Spring 2004 ECE 120B HW SOLUTION #3

11.1

(b) $0.7^n + 4(1.5)^n \longrightarrow \frac{z}{z-0.7} + \frac{4z}{z-1.5}$

(f) $20\cos(2n - \pi/4)$

use $\cos(A-B) \rightarrow \cos A \cos B + \sin A \sin B$

$$20\left[\cos(2n)\cos(\pi/4) + \sin(2n)\sin(\pi/4)\right]$$

$$= 20\left[\cos(2n)\frac{\sqrt{2}}{2} + \sin(2n)\frac{\sqrt{2}}{2}\right]$$

$$= 20\left(\frac{\sqrt{2}}{2}\right)\left[\cos(2n) + \sin(2n)\right]$$

$$= 10\sqrt{2}\cos(2n) + 10\sqrt{2}\sin(2n)$$

$$= \frac{10\sqrt{2}\; z(z-\cos(2))}{z^2 - 2z\cos(2) + 1} + \frac{10\sqrt{2}\; z\sin(2)}{z^2 - 2z\cos(2) + 1}$$

11.2

(f) $5e^{-t}\cos(t)$

$t = nT$
$T = 0.05\Omega$

$$f[n] = 5e^{-n(0.05)} \cos(n0.05)$$

$$= 5\left(e^{-0.05}\right)^n \cos(0.05n)$$

$$F(z) = \frac{5\ z(z - e^{-0.05}\cos(0.05))}{z^2 - 2(e^{-0.05})z\cos(0.05) + (e^{-0.05})^2}$$

$$= \frac{5z(z-(0.951)(0.99875))}{z^2 - 2(0.951)z(0.99875) + 0.9048}$$

$$= \frac{5z(z-0.949811)}{z^2 - 1.89962z + 0.9048}$$

$$= \frac{5z(z-0.95)}{z^2 - 1.9z + 0.905}$$

$$= \frac{5z^2 - 4.75z}{z^2 - 1.9z + 0.905}$$

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11.4

(a)

(i) $X = \sum_{n=0}^{\infty} 0.5^n = \sum_{n=0}^{\infty} 0.5^n z^{-n} \Big|_{z=1}$

$$= \frac{z}{z-0.5} \Big|_{z=1}$$

$$= \frac{1}{1-0.5}$$

$$= 2$$

(ii) $X = \sum_{n=2}^{\infty} 0.5^n = \sum_{n=0}^{\infty} 0.5^n - 0.5^0 - 0.5^1$

(from part(i) we know $\sum_{n=0}^{\infty} 0.5^n = 2$)

$\therefore \quad = 2 - 1 - 0.5$

$= 0.5$

(b) $X = \sum_{n=0}^{\infty} 0.5^n \cos(0.1n) = \sum_{n=0}^{\infty} 0.5^n \cos(0.1n) z^{-n} \Big|_{z=1}$

$$= \frac{z(z - 0.5\cos(0.1))}{z^2 - 2(0.5) z \cos(0.1) + (0.5)^2} \Big|_{z=1}$$

$$= 1.97$$

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11.9 $f[n] = a^n u[n]$ $F(z) = \frac{z}{z-a}$

(a) $f[n/3]$

Use Time Scaling Property:

$\mathcal{Z}[f[n/k]] = F(z^k)$, k a positive integer

$$\rightarrow F(z) = \frac{z^3}{z^3 - a}$$

(b) $f[n-3]u[n-3]$

Use Real Shifting Property:

$\mathcal{Z}[f(n-no)u(n-no)] = z^{-no} F(z)$, $no \geq 0$

$$\rightarrow z^{-3}\left(\frac{z}{z-a}\right) = \frac{z^{-2}}{z-a} = \frac{1}{z^2(z-a)}$$

Verifying: (using long division)

$$\begin{array}{rl}
 & z^{-3} + az^{-4} + a^2z^{-5} + a^3z^{-6} \; \dots \\
z^3 - az^2 \overline{)} & 1 \\
- & 1 - az^{-1} \\
\hline
 & 0 + az^{-1} \\
- & az^{-1} - a^2z^{-2} \\
\hline
 & 0 + a^2z^{-2} \\
- & a^2z - a^3z^{-3} \\
\hline
 & a^3z^{-3} \\
- & a^3z^{-3} - a^4z^{-4} \\
\hline
 & a^4z^{-4} \; \dots
\end{array}$$

$$\Rightarrow \frac{1}{z^3} + \frac{a}{z^4} + \frac{a^2}{z^5} + \frac{a^3}{z^6} \; \dots$$

(continued on next page)

11.9 (b) continued

Note that using equasion 11.4

$$F(z) = \sum_{n=0}^{\infty} f[n] z^{-n}$$

then $f[n-3]u(n-3] \longrightarrow z^{-3}\sum_{n=0}^{\infty} f[n] z^{-n}$ where $f[n]=a^n u(n)$

$$= z^{-3}\left[a^0 z^{-0} + a^1 z^{-1} + a^2 z^{-2} + a^3 z^{-3} \dots\right]$$

$$= z^{-3}\left[1 + \frac{a}{z} + \frac{a^2}{z^2} + \frac{a^3}{z^3} \dots\right]$$

$$= \frac{1}{z^3} + \frac{a}{z^4} + \frac{a^2}{z^5} + \frac{a^3}{z^6} \dots$$

same as result obtained from long division

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11.9

(c) $f[n+3]u[n]$ with $f[n] = a^n u[n]$ $F(z) = \frac{z}{z-a}$

use Time scaling property

$$\mathcal{Z}\left[f[n+no]u[n]\right] = z^{no}\left[F(z) - \sum_{n=0}^{no-1} f[n] z^{-n}\right]$$

$$\rightarrow F(z) = z^3\left[\frac{z}{z-a} - \sum_{n=0}^{2} a^n z^{-n}\right]$$

$$= z^3\left[\frac{z}{z-a} - \left[a^0 z^{-0} + a^1 z^{-1} + a^2 z^{-2}\right]\right]$$

$$= z^3\left[\frac{z}{z-a} - 1 - \frac{a}{z} - \frac{a^2}{z^2}\right]$$

$$= z^3\left[\frac{z(z^2) - z^2(z-a) - az(z-a) - a^2(z-a)}{(z-a)(z^2)}\right]$$

$$= \frac{z^{\cancel{3}}}{\cancel{z^2}(z-a)}\left[\cancel{z^3} - \cancel{z^3} + \cancel{az^2} - \cancel{az^2} + \cancel{a^2 z} - \cancel{a^2 z} + a^3\right]$$

$$= \frac{a^3 z}{z-a}$$

(continued on next page)

11.9 (c) Continued

Verifying:

$$\begin{array}{rl}
 & a^3 + a^4z^{-1} + a^5z^{-2} + a^6z^{-3} \;\cdots \\
z-a & \overline{\big)\, a^3z} \\
 & -\underline{a^3z - a^4} \\
 & \qquad a^4 \\
 & \qquad -\underline{a^4 - a^5z^{-1}} \\
 & \qquad\qquad a^5z^{-1} \\
 & \qquad\qquad -\underline{a^5z^{-1} - a^6z^{-2}} \\
 & \qquad\qquad\qquad a^6z^{-2} \\
 & \qquad\qquad\qquad -\underline{a^6z^{-2} - a^7z^{-3}} \\
 & \qquad\qquad\qquad\qquad a^7z^{-3} \;\cdots
\end{array}$$

$$\Rightarrow a^3 + \frac{a^4}{z} + \frac{a^5}{z^2} + \frac{a^6}{z^3} \cdots$$

Note that using equation 11.4 $F(z) = \sum_{n=0}^{\infty} f(n) z^{-n}$

then

$$f[n+3]u[n] \longrightarrow z^3\left[F(z) - \Sigma f(n) z^{-n}\right]$$

$$= z^3\left[\sum_{n=0}^{\infty} f(n) z^{-n} - \Sigma f(n) z^{-n}\right]$$

$$= z^3\left[\sum_{n=3}^{\infty} f(n) z^{-n}\right] \quad \text{where } f(n) = a^n u(n)$$

$$= z^3\left[a^3z^{-3} + a^4z^{-4} + a^5z^{-5} + a^6z^{-6} \cdots\right]$$

$$= a^3 + \frac{a^4}{z} + \frac{a^5}{z^2} + \frac{a^6}{z^3} \cdots$$

Same as result obtained from long division.

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11.9

(d) $b^{2n} f[n]$ (using two different procedures)

1) use Complex shifting

$$\mathcal{Z}[a^n f[n]] = F(z/a)$$

$$\rightarrow F(z) = \frac{z/(b^2)}{z/(b^2) - a} = \frac{z}{z - ab^2}$$

2) $b^{2n} f[n] = b^{2n}[a^n u[n]]$

$$= (b^2 a)^n u(n)$$

$$\rightarrow F(z) = \frac{z}{z - ab^2}$$