12.1-

a).(i). $f(nTs) = 8\cos(2\pi(0.1n)) + 4\sin[4\pi(0.1n)]$

$f[n] = 8\cos[0.2\pi n] + 4\sin[0.4\pi n]$.

$$F(\Omega) = 8\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - 0.2\pi - 2\pi k) + \delta(\Omega + 0.2\pi - 2\pi k)]$$
$$-j4\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - 0.4\pi - 2\pi k) - \delta(\Omega + 0.4\pi - 2\pi k)]$$

(ii). $g[n] = 4\cos[0.5\pi n]\, u[n]$

$$4\cos[0.5\pi n] \longleftrightarrow 4\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - 0.5\pi - 2\pi k) + \delta(\Omega + 0.5\pi - 2\pi k)]$$

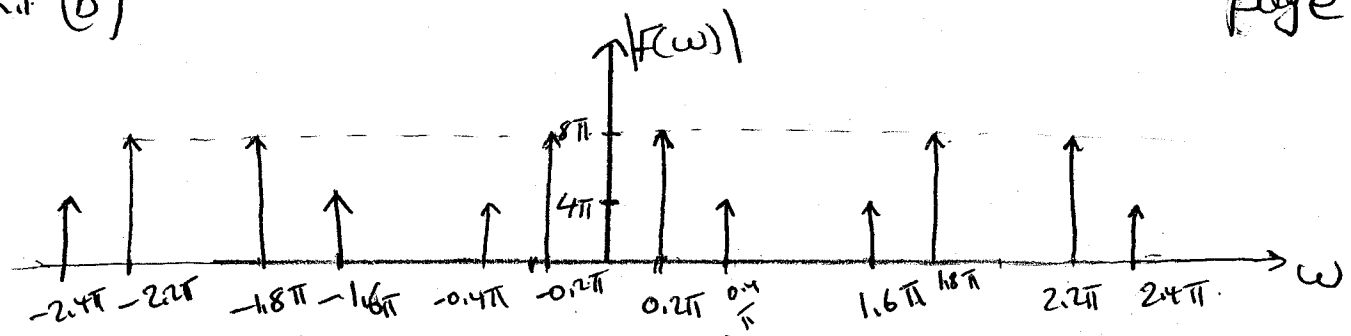
$$u[n] \longleftrightarrow \frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\Omega - 2\pi k)$$

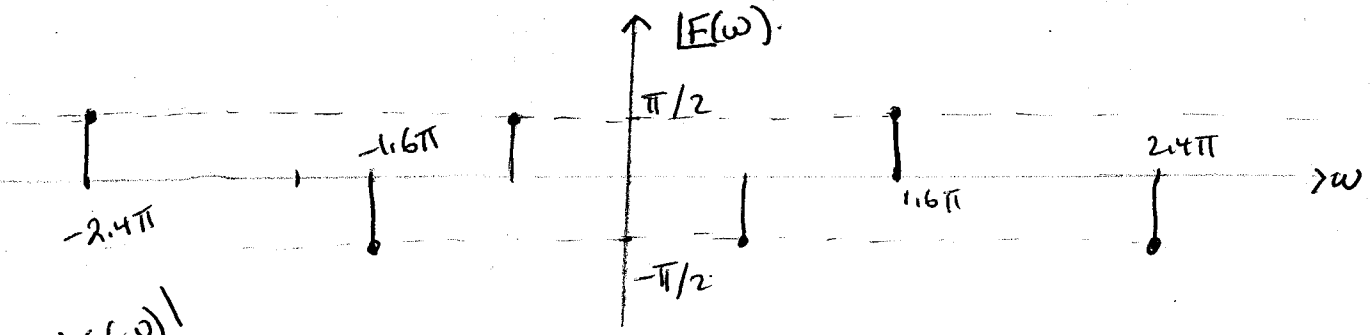
$$x[n]\,y[n] \longleftrightarrow \frac{1}{2\pi} X(\Omega) * Y(\Omega)$$

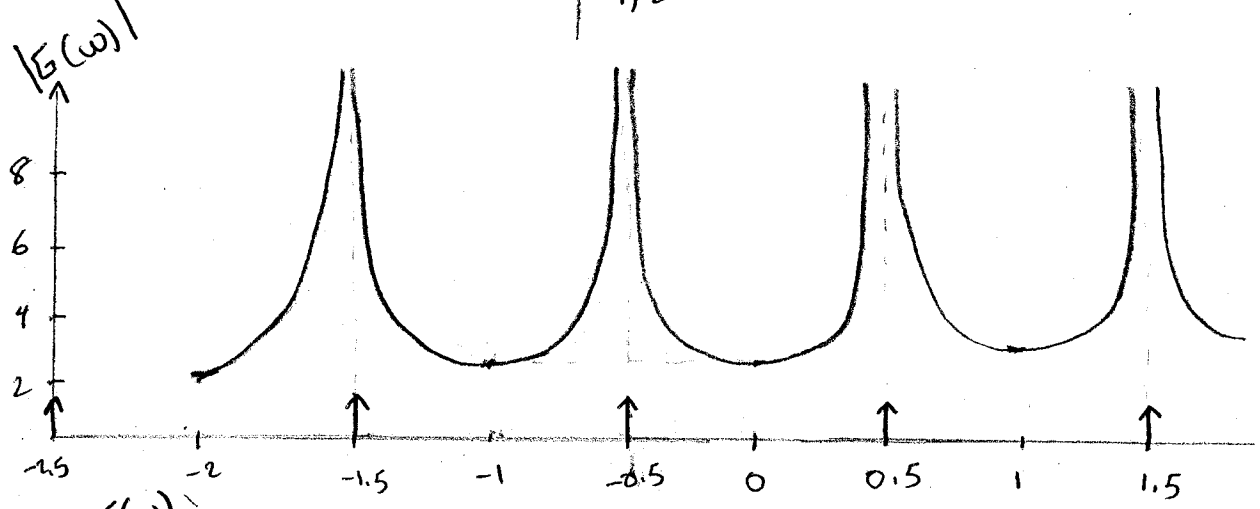
$$G(\Omega) = \frac{4e^{j2\Omega}}{1 + e^{j2\Omega}} + 2\pi \sum_{k=-\infty}^{\infty} [\delta(\Omega - 0.5\pi - 2\pi k) + \delta(\Omega + 0.5\pi - 2\pi k)]$$

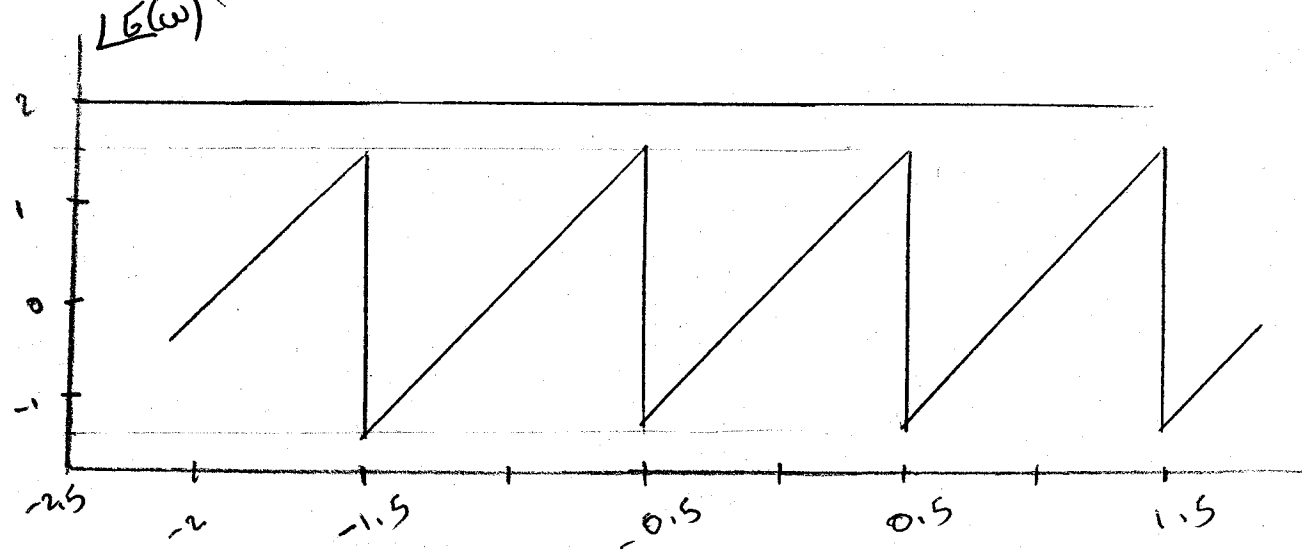
ECE 120B
HW #5 solution

12.1 (b)









12.2 (a).

$$x[n] = \begin{cases} (0.5)^n, & n \geq 0 \\ 0, & n < 0 \end{cases} \qquad X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\pi\Omega}$$

$$X(\Omega) = \sum_{0}^{\infty} (0.5)^n e^{-j\pi\Omega} = 1 + 0.5e^{-j\Omega} + (0.5e^{-j\Omega})^2 + \underbrace{(0.5e^{-j\Omega})^3 + \cdots}_{\text{geometric series}}$$

$$X(\Omega) = \frac{1}{1 - 0.5e^{-j\Omega}}$$

(b). $y[n] = n(0.5)^n u[n] \overset{DTFT}{\longleftrightarrow} Y(\Omega) = \sum n(0.5)^n e^{-j\pi\Omega}$

From table 12.1 $Y(\Omega) = \dfrac{0.5e^{j\Omega}}{(e^{j\Omega} - 0.5)^2}$

(c). $v[n] = 2[u[n] - u[n-5]]$

$$V(\Omega) = \sum_{n=0}^{4} 2e^{-j n\Omega} = 2[1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega}]$$

$$= 2e^{-j2\Omega}[e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}]$$

$$= 2e^{-j2\Omega}[1 + 2\cos\Omega + 2\cos 2\Omega]$$

OR from table 12.1 $V(\omega) = 2\dfrac{\sin(\frac{5}{2}\Omega)}{\sin(\frac{\Omega}{2})} e^{-j2\Omega}$ (with time-shift property)

(d). $w[n] = \text{rect}_5(n/4) + \text{rect}_5(n/10)$

$$W(\Omega) = \sum_{-5}^{5} 1 \cdot e^{-j n\Omega} + \sum_{-2}^{2} 1 \cdot e^{-j n\Omega}$$

$$= e^{j5\Omega} + e^{j4\Omega} + e^{j3\Omega} + \cdots e^{-j5\Omega}$$

or from table 12.1

$$W(\Omega) = \frac{\sin(\frac{5}{2}\Omega)}{\sin(\frac{\Omega}{2})} + \frac{\sin(\frac{11\Omega}{2})}{\sin(\frac{5\Omega}{2})}$$