^{\varepsilon}\dfrac{\partial g^{*}}{\partial\sigma^{*}_{ij}}$. (5)

•Yield function: $f(\sigma^{*}_{ij} + \Delta\sigma^{\varepsilon}_{ij}, H^{*}_{ij} + \Delta H_{ij}) \le 0$ (6)

where H_{ij} is a hardening tensor. The Taylor's expansion suggests that

$$f = f^{*} + \frac{\partial f^{*}}{\partial\sigma^{*}_{ij}}d\sigma^{\varepsilon}_{ij} + \frac{\partial f^{*}}{\partial H_{ij}}dH_{ij} \le 0; \qquad f^{*} = f(\sigma^{*}_{ij}, H^{*}_{ij}). \tag{7}$$

•Hardening law: $H_{ij} = H_{ij}(\varepsilon^{p}_{kl}) \;\Rightarrow\; dH_{ij} = \dfrac{\partial H_{ij}}{\partial\varepsilon^{p}_{kl}}d\varepsilon^{p}_{kl} \approx d\lambda^{\varepsilon}F_{ijkl}\dfrac{\partial g^{*}}{\partial\sigma^{*}_{kl}}; \quad F_{ijkl} = \dfrac{\partial H_{ij}}{\partial\varepsilon^{p}_{kl}}$ (8)

Combining the inequality (7) with Eqn(8) yields

$$f = f^{*} + \frac{\partial f^{*}}{\partial\sigma^{*}_{ij}}d\sigma^{\varepsilon}_{ij} + d\lambda^{\varepsilon}F_{ijkl}\frac{\partial g^{*}}{\partial\sigma^{*}_{kl}}\frac{\partial f^{*}}{\partial H_{ij}} \le 0 \tag{9}$$

The total constitutive law is now given by

$$\Delta\sigma^{\varepsilon}_{ij} = E^{\varepsilon}_{ijkl}\Delta\varepsilon^{\varepsilon}_{kl} - d\lambda^{\varepsilon}\sigma^{g}_{ij}; \qquad \sigma^{g}_{ij} = E^{\varepsilon}_{ijkl}\frac{\partial g^{*}}{\partial\sigma^{*}_{kl}}. \tag{10}$$

ASYMPTOTIC EXPANSION AND HOMOGENIZATION THEORY

Suppose the displacement increment is expanded as

$$\Delta u_i^\varepsilon(\boldsymbol{x};t) = \Delta u_i^0(\boldsymbol{x},\boldsymbol{y};t) + \varepsilon \Delta u_i^1(\boldsymbol{x},\boldsymbol{y};t) + \varepsilon^2 \Delta u_i^2(\boldsymbol{x},\boldsymbol{y};t) + \cdots \tag{11}$$

where $\Delta u_i^\alpha(\boldsymbol{x},\boldsymbol{y};t) = \Delta u_i^\alpha(\boldsymbol{x},\boldsymbol{y}+\boldsymbol{Y};t)$ $(\alpha = 0,1,2,\cdots)$ are $\boldsymbol{Y}$-periodic functions. The chain rule of differentiation implies $\partial(\bullet)/\partial x_i \;\Rightarrow\; \partial(\bullet)/\partial x_i + (1/\varepsilon)\partial(\bullet)/\partial y_i$, so the incremental form of the geometrical relation can be expressed as

$$\Delta\varepsilon_{ij}^\varepsilon = \frac{1}{\varepsilon}\Delta\varepsilon_{ij}^{0y} + [\Delta\varepsilon_{ij}^{0x} + \Delta\varepsilon_{ij}^{1y}] + \varepsilon[\Delta\varepsilon_{ij}^{1x} + \Delta\varepsilon_{ij}^{2y}] + \cdots \tag{12}$$

$$\Delta\varepsilon_{ij}^{\alpha x} = \frac{1}{2}[\frac{\partial \Delta u_i^\alpha}{\partial x_j} + \frac{\partial \Delta u_j^\alpha}{\partial x_i}], \quad \Delta\varepsilon_{ij}^{\alpha y} = \frac{1}{2}[\frac{\partial \Delta u_i^\alpha}{\partial y_j} + \frac{\partial \Delta u_j^\alpha}{\partial y_i}], \quad (i,j=1,2,3;\quad \alpha = 0,1,\cdots)$$

We set specially $\Delta\varepsilon_{ij}^0 \equiv \Delta\varepsilon_{ij}^{0x}$. Let us suppose the stress increment is of the same form as the strain increment (12):

$$\Delta\sigma_{ij}^\varepsilon(\boldsymbol{x};t) = \frac{1}{\varepsilon}\Delta\sigma_{ij}^0(\boldsymbol{x},\boldsymbol{y};t) + \Delta\sigma_{ij}^1(\boldsymbol{x},\boldsymbol{y};t) + \varepsilon\Delta\sigma_{ij}^2(\boldsymbol{x},\boldsymbol{y};t) + \cdots \tag{13}$$

Then, the constitutive law given by Eqn(10) satisfies

$$\begin{aligned} &\Delta\sigma_{ij}^0(\boldsymbol{x},\boldsymbol{y};t) = E_{ijkl}^\varepsilon \Delta\varepsilon_{kl}^{0y}, && \Delta\sigma_{ij}^1(\boldsymbol{x},\boldsymbol{y};t) = E_{ijkl}^\varepsilon\left\{\Delta\varepsilon_{kl}^{0x} + \Delta\varepsilon_{kl}^{1y}\right\} - d\lambda^\varepsilon\sigma_{ij}^g \\ &\Delta\sigma_{ij}^2(\boldsymbol{x},\boldsymbol{y};t) = E_{ijkl}^\varepsilon\left\{\Delta\varepsilon_{kl}^{1x} + \Delta\varepsilon_{kl}^{2y}\right\}, && \cdots \end{aligned} \tag{14}$$

because σ_{ij}^g is known at the present step $[t, t+\Delta t]$, and the equilibrium equation is

$$\frac{\partial\Delta\sigma_{ij}^\varepsilon}{\partial x_j} = \frac{1}{\varepsilon^2}\frac{\partial\Delta\sigma_{ij}^0}{\partial y_j} + \frac{1}{\varepsilon}\left(\frac{\partial\Delta\sigma_{ij}^0}{\partial x_j} + \frac{\partial\Delta\sigma_{ij}^1}{\partial y_j}\right) + \left(\frac{\partial\Delta\sigma_{ij}^1}{\partial x_j} + \frac{\partial\Delta\sigma_{ij}^2}{\partial y_j}\right) + \cdots = -\Delta f_i^\varepsilon - \left(f_i^{*\varepsilon} + \frac{\partial\sigma_{ij}^{*\varepsilon}}{\partial x_j}\right)$$

We discuss each term of this equation.

<u>$\mathcal{O}(\varepsilon^{-2})$ term</u> :

$$\frac{\partial\Delta\sigma_{ij}^0}{\partial y_j} = 0 \quad\Rightarrow\quad \Delta\sigma_{ij}^0(\boldsymbol{x},\boldsymbol{y};t) = \Delta\sigma_{ij}^0(\boldsymbol{x};t)$$

$\Delta\sigma_{ij}^\varepsilon$ must have the limit by Eqn(13) when $\varepsilon\to 0$, so $\Delta\sigma_{ij}^0(\boldsymbol{x};t)\equiv 0$. Then, we have

$$\Delta u_i^0(\boldsymbol{x},\boldsymbol{y};t) = \Delta u_i^0(\boldsymbol{x};t) \tag{15}$$

<u>$\mathcal{O}(\varepsilon^{-1})$ term</u> (Local problem):

$$\frac{\partial\Delta\sigma_{ij}^0}{\partial x_j} + \frac{\partial\Delta\sigma_{ij}^1}{\partial y_j} = 0 \quad\Rightarrow\quad \frac{\partial\Delta\sigma_{ij}^1}{\partial y_j} = 0 \tag{16}$$

because $\Delta\sigma_{ij}^0(\boldsymbol{x};t)\equiv 0$. Let us substitute Eqn$(14)_2$ into Eqn(16), and get

$$\frac{\partial}{\partial y_j}\{E_{ijkl}^\varepsilon\Delta\varepsilon_{kl}^{1y} - d\lambda^\varepsilon\sigma_{ij}^g\} = \frac{\partial}{\partial y_j}\{E_{ijkl}^\varepsilon\frac{\partial\Delta u_k^1}{\partial y_l} - d\lambda^\varepsilon\sigma_{ij}^g\} = -\frac{\partial}{\partial y_j}\{E_{ijkl}^\varepsilon\Delta\varepsilon_{kl}^0\} \tag{17}$$

It is understood that the RHS of Eqn(17) is a function of only $\boldsymbol{x}$, so Eqn(17) gives a PDE in terms of $\boldsymbol{y}$ for obtaining $\Delta u_k^1(\boldsymbol{x},\boldsymbol{y};t)$ in the unit cell $\boldsymbol{Y}$ under the periodic boundary condition such that

$$\Delta u_i^1(\boldsymbol{x}, \boldsymbol{y}; t) = \Delta u_i^1(\boldsymbol{x}, \boldsymbol{y} + \boldsymbol{Y}; t). \tag{18}$$

Eqn(17) with the boundary condition (18) is called the ***local*** or ***unit cell problem.*** Note the second term of the LHS of Eqn(17) implies a plastic constraint (9), which is rewritten as

$$\left\{ f^* + \frac{\partial f^*}{\partial \sigma_{ij}^*} E_{ijkl}^{\varepsilon} \Delta \varepsilon_{kl}^0 \right\} + \frac{\partial f^*}{\partial \sigma_{ij}^*} E_{ijkl}^{\varepsilon} \Delta \varepsilon_{kl}^{1y} - d\lambda^{\varepsilon} T^{\varepsilon} \leq 0 \tag{19}$$

$$T^{\varepsilon} = \frac{\partial f^*}{\partial \sigma_{ij}^*} E_{ijkl}^{\varepsilon} \frac{\partial g^*}{\partial \sigma_{kl}^*} - F_{ijkl} \frac{\partial g^*}{\partial \sigma_{kl}^*} \frac{\partial f^*}{\partial H_{ij}}$$

$\mathcal{O}(\varepsilon^0)$ term (Global problem):

$$\frac{\partial \Delta \sigma_{ij}^1}{\partial x_j} + \frac{\partial \Delta \sigma_{ij}^2}{\partial y_j} = -\Delta f_i^{\varepsilon} - \left(f_i^{*\varepsilon} + \frac{\partial \sigma_{ij}^{*\varepsilon}}{\partial x_j} \right) \tag{20}$$

We introduce a volume average operator over a unit cell $\boldsymbol{Y}$ by $\langle \bullet \rangle = \int_Y \bullet \, dy/|Y|$ ($|Y|$: volume of the unit cell). Then, averaging Eqn(20) yields

$$\frac{\partial < \Delta \sigma_{ij}^1 >}{\partial x_j} = - < \Delta f_i^{\varepsilon} > - < f_i^{*\varepsilon} > - \frac{\partial < \sigma_{ij}^{*\varepsilon} >}{\partial x_j}, \tag{21}$$

because the second term of the LHS of Eqn(20) vanishes under the periodic condition. Let us discuss the implication of $< \Delta \sigma_{ij}^1 >$. By using the solution of the local problem (17), we can introduce a strain concentration tensor D_{ijkl} such as

$$\Delta \varepsilon_{ij}^{1y} = D_{ijkl} \Delta \varepsilon_{kl}^0 \tag{22}$$

(It is not easy to obtain the tensor D_{ijkl} explicitly. We will show a numerical procedure to calculate $\Delta \varepsilon_{ij}^{1y}$ as a function of $\Delta \varepsilon_{ij}^0$.) Recalling Eqn$(14)_2$, we have

$$\Delta \sigma_{ij}^1 = (E_{ijkl}^{\varepsilon} + E_{ijrs}^{\varepsilon} D_{rskl}) \Delta \varepsilon_{kl}^0 - d\lambda^{\varepsilon} \sigma_{ij}^g \tag{23}$$

This suggests that

$$< \Delta \sigma_{ij}^1 > = E_{ijkl}^h \Delta \varepsilon_{kl}^0 - < d\lambda^{\varepsilon} \sigma_{ij}^g >; \qquad E_{ijkl}^h = < E_{ijkl}^{\varepsilon} + E_{ijrs}^{\varepsilon} D_{rskl} > \tag{24}$$

Substituting this into Eqn(21) yields

$$\frac{\partial \{ E_{ijkl}^h \Delta \varepsilon_{kl}^0 \}}{\partial x_j} = F_i; \qquad F_i = \frac{\partial < d\lambda^{\varepsilon} \sigma_{ij}^g >}{\partial x_j} - < \Delta f_i^{\varepsilon} > - < f_i^{*\varepsilon} > - \frac{\partial < \sigma_{ij}^{*\varepsilon} >}{\partial x_j} \tag{25}$$

Eqn(25) called the ***global problem*** gives a PDE for obtaining $\Delta u_i^0(\boldsymbol{x}; t)$ in the global coordinates $\boldsymbol{x}$. Note that F_i gives a 'body force' which is composed of three terms: 1) the physical body force increment $< \Delta f_i^{\varepsilon} >$, 2) the unequilibrium force ($< f_i^{*\varepsilon} > + \partial < \sigma_{ij}^{*\varepsilon} > / \partial x_j$) at the previous step, and 3) the deviation of the plastic force from the elastic one ($-\partial < d\lambda^{\varepsilon} \sigma_{ij}^g > / \partial x_j$).

The method to calculate the volume average of $d\lambda^{\varepsilon} \sigma_{ij}^g$ is as follows: Let us define

$$< d\lambda^{\varepsilon} \sigma_{ij}^g > = \sum_{r=1}^{m} d\lambda_{(r)}^{\varepsilon} \sigma_{ij(r)}^g f_{(r)} \tag{26}$$

where r indicates the r-th material constituent in a unit cell, $f_{(r)}$ its volume fraction, and m the total number of material constituents. Then, we assume that

$$< d\lambda^\varepsilon \sigma^g_{ij} >= d\lambda < \sigma^g_{ij} >; \qquad d\lambda = \frac{< d\lambda^\varepsilon \sigma^g_{ij} >}{< \sigma^g_{ij} >} \qquad \text{if} \qquad < \sigma^g_{ij} > \neq 0. \tag{27}$$

COMPUTATIONAL ALGORITHM: DOMAIN DESSOLUTION & LOCAL ITERATION

Let us discuss a scheme to solve the nonlinear boundary value problem given by Eqn(17) and Eqn(25) under the constraint (19). The main idea is to introduce a two stage iteration procedure for determining $\Delta\varepsilon^0_{kl}$ in Eqn(25) and for determinig $\Delta\varepsilon^{1y}_{kl}$ in Eqn(17) under the constraint (19). The former procedure is called the Global Problem, and the latter the Local Problem. The iteration scheme is shown in Figure 2. Because the Global Problem is same as the usual nonlinear elastic problem, we here discuss only the method to solve the Local Problem by using the ***domain dissolution method*** under a local iteration procedure.

Let us resolve Δu^1_i into two parts such as

$$\Delta u^1_i = \Delta u^{11}_i + \Delta u^{12}_i \qquad \Longrightarrow \qquad \Delta\varepsilon^{1y}_{kl} = \Delta\varepsilon^{11}_{kl} + \Delta\varepsilon^{12}_{kl} \tag{28}$$

$$\Delta\varepsilon^{11}_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u^{11}_i}{\partial y_j} + \frac{\partial \Delta u^{11}_j}{\partial y_i}\right), \qquad \Delta\varepsilon^{12}_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u^{12}_i}{\partial y_j} + \frac{\partial \Delta u^{12}_j}{\partial y_i}\right) \tag{29}$$

$$\Delta u^{11}_i(\boldsymbol{x}, \boldsymbol{y}; t) = \Delta u^{11}_i(\boldsymbol{x}, \boldsymbol{y} + \boldsymbol{Y}; t), \qquad \Delta u^{12}_i(\boldsymbol{x}, \boldsymbol{y}; t) = \Delta u^{12}_i(\boldsymbol{x}, \boldsymbol{y} + \boldsymbol{Y}; t) \tag{30}$$

where $\Delta\varepsilon^{11}_{ij}$ satisfies the local elastic problem given by

$$\frac{\partial}{\partial y_j}[E^\varepsilon_{ijkl}\Delta\varepsilon^{11}_{kl}] = -\frac{\partial}{\partial y_j}[E^\varepsilon_{ijkl}\Delta\varepsilon^0_{kl}] \tag{31}$$

and $\Delta\varepsilon^{12}_{ij}$ satisfies the local plastic problem given by

$$\frac{\partial}{\partial y_j}[E^\varepsilon_{ijkl}\Delta\varepsilon^{12}_{kl}] = -\frac{\partial}{\partial y_j}[d\lambda^\varepsilon \sigma^g_{ij}] \tag{32}$$

$$\frac{\partial f^*}{\partial \sigma^*_{ij}} E^\varepsilon_{ijkl}\Delta\varepsilon^{12}_{kl} - d\lambda^\varepsilon T^\varepsilon + f_* \leq 0; \qquad f_* = f^* + \frac{\partial f^*}{\partial \sigma^*_{ij}} E^\varepsilon_{ijkl}(\Delta\varepsilon^0_{kl} + \Delta\varepsilon^{11}_{kl}). \tag{33}$$

Eqn(31) is same as the elastic problem, so we introduce a characteristic function W^{kl}_i by

$$\Delta u^1_i(\boldsymbol{x}, \boldsymbol{y}; t) = -W^{kl}_i(\boldsymbol{y})\frac{\partial \Delta u^0_k}{\partial x_l} + c(\boldsymbol{x}), \qquad (c_k(\boldsymbol{x});\ \text{a constant in } \boldsymbol{y}), \tag{34}$$

Substituting this into Eqn(31) yields the following PDE to determine W^{rs}_i:

$$\frac{\partial}{\partial y_l}\left[E^\varepsilon_{klij}(\delta_{ri}\delta_{sj} - \frac{\partial W^{rs}_i}{\partial y_j})\right] = 0. \tag{35}$$

This PDE defined in the unit cell can be solved, for example, by a finite element method under the periodic boundary condition (30). Note that the homogenized Young's modulus E^h_{ijkl} given in Eqn(24) is now explicitly written as

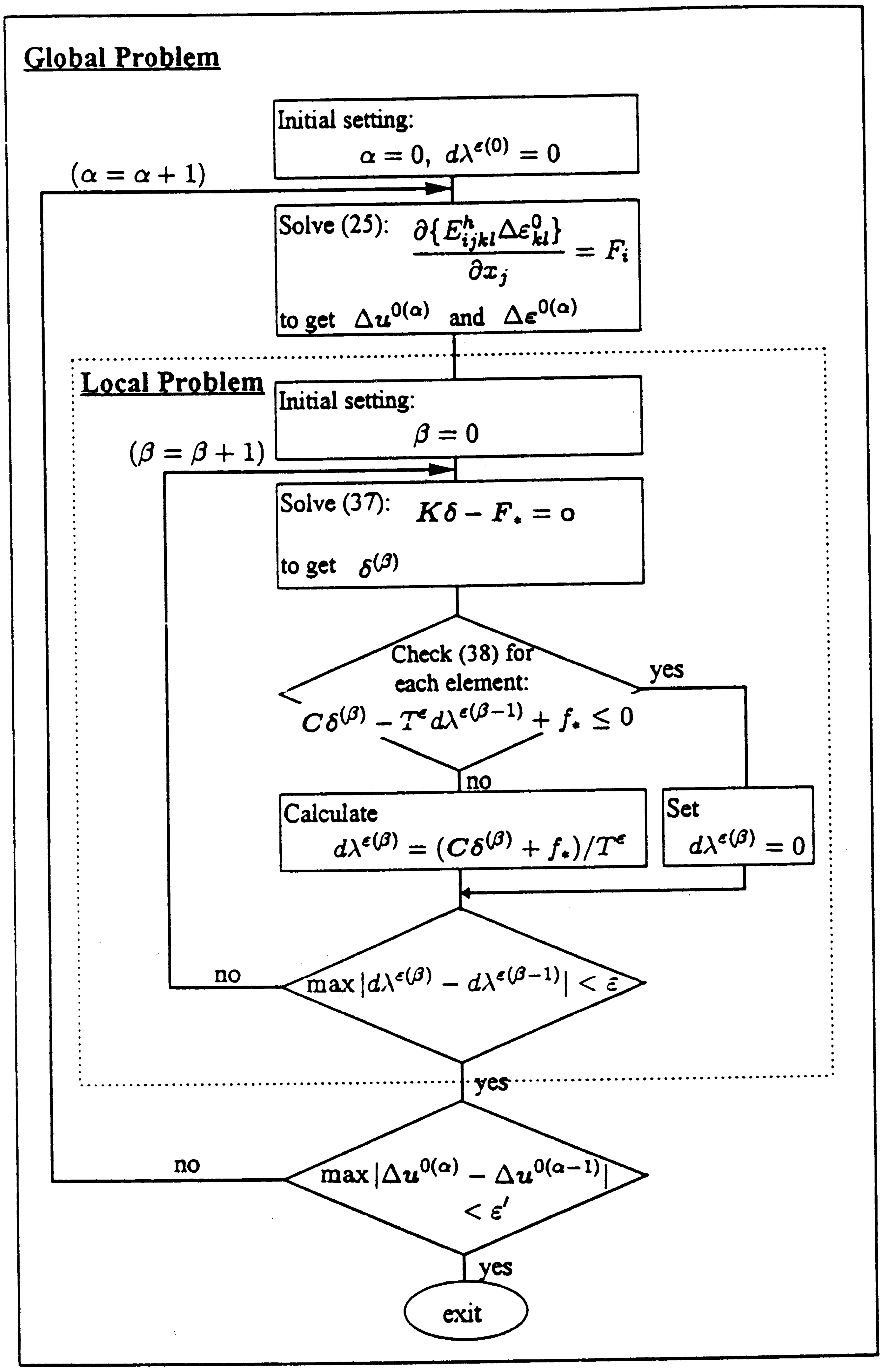


Figure 2. A local iteration scheme for the elastoplastic homogenization problem.

$$E^h_{ijkl}(\boldsymbol{x}, \varepsilon^0_{rs}; t) = \frac{1}{|Y|} \int_Y E^\varepsilon_{qnkl}(\boldsymbol{x}, \varepsilon^0_{rs}; t) \left[\delta_{qi}\delta_{nj} - \frac{\partial W^{ij}_q}{\partial y_n} \right] dy. \tag{36}$$

In order to solve Eqn(32) under the constraint (33) we also introduce a finite element approximation, so we have

$$\boldsymbol{K\delta} - \boldsymbol{F}_* = \boldsymbol{o} \tag{37}$$

$$\boldsymbol{C\delta} - T^\varepsilon d\lambda^\varepsilon + f_* \leq 0 \tag{38}$$

where the vector $\boldsymbol{\delta}$ consists of nodal values of Δu_i^{12} discretized by using the shape function $\boldsymbol{N}$, and others are

$$\boldsymbol{K} = \int_{Y^e} \boldsymbol{B}^T \boldsymbol{D}\boldsymbol{B} dY, \qquad \boldsymbol{F}_* = \int_{Y^e} \boldsymbol{B}^T d\lambda^\epsilon \boldsymbol{\sigma}^g dY, \qquad \boldsymbol{C} = \boldsymbol{df}_\sigma^* \boldsymbol{D}\boldsymbol{B}.$$

Here $\boldsymbol{B}$ is the strain-displacement matrix, $\boldsymbol{D}$ the matrix form of E_{ijkl}^ϵ, $\boldsymbol{\sigma}^g$ the vector form of σ_{ij}^g, and $\boldsymbol{df}_\sigma^*$ the vector form of $\partial f^*/\partial \sigma_{ij}^*$. Eqn(37) under the constraint (38) can be solved by using a local iteration scheme which is shown in Figure 2.

A NUMERICAL EXAMPLE FOR MIXED SOIL

A mixed soil is studied here whose matrix is elastoplastic. For the compacted clay part, the Tsinghua model is used and its model parameters are as follows:

Elastic part: The bulk modulus B and shear modulus G are used. They are given by

$$B = 157.8p, \qquad G = 103.33P_a \left(\frac{\sigma_3}{P_a}\right)^{0.8514}$$

where $p = \sigma_{ii}/3$, and P_a is the atmospheric pressure whose value can be taken as $1kgf/cm^2$.

Plastic part: An associated flow law is employed, so

$$f = g = \left(\frac{p - H}{1.206H}\right)^2 + \left(\frac{q}{1.46H}\right)^2 - 1 = 0; \qquad q = \sqrt{\frac{3}{2}(\sigma_{ij} - p\delta_{ij})(\sigma_{ij} - p\delta_{ij})}$$

and the hardening function is given by

$$H = \frac{P_a}{2.206}\left(\frac{\varepsilon_v^p - 0.06\bar{\varepsilon}^p}{0.0166}\right)^2 + \frac{P_r}{2.206},$$

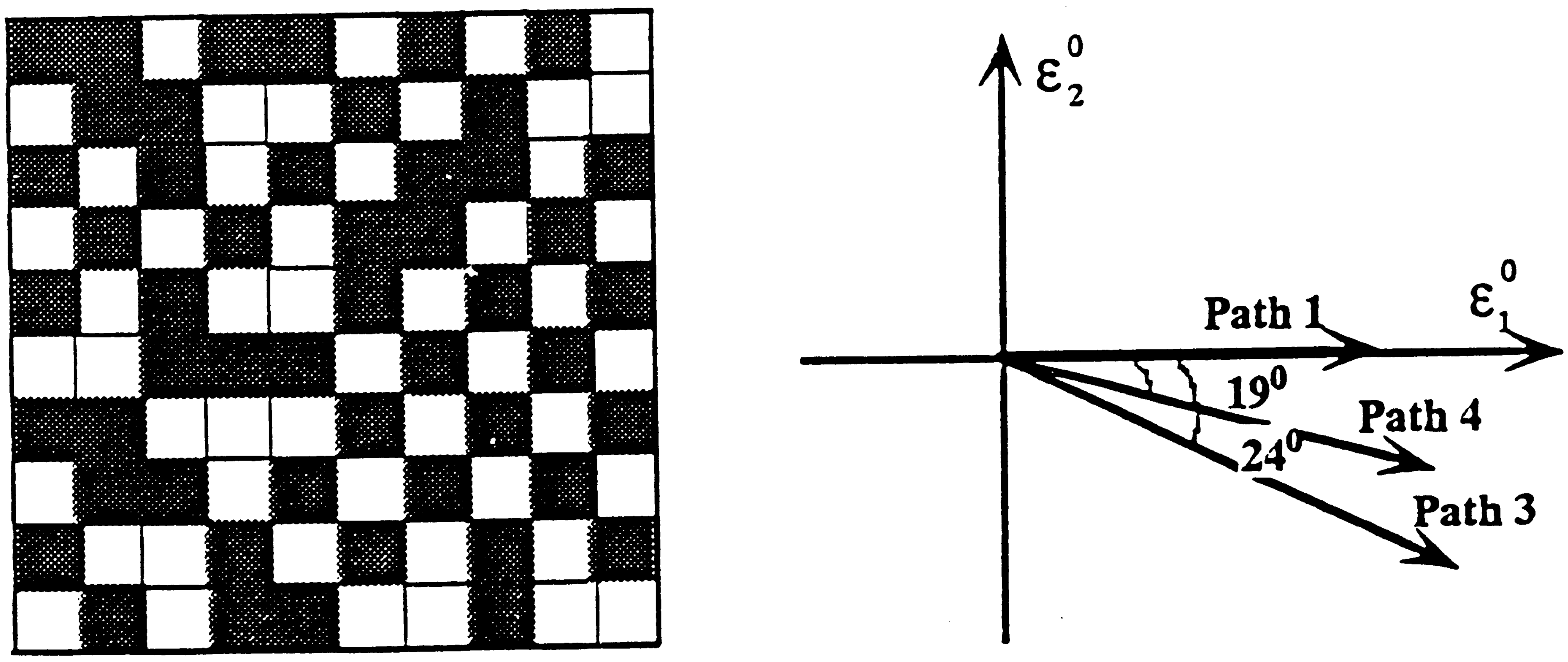
so the loading/unloading criterion is

$$\begin{cases} H \geq H_{max} & : \text{loading} \\ H < H_{max} & : \text{unloading/reloading} \end{cases}$$

where $P_r = 0.337P_a$ is the pre-consolidated pressure.

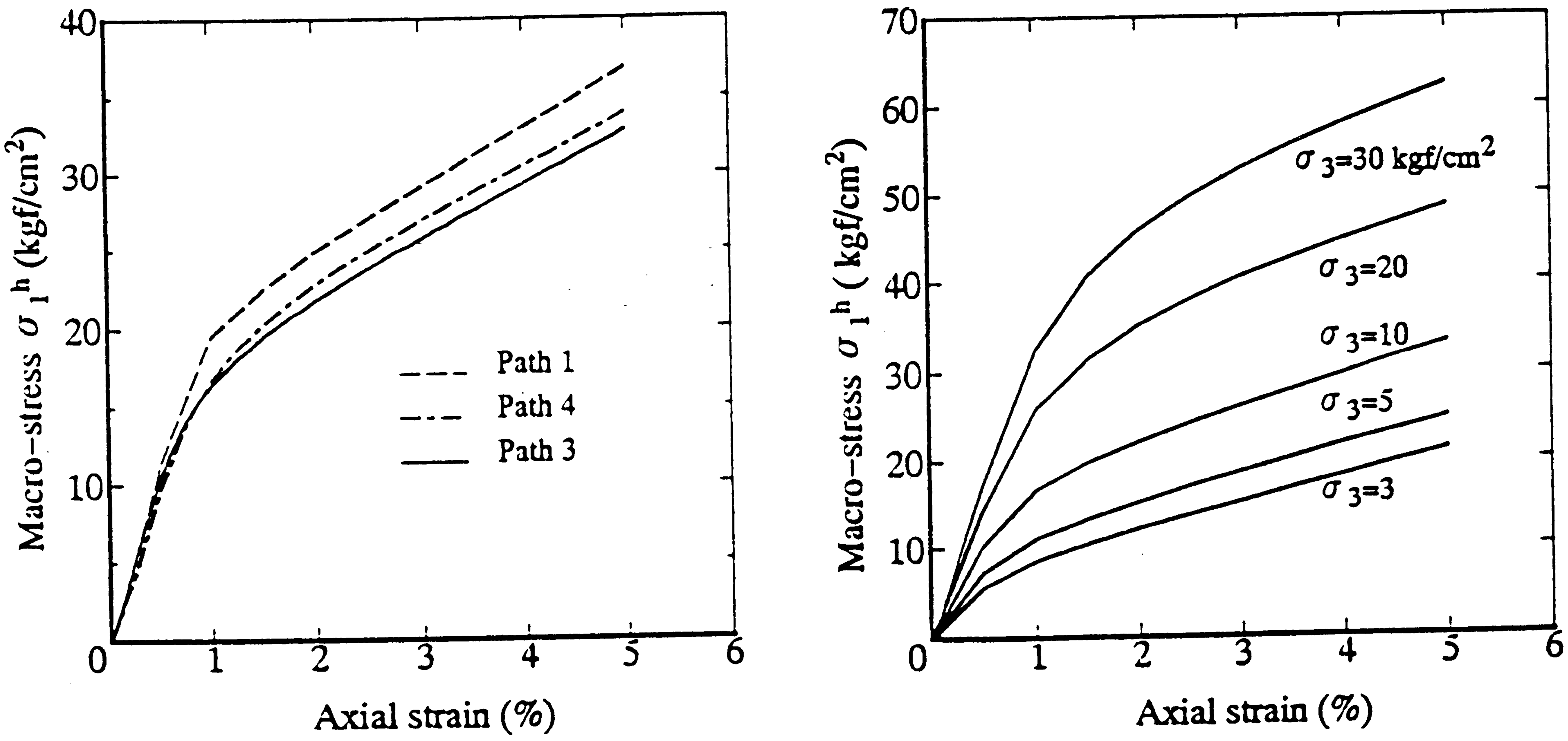
Figure 3(a) shows the microstructure of mixed soil to study the effect of strain paths. Here the black part is elastic inclusions. The volume fraction of the inclusions is 50%. The elastic constants of inclusions are assumed to be as follows: Young's modulus $E = 1000kgf/cm^2$ and Poisson's ratio $\nu = 0.3$. Strain paths are shown in Figure 3(b). Two cases are studied here. One is for checking the effect of strain paths on homogenized stress/strain relationship under initial stress $\sigma_3 = 10kgf/cm^2$. The other is for checking the effect of different initial stress states. These initial stress states are $\sigma_3 = 3, 5, 10, 20, 30kgf/cm^2$ under path 4. A plane strain condition is assumed.

Figure 4 shows the simulated curves. It is observed that both strain path and confining pressure have a vital effect on the homogenized stress-strain relation.



(a) (b)

Figure 3. Microstructure of a mixed soil and strain paths.



(a) Strain path effect (b) Confining pressure effect

Figure 4. Simulation for the mixed soil.

REFERENCES

Chen, W.F., and Saleeb, A.F.(1981, 1982), *Constitutive Equations for Engineering Materials, Vol.1, Elasticity and Modeling; Vol.II, Plasticity and Modeling*, Wiley, New York.

Roscoe, K.H., Burland, J.B.(1968), "On the generalized stress-strain behavior of 'wet clay'," *Engineering Plasticity*, Ed. J. Heyman and F.A. Leckie, Cambridge University Press.

Wang, J.G., and Chen, B.(1993), "Consideration on characteristics and recognition theories of soils," *J. of Chongqing University*, **16**(3), 25-29.