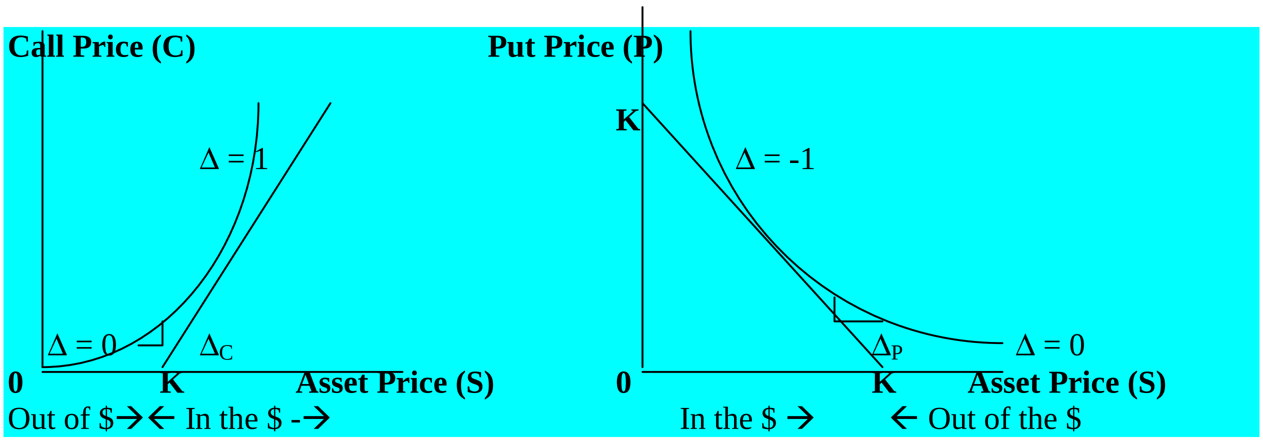
OPTIONS

1. *Valuation of Options Contracts*

a. Introduction

- The Value of an Option can be broken down into 2 Parts
  1. **INTRINSIC Value**, which depends only upon the price of the asset underlying the option in relation to the option's exercise price.
     Call = S – K, as long as the difference is a positive value, else C=0
     Put = K – S, as long as the difference is a positive value, else P=0
     Where S = the price of the underlying asset; and
     K = the exercise price of the option
  2. **TIME Premium** is a function of the probability that the option could change in the value by the time it expires. The time premium depends upon 4 variables, including the amount of time before expiration is reached, the volatility of the underlying stock, the time value of money, and the yield on the underlying asset. At expiration, the value of the option will be it's intrinsic value, and the time value will be zero



- Six Factors Determine the VALUE of Options (#1 determines the Intrinsic Value, 2-6 determine the Time Premium)
  1. **Exercise Price**: Ceteris Paribus, the higher the K, the less calls are worth, and the more puts are worth
  2. **Price of the Underlying Asset**: For any given K, the value of a Call rises as the price of S rises, and the value of a Put rises as the Price of S falls
  3. **Volatility**: The greater the volatility of the underlying asset (S), the more a put or call will be worth, ceteris paribus
  4. **Time Until Expiration**: The longer the time 'til expiration, the higher the value of the put or call, ceteris paribus
  5. **The Level of Interest Rates**: The Higher the Level of Interest Rates, the Higher the value of the Call and Lower the value of a Put, ceteris paribus. (NOTE: when rates rise, 2 factors affecting the value of options are affected #1 S (impacting intrinsic value), #2

the Time Value of Money (affecting time premium). The concept that rising interest rates increase the value of calls and lower puts only relates to the time premium. This can be offset by the impact on the intrinsic value

6. **Rising Dividends**: cause CALLS to fall, and PUTS to rise, ceteris paribus. This is because stock prices are adjusted downward by the amount of the dividend when the stock goes ex-dividend (but the strike prices of calls and puts are not adjusted).
7. **Type of Option**: AMERICAN options can be exercised at any time up to Strike; EUROPEAN options can only be exercised on the expiration date. Thus, American options are usually more valuable (but never worth less than European)

b. Valuing Call Options using the Single-Period Binomial Model

- The Value of a CALL can be APPROXIMATED by using the Single-period Model. This is a type of Riskless Hedge Model illustrated by the following example (Using a **3 Step Procedure**)

For Example: The Common Shares of XYZ, Inc. are trading at 24. Determine the Fair Market Value of a Call Option on XYZ shares under the following conditions:

Exercise Price	20
Expiration	3 months
Risk free rate	8% (annualized)
Range of possible prices within 3 months	Current Price +/- 6

1. Determine the Hedge Ratio

To do this, Determine a Hedged Position consisting of one share of stock, plus a certain number of Options (X) on a share that will produce the same wealth position at the end of the period, whether the stock is at its possible high or low (S +/- Range)

S = 24, +6 → 30, -6→ 18

(S – range)	(Lowest Option(K-S))	(X)	=		(S + range)	(Highest Option (S-K))	(X)
18	0	X	=	30	10		X

18 + (0)(X) = 30 + (10)(X)

X = -1.2

The Negative sign means that to produce the total hedge, the Call Option must be sold short against the stock

2. Discount the Hedged Ending Wealth to Its Present Value using the risk-free rate as the discount rate

$PV_{Ending\ Wealth}$=[Ending Stock Price – (Ending Option Price)(#Options used)] / $(1+r_F)^t$

Where t is the fraction of the year where the bond is ALIVE

$PV_{ending\ wealth}$= $[18 – (0*1.2)]/(1.08)^{.25}$ = $[30 –(10*1.2)]/(1.08)^{.25}$ = \$17.66 → both high & low produce same answer

3. Determine the Value of the Call Option based upon the CURRENT price of the stock

$PV_{Ending\ Wealth}$ = Current Stock Price – (#Calls Shorted)(Call Price)

\$17.66 = 24 – (1.2)(Call Price)

Call Price = \$5.28

c. Valuing Put Options with Put-Call Parity

- Once the Value of a Call Option is determined, the Value of a Put Option with the Same Parameters can be determined from Put-Call Parity
- Put-Call Parity is based upon a Riskless Hedge→ Owning 1 Share of Stock plus a Put on One share, and being short a call on one share (with strike prices and expiration dates of both options being the same) will guarantee an ending wealth value at the expiration date of the option equal to the strike price of the options, plus any dividend earned on the stock during the time the options are ALIVE

$S + P - C = K + D_P$

S = Stock Price; P = Put Price; C = Call Price, K = Strike; D_P = Div. Paid while option alive

For Example:

Stock Price	$50.00
Dividend per Quarter	$ 0.50
Strike Price on Options	$45.00
Expiration	3 months

If the stock falls to 40 in 3 months, the value of the hedge will be $45.50

Value of Stock	$40.00
Dividend	$ 0.50
Put	$ 5.00
Call	$ 0.00
Value of Portfolio	$45.50

If the stock rises to 65 in 3 months, the value of the hedge will still be $45.50

Value of Stock	$65.00
Dividend	$ 0.50
Put	$ 0.00
Call	-20.00
Value of Portfolio	$45.50

Since this hedge will always produce the same result at the time the option expire, one should always be willing to pay the PRESENT VALUE of the STRIKE PRICE + the PRESENT VALUE of any DIVIDEND to be Paid during the option's Life to Acquire it Today

$S + P - C = [K/(1+r_F)^t] + [D_P/(1+r_F)^t]$

Applied to the Previous example, if no dividend is paid while the option is alive, the price of the Put would be

$24 + P - C = [20/(1.08)^{.25}] + 0$

P = $0.90

d. Illustrating the Impact of Selected Factors on the Value of Options

- One Can Test All of these using the Single Period Binomial Model:
  Stock's Low Value +[(Option Low Value)X] = Stock's High Value + [(Option's High Value)X]
  And then, measure the Present Value of the Ending Wealth; then use Put-Call Parity to figure the rest out.

- ***The <u>Price</u> of the Underlying Stock (Delta & Gamma)***
  1. ***Δ***; The Sensitivity of the Call & Put prices to Changes in the Price of the underlying Security. As the Price of the underlying stock rises, the value of the call rises and the value of the put falls, ceteris paribus
     $\Delta_{Call} = [\Delta_{Call\ price} / \Delta S]$ $\Delta_{Put} = [\Delta_{Put\ Price} / \Delta S]$
     $\Delta_{Call} - \Delta_{Put} = 1$
  2. ***Φ:*** The Rate at which the Δ of an Option changes as the price of the underlying security changes. The Φs of a Call & Put option with Identical parameters are equal. The HIGHEST Absolute value of an option's Φ occurs when the option is "AT THE MONEY"; option Φs are 0 when the option is either deep in, or deep out of the money
     $\Phi_{Call} = [\Delta\Delta_{Call} / \Delta S]$ $\Phi_{Put} = [\Delta\Delta_{Put}/\Delta S]$

- ***The <u>Volatility</u> of the Underlying Stock (Vega)***
  1. ***Vega***: As volatilities of the underlying stock rise, the call prices rises and the put price rises (Both Call & Put moves in the Same direction). The Vega tends to be highest when an option is "At the Money" and it moves toward Zero as the option becomes deep in, or deep out, of the money. Note: both call & put with same parameters have equal Vegas.
     $V_{Call} = [\Delta \text{Call Price} / \Delta\sigma_S]$ $V_{Put} = [\Delta \text{Put Price} / \Delta\sigma_S]$

- ***The Level of the <u>Risk-Free</u> Rate (Rho)***
  1. ***ρ :*** The ρ of a CALL option tends to be LOW for Deep Out-of-the-money Call options; it tends to be most sensitive to the underlying stock's price when a Call is close to being 'at the money'. As the underlying stock's price moves deep into the money, the ρ tends to level off at a high value. The ρ of a PUT option tends to level off at a highly negative value when the put is deep in the money; it levels off at a less negative value when the underlying stock is deep out of the money. The ρ is most sensitive to the underlying stock's price when the put is close to being at the money. The ρ also depends on the time until expiration. For both Calls & Puts, the value of the ρ moves toward zero as the time until expiration decreases.
     $\rho_{Call} = [\Delta \text{Call Price} / \Delta r_F]$ $\rho_{Put} = [\Delta \text{Put Price} / \Delta r_F]$

- ***The <u>Time</u> until Expiration (Theta)***
  1. ***θ***: The sensitivities of option prices to changes in the time until expiration is called the θ of the option. The θ tends to have its maximum (negative) value when an option is 'at the money'; it moves toward zero as the option becomes deep in, or deep out of the money. Note that thetas are normally considered to be NEGATIVE values in order to indicate that, as the time until expiration of an option declines Toward zero, the value of the option undergoes a Time decay; i.e., the time premium component of an option's value declines with time.
     $\theta_{Call} = [-\Delta Call\ Price / \Delta t]$ $\theta_{Put} = [-\Delta Put\ Price / \Delta t]$

- ***Dividends***
  1. To calculate the price of call options on a stock that pays a dividend, the single-stage binomial model can be used. However, in the last step of the process, subtract the present value of the dividend, paid during the time the option was 'alive,' from the price of the stock when performing the calculation. This is done because the price of the stock will be reduced by the amount of the dividend paid when the stock goes ex-dividend, but the strike price of the option will not be adjusted accordingly
  2. The Existence of a dividend, ceteris paribus, LOWERS the value of the CALL and RAISES the value of a PUT (relative to their prices if the stock did not pay a dividend)

e. **Valuing Call Options Using the Multi-Period Binomial Model**

- When using the multi-period model, just go through the same steps as the single period model, except work backwards from the 2-period forward point back to the current time to determine the call price

f. Valuing Call Options with the Black-Scholes Model

- The Binomial Model is a Discrete function. Stock prices, however, are continuous variables. Thus, a better model for valuing options would be one that is based on a continuous probability distribution for describing the value of the underlying stock when the options contract expires. The Black-Scholes Model does this. It uses the following ASSUMPTIONS
  1. The options are EUROPEAN
  2. The RISK-FREE interest rate and the VOLATILITY of the UNDERLYING STOCK are CONSTANT over the life of the option
  3. The Stock Underlying the Call Option DOES NOT PAY a DIVIDEND
  4. TRANSACTION COSTS are ZERO
  5. The EXPECTED PRICE of the stock underlying the option on the expiration date of the option can be determined by assuming that the percentage rise or fall in its price from current levels can be described by a LOGNORMAL probability distribution (stock price movements can be described by GEOMETRIC-BROWNIAN motion)
- Using these assumption

  $C = SN(d_1) – Ke^{-rt}N(d_2)$

  C = Value of Call
  S = Stock's Price
  K = Strike Price
  N(d) = The Value of the area under a normal curve from the Extreme Left Tail to (d)
  $d_1 = [\ln(S/K) + (r+.5\sigma^2)t] / \sigma(t)^{.5}$
  $d_2 = d_1 – \sigma(t)^{.5}$
  r = Risk-free rate continuously compounded
  t = time to expiration, measured in fractions of a year
  σ = Annual σ of the Stock's return (volatility)
- Empirical Studies have shown that Black-Scholes is Reasonably accurate in valuing call options; HOWEVER,
  1. The Model works best for AT THE MONEY options. Actual price deviations from theoretical values become greater as the option moves deeper into, or out of, the money. Deep in the money options tend to sell at lower prices, while deep out of the money options tend to sell at higher prices than the model predicts
  2. The Model tends to undervalue American Options, since it assumes the options are European
  3. The Model tends to OVERVALUE call options, while Undervaluing Put options on stocks that pay dividends, because it ASSUMES NO dividends
  4. The Model uses as a component the Volatility of the underlying stock. There are several methods (none of them great) to attempt to do this

- *The Historical Approach*: This can be done by computing the Daily volatility in the underlying stock's return over N days
  $\sigma^2_{Daily} = [1/(N-1)]\Sigma(r_i - r_{mean})^2$
  $\sigma^2_{Yearly} = 250\sigma^2_{Daily}$
  Advantages: It is an easy calculation without any assumption regarding market efficiency
  Disadvantages: It assumes past volatility and future volatility will be equal
- *The Scenario Approach*: The requires the analyst to determine several possible scenarios about the state of the economy or market and the effect each scenario will have on the return on the underlying stock. Then, the analyst must assign a probability to each scenario. The variance of the returns is then determined from this relationship. However, this approach is HIGHLY Subjective and difficult to employ
  $\sigma^2_R = E(_R{}^2) - [E_{(R\ mean)}]^2$
- *The Implied Volatility Approach*: This uses the Black-Scholes Model itself to determine σ. The analyst inputs the current prices of the underlying stock and call option, along with the known parameters of the option into the Black-Scholes formula and solves for σ. This can only be done with a computer program to generate the solution via a trial & error method. This is then done with different parameters and the various implied volatilities can then be averaged to produce the WEIGHTED IMPLIED σ of the stock's returns. The weights are based on how close the options are to being At the Money. If the implied volatility is higher than the historical or expected volatility of the stock's price, assuming an efficient market, the inference can be made that the call option is overpriced; if the calculated implied volatility is lower than normal, the option would appear to be undervalued. Empirical evidence suggests that implied volatilities tend to REGRESS toward the long-term mean values.

_g._ Valuing Puts with the Black-Sholes Model

- The Black-Scholes Model is used to value call options. Once the value of the call is determined, put-call parity can be used to value the comparable put by simply substituting the Black-Scholes formula for C in the following relationship.
  $S + P - C = Ke^{-rt}$

h. Adjusting the Black-Sholes Model to Include Dividends (the MERTON Model)

- The Merton Model adjusts the Black-Scholes Model to incorporate dividends into the analysis. The assumption is that dividends are paid continuously in order to simplify the mathematics. While not realistic for stock options, it is very good with regard to foreign-currency options on which a continuously compounded risk-free rate can be earned. It also works for Stock index options to the extent that the basket of stocks underlying the index pays a dividend stream that resembles a continuous dividend
- To adjust for the continuous dividend, Merton substituted the term $\boxed{Se^{-\delta t}}$ for S in the Black-Scholes model

i. Using the Merton Model to Value Foreign Currency Options

- The Merton model can be used to value foreign currency options by simply letting S equal the spot exchange rate of the underlying currency and δ equal the interest rate in the country that issues the foreign currency.

j. The Binomial Model & the Merton Dividend Model

- The Binomial model modifies for dividends by subtracting the present value of a dividend paid during the option's life from the price of the stock used in the binomial calculations. Alternatively, the continuous dividend rate could be subtracted from the risk-free rate used in the binomial model calculations. When this adjustment is made, the binomial model and the Merton model are analogous and can be used for options paying a continuous dividend

2. ***Strategies Employing Options***

a. Reasons for Trading Options

- To SPECULATE by Buying the Option at one Price and Selling it for another, in the hopes of making a profit. Speculators like the potential for high returns on small investment requirements
- To ALTER RISK and RETURN Characteristics of a Portfolio. Unlike the outright purchase (or short sale) of assets, forward, or futures contracts which tend to produce payoff patterns and distributions that are linear and normally distributed, OPTIONS tend to Produce Payoff Patterns and Distributions that are NONLINEAR and SKEWED
- For short-term investment horizons, options trading can produce LOWER TRANSACTION costs than outright purchase and sale of the assets themselves
- Options can be used to execute some TAX STRATEGIES
- Options can be used to AVOID CERTAIN STOCK RESTRICTIONS

b. Payoff Possibilities to Analyze Options

- Determine the MAXIMUM Loss the strategy can produce
- Determine the MAXIMUM Gain that the strategy can produce
- Determine the BREAK-EVEN point of the strategy
- Gain/Loss produced by strategy if the Stock closes at certain price

c. Speculative Strategies

- **Buying a Naked Call**

1. Buy a Call option outright.

For Example: Buy a Call option with a Strike Price of 40 on a stock currently trading at 39 ¼. Pay 3 for the call. Analyze the payoff possibilities.

Stock Price (S)	**Intrinsic Value [c=max; (S-K),0]**	**Portfolio Value V_P =$100C**	**Profit (per option) $\Pi = 100(C_t - C_0)$**
38	0	0	(300)
39	0	0	(300)
40	0	0	(300)
41	1	100	(200)
42	2	200	(100)
43	3	300	0
44	4	400	100
45	5	500	200

From the Payoff Analysis Table, it is apparent that;

The MAXIMUM LOSS is $300.

The Most one can lose by BUYING a NAKED CALL is the PRICE PAID for the CALL

The MAXIMUM GAIN is unlimited.

The MOST one can Gain by BUYING a NAKED CALL is UNLIMITED

The BREAKEVEN point is 43.

$S_{BE(naked\ call)} = K+C$

2. Buying a Naked Call is a Strategy that requires a BULLISH outlook for the Underlying Stock

- **Writing a Naked Call**

1. Writing (Selling) a Call

For Example: Write a Call Option with a Strike Price of 40 on a Stock Currently trading at 39 ¼. Receive 3 as a premium. Analyze the payoff possibilities.

Stock Price (S)	**Intrinsic Value [c=max; (S-K),0]**	**Portfolio Value V_P = - $100C**	**Profit (per option) $\Pi = 100(C_0 - C_t)$**
38	0	0	300
39	0	0	300
40	0	0	300
41	1	(100)	200
42	2	(200)	100
43	3	(300)	0
44	4	(400)	(100)
45	5	(500)	(200)

From the Payoof Analysis Table, it is apparent that:

The MAXIMUM LOSS is UNLIMITED

The MOST one can lose by WRITING a NAKED CALL is UNLIMITED

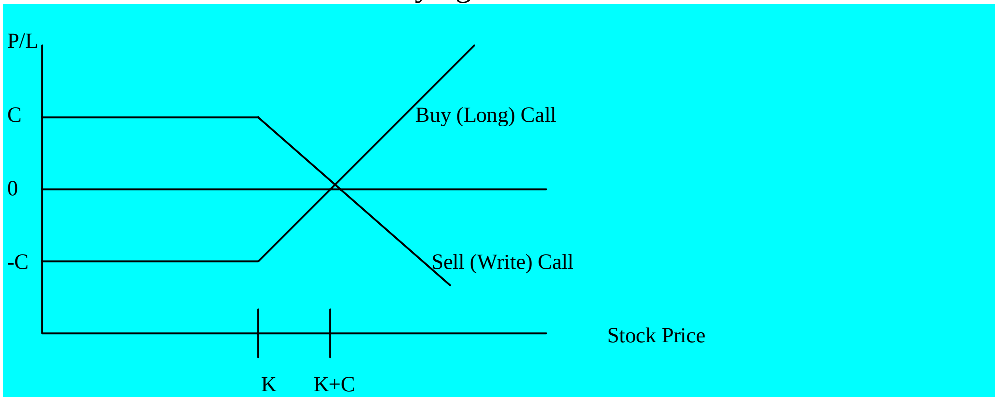
The MAXIMUM GAIN is $300

The MOST one can GAIN by WRITING A NAKED CALL is the CALL PREMIUM

The BREAKEVEN point is 43

$S_{BE\ (naked\ call)} = K + C$

2. Writing a Naked Call is a Strategy the requires a BEARISH outlook for the Underlying Stock



- **Buying a Naked Put**
  1. The Outright Purchase of a Put

For Example: Suppose an investor buys a put on a stock that is trading at 39 ¼. The Strike price is 40 and the option is priced at 2.

Stock Price (S)	Intrinsic Value [P=max; (K-S),0]	Portfolio Value V_P =$100P	Profit (per option) $\prod = 100(P_t-P_0)$
35	5	500	300
36	4	400	200
37	3	300	100
38	2	200	0
39	1	100	(100)
40	0	0	(200)
41	0	0	(200)
42	0	0	(200)

The MAXIMUM LOSS is $2

MAX LOSS for a BUYER of NAKED PUT is PRICE PAID for PUT

The MAXIMUM GAIN is $3800 (38/share)

MAX PROFIT is $\prod_{max} = K - P_0$

The BREAK EVEN is 38

$S_{BE\ (naked\ put)} = K - P$

  2. BUYING a NAKED PUT is a Strategy that Requires a BEARISH outlook for the underlying Stock

- **Writing a Naked Put**
  1. Selling (Writing) a Put

For Example: Suppose an investor buys a put on a stock that is trading at 39 ¼. The Strike price is 40 and the option is priced at 2.

Stock Price (S)	Intrinsic Value [P=max; (K-S),0]	Portfolio Value V_P = - $100P	Profit (per option) $\prod = 100(-P_t+P_0)$
35	5	(500)	(300)
36	4	(400)	(200)
37	3	(300)	(100)
38	2	(200)	0
39	1	(100)	100
40	0	0	200
41	0	0	200
42	0	0	200

The MAXIMUM LOSS is $3,800 (38/share)

$\prod_{min\ (write\ naked\ put)} = -(K-P_0)$

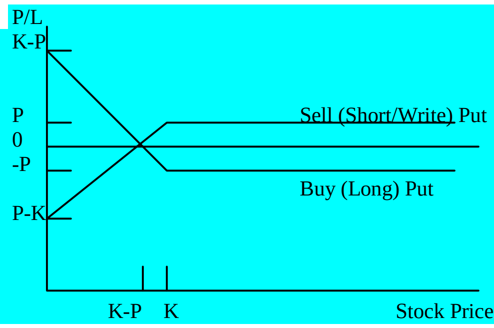
The MAXIMUM GAIN is $200 (2/share)

MAX GAIN from WRITING a PUT is the PREMIUM

The BREAK EVEN point is 38

$S_{BE\ (naked\ put)} = K - P_0$

  2. WRITING a NAKED PUT is a Strategy that requires a BULLISH outlook for the underlying Stock



d. Writing Covered Calls

- An Investor can Write Calls on a Stock Already owned.

For Example: An investor owns 100 shares of Stock trading at $50 (value of $5,000). He writes a covered call (selling 1 6-month call option with Strike price of $45 currently trading at $8).

Stock Price (S)	Intrinsic Value Call C = Max: (S-K), O	Portfolio Value V_P= 100(S-C)	Profit (per call) $\prod$=100($S-C_t-S_0+C_0$)
40	0	4,000	(200)
41	0	4,100	(100)
42	0	4,200	0
43	0	4,300	100
44	0	4,400	200
45	0	4,500	300
46	1	4,600	300
47	1	4,700	300

- Compared to owning stock alone, A Covered Call Strategy LOWERS the BREAK EVEN Point, LIMITS the UPSIDE POTENTIAL, and has the possibility of a Large Downside Loss
- SHOULD USE ONLY WHEN BELIEVE STOCK PRICE WILL REMAIN STABLE

e. Protective Put Strategy (Buy a Stock & a Put)

- An investor can Buy Puts on Stock Already Owned

For Example: an investor owns 100 shares of stock trading at $50. He buys 1 6-month put with a strike at 50 and priced at 3.

Stock Price (S)	Intrinsic Value Put P = Max: (K-S),0	Portfolio Value V_P=100(S+P)	Profit (per put) $\prod$=100($S_t+P_t-S_0-P_0$)
47	3	5,000	(300)
48	2	5,000	(300)
49	1	5,000	(300)
50	0	5,000	(300)
51	0	5,100	(200)
52	0	5,200	(100)
53	0	5,300	0
54	0	5,400	100
55	0	5,500	200

- Compared to owning the stock alone, there is a LIMITED MAXIMUM LOSS = Premium paid for the Put, an UNLIMITED UPSIDE POTENTIAL, and a rise in the breakeven point.
- Useful in situations when one thinks the stock will either RISE or FALL Significantly. If the price remains flat, it is a bad strategy.
- It is a form of PORTFOLIO INSURANCE. The cost of the insurance is the cost of acquiring the put.
- The PROBABILITY Distribution of Returns from the Protective Put strategy over the 6-month time horizon can be determined from the probability distributions of the returns of the underlying stock.

For Example: Assume the underlying stock has an expected return of 5% over the 6-month life of the option, with a σ +/- 15%. Buying a protective put at 3 will limit the loss potential of the strategy to $300, which occurs if the ending value of the stock on expiration is $50 or less. This occurs when there is 0% return on the stock, and one can use a normally distributed probability distribution to figure out this possibility.
The Z-Score associated with a ZERO return, in this case is:

$Z_0 = [(r_0 - \mu_r) / \sigma_r]$ = [(0-5)/15] = -.3333

From the Z Table, at .3333 is 12.93. Thus, the area to the left of that, which here is the area that there will be a loss, is 37.07%. thus, there is a 37.07% probability of earning a $300 loss.

- It is also POSSIBLE to Calculate the PROBABILITY of Earning a PROFIT with this Strategy

For Example: The Break-even point of this strategy is 53. This is a 6% return (53-50/50), The Z-Score associated with a 6% return is $Z_6 =[(r_6-\mu_r) / \sigma_r]$ = [(6-5)/15]=.0666
From the Z Table, this is an area of 2.65%. Thus, there is a 47.35% probability of a profit.

- Compared to a Stock Only portfolio, which can produce a 100% loss, a protective put portfolio has a maximum loss of 5.66% (here, where it is –300/5000). Also, the probability of this maximum loss is 37.07%, whereas the probability that the stock only portfolio would generate a loss of 5.66% is 23.88% [$Z_{-5.66}$= [$(r-\mu_r)/\sigma_r$] = [(-5.66-5)/15]=-.71, which is area between Z=0 and Z=.71 is .2612 (thus, the area to the left of Z=-71 is 23.88%].
- Using Portfolio Insurance INCREASES the probability of incurring a SMALL LOSS while ELIMINATING The PROBABILITY of INCURRING a LARGE LOSS

3. ***Mimicking & Synthesizing Portfolio Characteristics Using Put-Call Parity***

- Put-Call Parity states that being LONG a STOCK, LONG a PUT, and SHORT a CALL, assuming similar parameters, will always produce a terminal value = the strike price on the options (excluding effect of dividends). In an efficient market, then,
  $S+P-C = [K/(1+r_f)^t]$

- Therefore, a **SYNTHETIC CALL** can be Replicated as follows
  $C = S+P-[K/(1+r_f)^t]$
- This means OWNING a STOCK & a PUT on the same stock, and BORROWING the present value of K dollars at the Risk-free rate, creates a Portfolio that will replicate the behavior of a call option on the stock with a strike equal to the Future value of the dollars borrowed (K) and an exercise date equal to the exercise date on the put (which is the time when the borrowed funds must be repaid)

  For Example: a Manager wishes to purchase a 6-month call on XYZ stock with a Strike of 30. But, the market for such calls is not liquid. Given the following data, what can the manager do? XYZ at 45; 6-month puts with K=30 are trading at 1/8; 6-month risk free rate is 5%
  Answer: The manager can create a synthetic call on XYZ using the put-call parity relationship.
  $C = S+P-[K/(1+r_f)^t] = 45 + 1/8 - [30/(1.05)^{.5}] = 15.848$
  This can be interpreted to mean a synthetic call is created by purchasing the stock at 45, purchasing the put at 1/8, and financing the purchases by borrowing $29.277 at the risk-free rate for 6 months and incurring out-of pocket expenses of $15.858, which is the cost of the synthetic call

- Similarly, a **SYNTHETIC PUT** can be created as follows
  $P = [K/(1+r_f)^t]+C-S$

  For Example: A manager wishes to purchase a 3-month put on a stock with a strike of 40. But, no such puts are trading. Given the following data, what should he do?
  Stock at 45; 3-month call with strike at40 at 7; 3-month risk-free rate at 5%
  Answer: The manager can create a 3-month synthetic put on the stock with a strike price of 40 using put-call parity.
  $P = [K/(1+r_f)^{.25}]+C-S = [40/(1.05)^{.25}]+7-45 = 1.515$
  This means a synthetic put can be created by Selling the stock SHORT at 45, Buying a 3 month call with strike of 40 at 7, and investing $39.515 in a 3-month bill at the risk free rate. Plus, the manager must incur a cost of $1.515 which is the cost of the synthetic put

- Also, a **SYNTHETIC STOCK** can be created as follows
  $S = [K/(1+r_f)^t]-P+C$

- These Relationships can be used to determine whether or not the PUT, CALL and UNDERLYING STOCK are PROPERLY PRICED in relation to each other, given the risk-free rate, strike price, and time 'til expiration. This is Known as **Arbitrage Testing**

For Example: A non-dividend stock is selling at 30 when the risk free rate is 5%. A 6-month call option with a strike of 35 is trading at 3 ½ and a 6-month put option with a strike of 35 is trading at 7 ½. Are these securities properly priced?
Answer: According to Put-call parity

$S+P-C=[K/1+r_f)^t]$

$30+7.5-3.5 \ ?= 35/(1.05)^{.5}$

34 , 34.1565

This suggests that buying the stock at 30, buying the put at 7.5 and selling the call at 3.5 will cost 34; compared to the theoretically correct value for this portfolio at 34.1565. Since the combination portfolio is UNDERVALUED, an Arbitrageur should buy it. If purchased at 34, the combination portfolio will produce a guaranteed value of 35 on the date the expirations expire, according to put call parity. This represents a riskless profit of $1 on a $34 investment. (2.94% return in 6 months) This is an annualized rate of 5.88% compared to the risk-free rate of 5%. By financing the purchase at the risk-free rate, the arbitrageur can earn a net riskless rate of return of 0.88% (annualized) without investing any of his own cash

- The Lure of Free Money causes arbitragers to perform this arbitrage. But, as they buy the stock & put, prices will rise: and as they sell the call, prices will fall. Thus, arbitrageurs force the stock and option prices to move back to their proper alignments. Note, the arbitrageur buys each of the components of the portfolio, not just the one that is theoretically mis-priced. As realignment occurs, the whole portfolio converges to the theoretically sound price, and the arbitrageur still profits.
- If, on the other hand, the portfolio was overpriced, the arbitrageur would do the opposite and short the stock, write a put, and buy a call while investing the proceeds at the risk-free rate.

- ALSO, PROTECTIVE PUT Strategy can be replicated Synthetically by

  $\text{Protective Put}_{\text{Synthetic}} = S+P = [K/(1+r_F)^t]+C$

  As a Protective Put is Owning the Stock, and Owning a Put on the stock, it can be created synthetically by Investing at the Risk Free Rate and Buying a Call

- Plus, COVERED CALL Strategy can be replicated Synthetically by

  $\text{Covered Call}_{\text{Synthetic}} = S-C = [K/(1+r_f)^t] - P$

  Having funds invested at the risk free rate and writing a put will replicate the Covered Call strategy of owning the stock and writing a call on it

- When HOLDING PERIODS are SHORT, the effect of the TIME PREMIUM on OPTION PRICES can be IGNORED

  If $t=0$, $[K/(1+r_f(t/360))] = K$

  So, if a an investor holds the combination portfolio S+P-C for only a short period of time so that Δt is virtually 0, put-call parity suggests, that as the strike price, K, is a constant ($\Delta K = 0$), then

  $\Delta S + \Delta P - \Delta C = 0$ (when $t=0$)

  Over the short-term holding period, owning the stock, a put, and selling a call is the same as having a riskless hedge that will produce virtually no net gain or loss.

This can be algebraically manipulated to produce a variety of stock/option combinations that will exhibit different characteristics over short holding periods

$\Delta C = \Delta P + \Delta S$

Owning a Put and a Stock will replicate the performance of a call option in the short term.

$\Delta P = \Delta C - \Delta S$

Owning a Call and Shorting a Stock will replicate the performance of a put in the short term

$\Delta S = \Delta C - \Delta P$

Owning a Call and Writing a Put will generate a payoff in the short term that replicates the payoff of owning the underlying stock

$-\Delta P = \Delta S - \Delta C$

Owning a Stock and Writing a Call can replicate the performance of Writing a Put

$-\Delta C = -\Delta P - \Delta S$

Writing a Put and Shorting a Stock can replicate the performance of Writing a Call

$-\Delta S = \Delta P - \Delta C$

Buying a Put and Writing a Call can replicate the performance of shorting a stock

4. *Using Stock Options to Hedge Stock Holdings*

a. There are 2 Different types of Hedges that can be used to protect Stock Holdings: A Total Hedge & an Insurance Hedge

b. **TOTAL HEDGE**

- The Objective of a Total Hedge is to INSULATE the value of an investor's holding in a stock, even if the price of the stock changes
- To do this with options, a COMBINATION portfolio is constructed, consisting of the stock that is being hedged and a position in an option contract, such that the value of the combination of the two assets will remain unchanged, even if the price of the stock being hedged does change.
- The mathematics of options hedging is the same as that of futures hedging; with the only difference being that there are two types of options (puts & calls) and only 1 type of futures contract
- When options are used as the Hedging Vehicle, the mathematical condition to be satisfied is known as the **Δ NEUTRAL POSITION**

  $\Delta S Q_S + N_o Q_o \Delta o = 0$

  where S is the price of the security being hedged
  Q_S is the number of units of the security to be hedged
  o is the price of the option being used as Hedging vehicle
  Q_o is the contract size of the option
  N_o is the number of options required for total hedge

  Thus, the number of options on a stock that are required to hedge a given number of shares of the same stock is:

  $N_o = -(\Delta S/\Delta o)\,(Q_S/Q_o)$

  The Term (ΔS/Δo) is also called the HEDGE RATIO. Thus, the basic hedging relationship when options are used as the hedging vehicle (like a futures hedging vehicle) is

  $N_o = -\,HR\,(Q_S/Q_o)$

  Notice, the Hedge Ratio is the RECIPROCAL of the Δ of the OPTION

  $HR = 1 / \Delta o$ → $\Delta = (\Delta\text{Option Price} / \Delta\text{Stock Price}) = \Delta o/\Delta S$

  $\Delta_{Call} - \Delta_{Put} = 1$

  For Example: A Stock is selling at 27. A 3-month call on the stock, with strike of 25, is selling at 2 3/4 . The Δ of the Call is .8641. How many calls are needed to hedge a 1,000 share position in this stock?
  Answer: The basic Hedging Relationship is:
  $N_C = -\,HR\,(Q_S/Q_C)$
  $HR = 1/\Delta_{call} = 1/.8461 = 1.18$
  $N_C = -1.18(1000/100) = -11.8$ Calls
  The Negative sign means that 12 (rounded to a whole) calls should be SOLD SHORT in order to perform the hedge. If call options are being used as the hedging vehicle to hedge a long position in a stock, it is necessary to sell the call options shor.t

- Unlike using Futures as a Hedging Vehicle, producing a Total Hedge with Options is NOT SET IT & FORGET IT (since, as price of stock shifts, so does the Δ, and hence, so does the Hedge Ratio). When Options are used as a Hedging Vehicle, the Hedge must CONSTANTLY be REBALANCED as the price of the stock being hedged changes. ONLY by using deep in the money call options, whose Δ is virtually 1.0 for a wide range of prices of the underlying stock, will constant rebalancing not be necessary; even then, rebalancing will be required if the price of the stock being hedged falls substantially or if other option pricing parameters change

- WITH THOSE DISADVANTAGES, why use OPTIONS to Hedge? Well, there are NO FUTURES CONTRACTS on INDIVIDUAL STOCKS. To hedge a particular stock, one must use options (otherwise, try a risky cross-hedge with futures)

For Example: Suppose in the above example, one wanted to use PUTS instead of calls. What would you do? How many 3-month put options with a strike of 25 should a portfolio manager own in order to hedge a 1,000-share portfolio?
Answer: Use the basic hedging relationship

$N_P = -HR\ (Q_S/Q_P)$
$HR = 1/\Delta_P$
$\Delta_C - \Delta_P = 1 \rightarrow .8461 - 1 = \Delta_P = -.1539$
$HR = 1/-.1539 = -6.5$
$N_P = -\ (-6.5)(1000/100) = 65$ puts

The positive sign means 65 puts should be purchased to effectuate this hedge. If Put options are being used as the hedging vehicle to hedge a long position in stock, it is necessary to buy put options.

- Both Calls & Puts can be used to Hedge a Stock Portfolio. Which type of option is best to use as a Hedging vehicle? The answer depends upon Several Factors the Hedger Must consider:
  - Selling CALLS SHORT against a Stock Holding Generates REVENUE for the HEDGER. Plus, as this is essentially writing a covered call, covered call strategies work best if the price of the stock does not change very much in either direction. When there is much volatility to be expected, it is a Poor Strategy
  - Buying PUTS (Long) will incur a COST for the HEDGER. This, like a protective put, works best when one expects great volatility in the underlying stock. When the price of the stock remains flat, this is a poor strategy
  - As the Price of the Stock falls Further below the Strike Price of a Call, the HEDGE RATIO moves TOWARD ∞, so calls cannot be used as a hedging instrument, and puts will then have to be used
  - As the Price of the stock rises above the Strike Price of a PUT, the Hedge Ratio moves toward ∞ , so puts cannot be used and calls will have to be employed
- Other Practical Problems Arise when Options are used as a Hedging Vehicle
  - OPTION DELTAS are not as predictable in reality as in theory, because option pricing models are NOT as accurate as futures pricing models
  - Since the Longest options available are generally 9-months, it is NOT possible to use Options for LONG Term Hedges
  - When using options as a Hedging Vehicle, the COST of the Hedge depends upon
    - The VOLATILITY of the STOCK
    - The PRICE PATH of the Stock, because the Time Premium of an option is a function of what the price of the underlying stock is in relation to the option's strike price
    - The Hedging Cost is NOT KNOWN with CERTAINTY in Advance because the amount of rebalancing will depend upon

the actual price path that the stock takes during the hedging horizon. The more rebalancing required, the higher the cost of hedging due to commissions

c. INSURANCE HEDGE

- An insured hedge is one that only attempts to protect an investor against the effects of a catastrophic decline in the value of the portfolio being hedged, while enabling him to participate in upside appreciation. In effect, the insured hedge attempts to prevent the value of a portfolio from falling below some floor value, without materially inhibiting its upside performance.
- The CLASSIC Portfolio Insurance Strategy is the PROTECTIVE PUT Strategy.

1. This Strategy simply requires the hedger to purchase a sufficient number of put options to fully cover the number of shares held long, with the strike price of the option set to EQUAL the FLOOR Value below which the hedger does NOT want the value of the price per share to fall.

2. NOTE: This is essentially the same as a Total Hedge (using puts) except than an EQUAL DOLLAR MATCHED HEDGE RATIO of –1.0 (rather than one based on the Δ of the option) is used, and NO Rebalancing is necessary.

For Example: a manager has a portfolio of 1,000 shares of XYZ trading at $100. It is desired to protect the portfolio against a decline of more than 5% ($5/share). This portfolio insurance can be obtained by simply buying a sufficient number of puts, with a strike of 95, to cover 1,000 share of the stock. Suppose XYZ put with strike at 25 is trading at 1 3/8. Since each put has a contract size of 100 share, one must purchase 10 put contracts

$N_P = -HR(Q_S/Q_P)$

HR = -1.0 (Equal Dollar Matched)

$N_P = -(-1.0)(1000/100) = 10$ Puts

When 10 puts are purchased, no matter how low the stock may fall, the stock can always be sold at 95 by exercising the puts. Therefore, the portfolio can always be liquidated for $95,000. When the stock rises above $100/share, the puts will lapse and the investor will reap the price appreciation, less the cost of the puts.

3. The Cost of This Insurance is the COST of Buying the Puts

Here, with the puts trading at 1 3/8, it would cost $1,375 to buy 10 put contracts. When the $1,375 is subtracted from the $100,000 portfolio, this will reduce the size of the portfolio to $98,625. Thus, when the portfolio rises in value, the manager will garner less than what otherwise would have been the potential gain. If the stock fails to rise at least 1 3/8, (to 101 3/8) the portfolio will suffer a loss due to the cost of the insurance.

4. NOTE: the Protective Put Portfolio Insurance Strategy will Properly insure a Portfolio against the possibility of a loss exceeding a specific limit, provided that the cost of the insurance is NOT excessive and than an appropriate put option can be purchased (proper strike & date, etc.) These conditions may NOT always be met due to:

- The lowest-cost insurance can only be obtained by using European Puts. American puts are more expensive. But, with Portfolio insurance, there is no need to exercise before expiration. Thus, the extra cost of an American put is a dead weight loss to an investor using puts for insurance

        - The Expiration dates of standard puts usually do not correspond with the time horizon of the insured hedger insofar as normal options expire in a relatively short time
        - Options are likely to be mispriced and have high basis risk
        - There may be no puts trades on the stocks that one desires to hedge. While specifically designed OTC put contracts could be considered, they are likely to be too expensive
        - Fractional number of puts may be required to create a perfect insured hedge. Since fractional puts cannot be purchased, perfect hedging may not be possible
    - Some portfolio insurance strategies have been created that do not require the purchase of put options. Instead, a SYNTHETIC PUT, whose payoff pattern behaves like a put with a given strike price, is Created. Combining this synthetic put with a long position in the stock being hedged will produce a synthetic protective put strategy
    - An Insured Hedged can be accomplished by USING CALLS Plus CASH (Cash/Call) Strategy which produces the same payoff pattern as does the Protective Put. It is possible to Synthesize a PROTECTIVE PUT Strategy without using any options at all via DYNAMIC HEDGING

d. **Dynamic Hedging Strategies**

- DYNAMIC HEDGING is based on the principle that when a stock gets close to its floor price, the investor moves out of the stock and into cash. As the price moves away from the floor, the investor moves into the stock and out of cash.
- As the VALUE of the PORTFOLIO rises above the FLOOR, INCREASE exposure to Stocks (risky asset); as the value of the portfolio falls toward the floor, reduce exposure to stocks (risky asset). Thus, Dynamic Hedging is a MOMENTUM Strategy → as the value of stocks in a portfolio rises, buy more stocks; as the value of the stocks in a portfolio falls, sell more stocks and hold more cash. By doing this, the investor reduces the probability that the portfolio will fall below the floor (because, at the floor, the portfolio will be all cash), yet as the value of the portfolio rises above the floor, it becomes more fully invested in stocks so that it can fully participate in the upside potential
- Rather than actually buying & selling stocks as the value of a portfolio rises above or falls toward the floor value, STOCK INDEX FUTURES contracts can be used to RAISE & LOWER a Portfolio's exposure to the stock market. To Effectuate DYNAMIC HEDGING via FUTURES CONTRACTS:
    1. REDUCE the portfolio's exposure to the stock market (LOWER ITS β) by SELLING Futures contracts against the portfolio in greater and greater numbers as the value of the portfolio falls toward the floor. When the value of the portfolio is AT or NEAR the floor, enough futures contracts should be sold to CREATE a

TOTAL HEDGE, reduce the β to ZERO and making it insensitive to further downward movements in the stock market index

2. INCREASE the Portfolio's Exposure to the Stock market (RAISE its β) buy BUYING FUTURES contracts previously sold against it as the value of the portfolio rises above the floor. When the value of the portfolio rises to a level that is so high above the floor that there is virtually no chance that a short-term decline in the market index will be large enough to drive the value of the portfolio below the floor, no futures contracts should be sold against it, leaving the portfolio completely unhedged.

3. In theory, dynamic hedging requires the futures position to be changed continually in response to changes in the value of the portfolio relative to the floor and the volatility of the portfolio. The Advantage of using Futures is that it is much CHEAPER to Buy & Sell Futures than it is to TRADE INDIVIDUAL Stocks of equal value, and, being highly liquid, trading stock index futures make it less likely to disturb security prices than the trading of individual stocks themselves.

4. RISKS of PORTFOLIO INSURANCE

- The Strategy may not Exactly replicate the Protective Put strategy because the Synthetic Put is too difficult to replicate. This could cause the portfolio to fall below the desired floor value or to fail to participate in market rallies as much as expected. Factors contributing to this risk include:
  - ***i.*** An INCORRECT ESTIMATE of PORTFOLIO VOLATILITY produces an incorrect option pricing calculation used to determine the Hedge Ratio needed to Produce the proper exposure to the risky asset
  - ***ii.*** An INCORRECT OPTION Pricing Model may be used to determine the Hedge Ratio. This is especially important risk for portfolios containing complex securities
  - ***iii.*** TRACKING ERRORS between the futures market and the underlying assets in the portfolio can cause errors and mispricings. The correlation between the S&P 500 Futures and other stock prices is high, but not so close to 100% to reduce tracking error to Zero. Tracking errors tend to be LARGEST when stock markets undergo severe corrections, thus the strategy might fail the most at the worst possible time.
  - ***iv.*** Since it is impractical to rebalance the hedge continually, the rebalancings occur at certain 'trigger points'. If the triggers are set too often, the

cost of the insurance rises; if set too infrequently, imbalances occur that reduce the effectiveness of the insurance

- Dynamic Hedging Strategies can also be applied to BOND PORTFOLIOS. Then, Treasury bond Futures Contracts can be used to RAISE & LOWER EXPOSURE (DURATION) to bonds as needed to effectuate the strategy
- There tends to be less risk when using bond futures to effectuate a Dynamic Hedging Strategy, because:
  1. The CORRELATION between the Prices of Treasury Bond Futures Contracts and the Prices of Bonds in a Portfolio tends to be high. This reduces tracking errors
  2. Bond Contracts tend to be Priced MORE CORRECTLY than Stock Index Futures Contracts. This means that mispricings are small, or effectively, non-existent.

e. Using Treasury Bond Futures Contracts to Effectuate a Portfolio Insurance Strategy by Using Dynamic Hedging (from CBOT Article)

For Example: A money manager has $100,000,000 portfolio that is 70% invested in long-term bonds with a Yield of 6.5% and a duration of 6 years; and 30% invested in T-Bills with a modified duration of .24 years. The manager wants to produce a Portfolio insurance strategy using dynamic hedging that will ensure that the value of the portfolio will not fall below a floor value of $100,000,000 by the end of the year, while still participating in any upside potential if rates decline. This will be done using Treasury Bond Futures contracts, whose basis point value has been estimated to be $76.30

In order to Effectuate this Strategy, the manager would employ the following procedure

1. THE PROTECTIVE PUT Strategy is to OWN the Portfolio and OWN a PUT OPTION on the portfolio that has a Strike Price equal to the Present Value of $100,000,000 in one year. The Δ of such a Portfolio at the current time can be Calculated by using an option pricing model that incorporates the current short-term interest rate (r), an assumed volatility of the portfolio(σ), the current value of the portfolio (S) and a strike price (K) that equals the present value of $100,000,000 in one year.
2. The Percentage of the Portfolio that should be exposed to the Risk Asset (Long-term bonds in this example) is the HEDGE RATIO of the Strategy. It is Equal to ONE (minus the absolute value of the calculated Δ of the put)

   %Exposure to Hedge = $1 - |\Delta_P|$

   Let us suppose that such a calculation produces a Δ of the synthetic put equal to .20, so that the desired exposure to the risky asset (bonds) is 80%
3. Calculate the BPV of the Current Portfolio, which is $70,000,000 invested in bonds and $30,000,000 invested in Treasury Bills

   $BPV_P = BPV_{bond} + BPV_{Bills}$

   $BPV_P = (.0001)(V_{Bond})(D^*_{Bond}) + (.0001)(V_{Bill})(D^*_{Bill})$

   $BPV_P = (.0001)(70,000,000)[6/(1+(.065/2))] + (.0001)(30,000,000)(.24)$

   $BPV_P = \$41,398$
4. Calculate the BPV of the desired portfolio, which, based upon the hedge ratio of the strategy is $80,000,000 bonds and $20,000,000 bills

   $BPV_{desired} = (.0001)(80,000,000)[6/(1+.065/2))] + (.0001)(20,000,000)(.24)$

   $BPV_{desired} = \$46,969$
5. Determine the number of Treasury Bond Futures Contracts needed to produce the desired exposure to the risk asset

   $N_F = [(BPV_{desired} – BPV_{current})/BPV_F] = [(46,969 – 41,398)/76.30] = 73$ Contracts

This Procedure effectively makes the duration of the portfolio equal to what it would be if it were allocated in the desired manner. Therefore, the Dynamically Hedged Portfolio will behave as if it were allocated in the appropriate manner Every time a trigger point is reached, this calculation is performed over again, and the portfolio's exposure to the risky asset is rebalanced by adjusting the number of futures contracts.