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Chaotic Exchange Rate Dynamics Redux

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Abstract

This article generalizes the results shown in De Grauwe, Dewachter, and Embrechts (1993) in a more sophisticated framework. In their model, the speculative dynamics resulting from the interaction between chartists and fundamentalists are incorporated into a Dornbusch-style model to generate a chaotic nominal exchange rate. Here the model of Obstfeld and Rogoff (1995, 1996) replaces the Dornbusch model, and chaotic solutions are still shown to be possible for sensible parameter values.

Introduction

A major advantage of exchange rate models that are able to generate chaotic solutions is their ability to replicate the random-like pattern observed in actual exchange rates. De Grauwe and Dewachter (1992) and De Grauwe, Dewachter, and Embrechts (1993, Chapter 5) provide examples of these models, where the traditional sticky price model of Dornbusch (1976) is extended to incorporate speculative dynamics and to give rise to an arbitrarily large number of equilibria, including chaotic ones.

An apparently successful attempt to update the Dornbusch model with microfoundations is the "new open economy macroeconomics" model of Obstfeld and Rogoff (1995, 1996, Chapter 10). Their "redux" model can rigorously justify the Keynesian assumption of the Dornbusch model that output is demand determined in the short run if prices are fixed.

The model presented here blends the speculative-dynamics side of the De Grauwe, Dewachter, and Embrechts model with the redux model along with a consideration of foreign exchange intervention. In particular, domestic producers are assumed to speculate in foreign exchange at the previous time period. They no longer have rational expectations as in the original redux model. Rather, they behave either like *chartists* (using backward-looking forecast rules) or *fundamentalists* (making forecasts by paying attention to the long-run purchasing-power-parity exchange rate implied by the fundamentals of the model).

The article is organized as follows. Section 1 presents the model and its solutions, Section 2 discusses results, and Section 3 concludes. Formal tests for chaos are relegated to an appendix.

1. The model

The assumption of Obstfeld and Rogoff (1995, 1996) that domestic agents have perfect foresight regarding the nominal exchange rate is relaxed in this model. Here, forecasts are assumed to be "near rational," in the sense that they may be either backward looking—using chart information—or based on the fundamentals of the model. The introduction of such speculative dynamics through chartists and fundamentalists means that efficiency in the foreign exchange market is not assumed from the start. Tastes, technology, and speculative dynamics are specified to derive individual decision makers' first-order optimality conditions. The model is then solved for the nominal exchange rate in the short run, where goods prices are rigid in local currency terms and output is demand determined. Central bank intervention is also introduced. Eventually, the resulting nonlinear nominal exchange rate equation is used to perform numerical simulations for alternative parameter values.

1.1. Basic features

It is assumed that there are two countries, populated by a large number (continuum) of individual monopolistic producers $z \in [0, 1]$, each of whom produces a single differentiated perishable good (also indexed by z) using its own labor as input, but consumes all goods produced in the world. Thus, there is no capital or investment, although this is not an endowment economy because labor supply is elastic.

Agents live indefinitely and maximize an intertemporal utility function; money conveys utility and acts as a store of wealth. Time period t output of good z, $y_t(z)$, is endogenous and is chosen in a manner that depends on the marginal revenue of higher production, the marginal utility of consumption, and the disutility of effort. Home producers lie on the interval $[0, n]$, and the remaining $(n, 1]$ producers are foreign.

The foreign exchange market is assumed not to be efficient from the start. Thus, the perfect-foresight setting of the original redux model is replaced with the speculative dynamics described in Section 1.7 below. Home producers are assumed to speculate in foreign currency at the previous time period. At the starting period, they are assumed to borrow the amount of domestic money needed to buy foreign money before producing. Accordingly, they make subsequent production decisions after their positions in foreign exchange have been taken. Home producers in the interval $[0, T_t]$ make forecasts of the nominal exchange rate based on technical analysis (charts); the rest $(T_t, n]$ make forecasts based on the fundamentals of the model. The weight of home charting T_t can change as the nominal exchange rate deviates from equilibrium, as will be explained in Section 1.7. (This method of introducing two types of speculator in the redux model is formally similar to the procedure adopted by Fender and Yip (2000) when modeling the introduction of a tariff in the model.)

1.2. Preferences

Individuals worldwide have identical preferences over a consumption index of all the individual goods produced, real money balances, and effort expended in producing output. Since all agents within a country have symmetrical preferences and constraints, the maximization problems of national consumer–producers can be analyzed by the intertemporal utility function of a representative home agent U_t given by

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{\sigma}{\sigma - 1} C_s^{\frac{\sigma-1}{\sigma}} + \frac{\delta}{\delta - 1} \left(\frac{M_s}{P_s} \right)^{\frac{\delta-1}{\delta}} - \frac{y_s(z)^2}{2} \right], \tag{1}$$

where $\beta \in (0, 1)$ is a fixed preference parameter that measures the individual's impatience to consume; $\sigma \in (0, \infty)$ stands for the elasticity of intertemporal substitution; and $\delta \in (0, \infty)$ will turn out to be the consumption elasticity of money demand, for which sensible values lie on the interval $\delta \in (0, 1)$.

In (1), the representative home consumer–producer obtains utility U from the present discounted value of a function that depends positively on consumption and real money balances and negatively on work effort, which is positively related to output of good z. Variable C is a home real-consumption index as defined below; M represents a representative home agent's holdings of nominal money balances; and P is a home consumer-price index, also defined below. A foreign representative individual's utility function is analogous to (1).

It might be noted that while home money is held only by home agents, foreign money is held by foreign (as in Obstfeld–Rogoff) *and* home agents in this model. Despite this situation, foreign money does not enter the home utility function (1). Instead, excess returns from holding foreign money will be embodied in the home individual's budget constraint (16) in Section 1.4.

Using $c(z)$ to denote the home individual's consumption of good z, the home real-consumption index C is defined as a generalization of a two-good constant-elasticity-of-substitution (CES) function that takes the form

$$C = \left[\int_0^1 c(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between different goods. It will also turn out to be the price elasticity of demand faced by each monopolist, as will be seen in Section 1.5. Parameter θ may also be thought of as the degree of competition in the economy; if $\theta \to \infty$, perfect competition holds, and if $\theta \to 1$, pure monopoly holds. The requirement $\theta > 1$ is to ensure an interior equilibrium with a positive level of output.

Each of the home real consumption indexes for chartists and fundamentalists is defined analogously to (2). The foreign real-consumption index C^* is defined similarly, i.e.,

$$C^* = \left[\int_0^1 c^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $c^*(z)$ is the foreign individual's consumption of good z. Throughout in this article, asterisks denote foreign variables.

The home consumer-price index P corresponding to (2) is given by

$$P = \left[\int_0^1 p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \tag{4}$$

where $p(z)$ is the home-currency price of good z. Index P is defined as the minimal expenditure of domestic money needed to purchase a unit of C. Formally, P solves the problem of minimizing the nominal budget constraint

$$Z = \int_0^1 p(z)c(z)\, dz \tag{5}$$

(where Z is any fixed total nominal expenditure on goods) subject to $C = 1$, as defined in (2). Equation (4) is an extension of the price index for the two-good CES case.

The foreign consumer-price index P^* corresponding to (3) is given analogously by

$$P^* = \left[\int_0^1 p^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \tag{6}$$

where $p^*(z)$ is the foreign-currency price of good z.

1.3. Purchasing power parity

It is assumed that there are no impediments or costs to trade between the two countries, so the law of one price holds for each individual good z, i.e.,

$$p(z) = Ep^*(z), \tag{7}$$

where E is the nominal exchange rate, defined as the home-currency price of foreign currency.

Since goods 0 to n are made at home and the rest are produced abroad, consideration of (7) allows us to rewrite both the home consumer-price index (4) as

$$P = \left[\int_0^n p(z)^{1-\theta} dz + \int_n^1 [Ep^*(z)]^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{8}$$

and the foreign consumer-price index (6) as

$$P^* = \left[\int_0^n \left(\frac{p(z)}{E} \right)^{1-\theta} dz + \int_n^1 p^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \tag{9}$$

Using $p_T(z)$ to denote the price of good z set by home producers who behaved as chartists at the previous time period, and $p_F(z)$ to stand for the price of good z set by home producers who behaved as fundamentalists, (8) can also be rewritten as

$$P = \left[\int_0^{T_t} p_T(z)^{1-\theta} dz + \int_{T_t}^n p_F(z)^{1-\theta} dz + \int_n^1 [Ep^*(z)]^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \tag{10}$$

Similarly, (9) can be rewritten as

$$P^* = \left[\int_0^{T_t} \left(\frac{p_T(z)}{E} \right)^{1-\theta} dz + \int_{T_t}^n \left(\frac{p_F(z)}{E} \right)^{1-\theta} dz + \int_n^1 p^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \tag{11}$$

Adopting a rationale similar to the one presented by Fender and Yip (2000), all home producers who act as chartists (fundamentalists) are assumed to set the same price. This allows us to rewrite (10) and (11) as

$$P = \left[T_t p_T^{1-\theta} + (n - T_t) p_F^{1-\theta} + (1-n)(Ep^*)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{12}$$

and

$$P^* = \left[T_t \left(\frac{p_T}{E} \right)^{1-\theta} + (n - T_t) \left(\frac{p_F}{E} \right)^{1-\theta} + (1-n) p^{*1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{13}$$

Comparing (12) and (13), it turns out that home and foreign consumer-price indexes are related by purchasing power parity (PPP):

$$P = EP^*. \tag{14}$$

The law of one price (7) along with PPP (14) also implies that the real price of any good z is the same at home and abroad, i.e.,

$$\frac{p(z)}{P} = \frac{p^*(z)}{P^*}. \tag{15}$$

This fact will be used in Section 1.5 when we derive the world demand for good z.

1.4. *Individual budget constraint*

There is an integrated world capital market in which both countries can borrow and lend. Here, an internationally traded asset is a riskless real bond denominated in the home real-consumption index. Since home consumer–producers also hold foreign currency for speculative purposes, returns or losses of such decisions at the former time period are assumed to impact the individual budget constraint at the current period. The period budget constraint for a representative home individual can then be written in nominal terms as

$$P_t B_{t+1} + M_t = P_t(1 + r_t)B_t + M_{t-1} + p_t(z)y_t(z) - P_t C_t + (1 + R_t)M^*_{t-1}, \tag{16}$$

where B_t and B_{t+1} denote the stock of bonds held by a home resident at time periods t and $t + 1$ respectively; M_{t-1} is a home agent's holdings of nominal money balances at $t - 1$; r_t denotes the consumption-based real interest rate earned on bonds between dates $t - 1$ and t; M^*_{t-1} is a home agent's holdings of foreign currency at $t - 1$; and R_t stands for returns (or losses for negative values) at date t resulting from holding the foreign currency at $t - 1$.

1.5. *Demand*

Consumers seek to allocate any given amount of money they spend on current consumption between different goods so as to maximize the consumption index. If we maximize the home real consumption index (2) subject to the nominal budget constraint (5), it turns out that for any two goods z and z', $c(z') = c(z)[p(z)/p(z')]^{\theta}$ is valid. Inserting this expression into (5) and using (4) yields $c(z) = [p(z)/P]^{-\theta}(Z/P)$. Since P is the minimum money cost of one unit of C, one obtains the representative individual's demand for good z in the home country as given by

$$c(z) = \left[\frac{p(z)}{P}\right]^{-\theta} C. \tag{17}$$

By a similar rationale, the representative foreign individual's demand for good z is given by

$$c^*(z) = \left[\frac{p^*(z)}{P^*}\right]^{-\theta} C^*. \tag{18}$$

For the home country, (17) can be further rewritten as

$$c(z) = \left[\frac{p_T(z)}{P}\right]^{-\theta} C \tag{19}$$

for $z \in [0, T_t]$, and

$$c(z) = \left[\frac{p_F(z)}{P}\right]^{-\theta} C \tag{20}$$

for $z \in (T_t, n]$.

Taking a population-weighted average of home and foreign demands—i.e., $nc(z) + (1 - n)c^*(z)$—after using (17), (18), and (15), one obtains the world demand for good z, $y^d(z)$, in the following CES form:

$$y^{\mathrm{d}}(z) = \left[\frac{p(z)}{P}\right]^{-\theta} C^{\mathrm{W}}, \tag{21}$$

where C^{W}, the world consumption, is given by

$$C^{\mathrm{W}} \equiv nC + (1 - n)C^*. \tag{22}$$

Thus, due to the fact that each individual producer has a degree of monopoly power, it turns out that each country faces a downward-sloping demand curve for its output in the aggregate.

1.6. Foreign-exchange market inefficiency

In the bonds market, producers have perfect foresight. Thus, the real interest rate links the nominal interest rate to inflation through Fisher parity identity (e.g., Obsfeld and Rogoff, 1996, pp. 516–517), i.e., the nominal interest rate for home-currency loans between dates t and $t + 1$, i_{t+1}, is defined by

$$1 + i_{t+1} \equiv \frac{P_{t+1}}{P_t}(1 + r_{t+1}), \tag{23}$$

where r_{t+1} is the consumption-based real interest rate earned on bonds between dates t and $t + 1$, and P_{t+1}/P_t gives the domestic inflation. Since the foreign-currency nominal interest rate has a definition similar to (23), the so-called Fisher hypothesis postulates that real rates of interest are equalized between the two countries.

In the foreign exchange market, however, producers do not have rational expectations. They may make forecasts in a manner that is internally consistent with the fundamentals of the model, or they may use charts to arrive at backward-looking expectations. In the first case, as will be seen in Section 1.7, they pay attention in particular to the equilibrium–PPP value. In the second case, they use a different information set and look at past exchange rates. So when producers forecast the nominal exchange rate by paying attention to PPP, they behave as fundamentalists. When they make backward-looking expectations, they behave as chartists.

Under such an assumption of heterogenous trading, there is no reason for efficiency in the foreign exchange market to take place from the start. In the original redux model, uncovered interest rate parity (UIP) holds (e.g., Obstfeld and Rogoff, 1995, p. 630), which means that the foreign exchange market is assumed to be efficient. Here the possibility of nonzero excess returns coming from speculation forces the replacement of UIP with

$$\frac{E_{t+1}^{\mathrm{e}}}{E_t} = \frac{1 + i_{t+1}}{1 + i_{t+1}^*}(1 + R_t), \tag{24}$$

where i_{t+1}^* is the nominal interest rate for foreign-currency loans between dates t and $t+1$, and E_{t+1}^{e} stands for the forecast made at time period t for the nominal exchange rate at $t+1$, without making any distinction whether such forecasts are made looking at PPP or using charts (this will be done in Section 1.7). Equation (24) is familiar from presentations in which departures from UIP are assumed, and R_t is viewed as giving excess returns from speculation.

1.7. Speculative dynamics

Nominal exchange rate expectations in the home country are now split between two components: the expectations based on charts, ${}_T E_{t+1}^{\mathrm{e}}$, and the expectations based on the fundamentals of the model, ${}_F E_{t+1}^{\mathrm{e}}$, i.e.,

$$\left(\frac{E_{t+1}^{\mathrm{e}}}{E_{t-1}}\right)^n = \left(\frac{{}_T E_{t+1}^{\mathrm{e}}}{E_{t-1}}\right)^{T_t} \left(\frac{{}_F E_{t+1}^{\mathrm{e}}}{E_{t-1}}\right)^{n-T_t}, \tag{25}$$

where E_{t-1} is the nominal exchange rate at time period $t-1$. It might be noted that E_{t-1} appears rather than E_t in (25) (and also in (26), (30), and (31) below) because of the assumption that producer–speculators take market positions at the former time period based on the forecasts they have made for the current period.

The expectation rule for the forecasts based on charts is defined by

$$\frac{{}_T E^{\mathrm{e}}_{t+1}}{E_{t-1}} = \left[\left(\frac{E_{t-1}}{E_{t-2}} \right) \left(\frac{E_{t-3}}{E_{t-2}} \right) \right]^{\nu}, \tag{26}$$

where E_{t-2} and E_{t-3} are the nominal exchange rates at time periods $t-2$ and $t-3$, respectively, and parameter $\nu \in (0, \infty)$ stands for the degree of past extrapolation used by technical analysis in the domestic country. Since $\nu > 0$, the greater ν, the more the past will be extrapolated into the future in nominal exchange rate forecasts, and home chartists will expect the nominal exchange rate at time period $t+1$ to fall short of the nominal exchange rate prevailing at $t-1$.

The rationale for (26) runs as follows. Producer–speculators expect an increase in the nominal exchange rate whenever a short-run moving average of past nominal exchange rates E^{S} crosses a long-run moving average of past nominal exchange-rates E^{L} from below. In such an event, producer–speculators give a buy order for the foreign currency. By contrast, they expect a decline of the nominal exchange rate whenever E^{S} crosses E^{L} from above. In the latter case, speculators order a selling of the foreign currency. Figure 1 illustrates this.

Thus, if $E^{\mathrm{S}} > E^{\mathrm{L}}$, the buy signal will imply $R_t > 0$ for the next time period; if $E^{\mathrm{S}} < E^{\mathrm{L}}$, the sell signal will lead to $R_t < 0$ on the next date. This outcome might be postulated as

$$\frac{{}_T E^{\mathrm{e}}_{t+1}}{E_{t-1}} = \left(\frac{E^{\mathrm{S}}}{E^{\mathrm{L}}} \right)^{2\nu}. \tag{27}$$

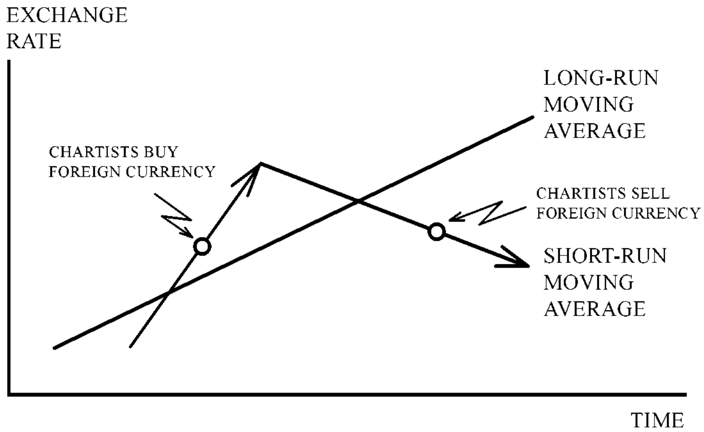


Figure 1. The chart used in the model forecasts. Speculators expect an increase in the nominal exchange rate whenever a short-run moving average of past exchange rates E^{S} crosses a long-run moving average of past exchange rates E^{L} from below; in such an event, they give a buy order for the foreign currency. By contrast, they expect a decline of the nominal exchange rate whenever E^{S} crosses E^{L} from above; in the latter case, speculators order a selling of the foreign currency.
Source: De Grauwe, Dewachter, and Embrechts (1993, p. 73), with minor modifications.

Equation (27) states that since $\nu > 0$, whenever $E^{\mathrm{S}} > E^{\mathrm{L}}$ $(E^{\mathrm{S}} < E^{\mathrm{L}})$, home chartists expect an increase (decrease) of the nominal exchange rate relative to the most recently observed value E_{t-1}.

The short-run moving average E^{S} is, by assumption, based on a one-period change, i.e.,

$$E^{\mathrm{S}} = \frac{E_{t-1}}{E_{t-2}}, \tag{28}$$

and the long-run moving average E^{L} is based on a two-period change, i.e.,

$$E^{\mathrm{L}} = \left(\frac{E_{t-1}}{E_{t-2}}\right)^{\frac{1}{2}} \left(\frac{E_{t-2}}{E_{t-3}}\right)^{\frac{1}{2}}. \tag{29}$$

Rule (26) can be obtained by plugging (28) and (29) into (27). A possible microeconomic foundation for such a chartist behavior is discussed by De Grauwe (1996, pp. 181–185).

While making forecasts based on the fundamentals of the model, producer–speculators are assumed to use the following rule:

$$\frac{{}_F E^{\mathrm{e}}_{t+1}}{E_{t-1}} = \left(\frac{E^{\mathrm{PPP}}_{t-1}}{E_{t-1}}\right)^{\lambda}, \tag{30}$$

where E^{PPP}_{t-1} represents the equilibrium-PPP exchange rate at time period $t-1$, and parameter $\lambda \in (0, \infty)$ stands for the expected speed of return of the current (at date $t-1$) nominal exchange rate toward its equilibrium-PPP value.

According to (30), whenever producers who behave as fundamentalists observe a market rate above (below) the PPP value, they will expect it to decline (increase) in the future. Since $\lambda > 0$, the greater is λ, the faster fundamentalists will expect the nominal exchange rate to increase (fall) toward its equilibrium-PPP value when $E_{t-1} < E^{\mathrm{PPP}}_{t-1} (E_{t-1} > E^{\mathrm{PPP}}_{t-1})$. Values of λ greater than one could mean that fundamentalists expect some sort of future overshooting of the nominal exchange rate. Values of λ greater than one could thus be interpreted as meaning that fundamentalists expect convergence toward PPP after a transitional period of nominal exchange rate volatility.

The weight of charting in the home country $T_t \in (0, n)$ is endogenized by

$$T_t = \frac{n}{1 + \iota\left(E_{t-1} - E^{\mathrm{PPP}}_{t-1}\right)^2}, \tag{31}$$

where parameter $\iota \in (0, \infty)$ stands for the speed at which forecasts based on charts in the domestic country switch to forecasts based on fundamentals. The higher ι is, the faster charting will decrease.

In (31), the technical analysis used by producers who behave as chartists is made dependent on the size of deviation of the current (at date $t-1$) nominal exchange rate from its equilibrium (fundamental)-PPP value. Whenever the deviation from PPP increases, the expectations based on charts among domestic speculators will be reduced, and whenever the deviation from PPP tends to be eliminated, charting will grow in importance among domestic speculators. In accordance with (31), LeBaron (1994, p. 400) points out that predictability appears to be higher during periods of lower volatility, a phenomenon used by chartists to achieve some small out-of-sample improvements in forecasts. Equations analogous to (25), (26), (30), and (31) are presented by De Grauwe, Dewachter, and Embrechts (1993).

Before we proceed to derive first-order conditions, it is appropriate to solve first for E^{e}_{t+1}, because this model is recursive. Insertion of (26), (30), and (31) into (25) produces, after assuming that the equilibrium-PPP exchange rate equals one,

$$E^{\mathrm{e}}_{t+1} = E^{f_1}_{t-1} E^{f_2}_{t-2} E^{f_3}_{t-3}, \tag{32}$$

where

$$f_1 \equiv \frac{1 + \nu + \iota(1-\lambda)(E_{t-1}-1)^2}{1 + \iota(E_{t-1}-1)^2}, \tag{33}$$

$$f_2 \equiv \frac{-2\nu}{1 + \iota(E_{t-1}-1)^2}, \tag{34}$$

and

$$f_3 \equiv \frac{\nu}{1 + \iota(E_{t-1}-1)^2}. \tag{35}$$

Equation (32) thus shows that nominal exchange rate forecasts depend on past nominal exchange rates in a nonlinear way. This completes the description of the model.

1.8. First-order conditions

We can now derive the first-order optimality conditions for the representative home consumer–producer. Since $y^{\mathrm{d}}_t(z) = y_t(z)$ in equilibrium, the world demand function (21) implies $p_t(z)y_t(z) = P_t y_t(z)^{(\theta-1)/\theta}(C^{\mathrm{W}}_t)^{1/\theta}$. Substituting this in the period budget constraint (16) yields an expression for C, which when inserted into the utility function (1) produces the unconstrained maximization problem of the home individual.

The first-order conditions with respect to $B_{t+1},\ M_t$, and $y_t(z)$ are, respectively,

$$C^{\frac{1}{\sigma}}_{t+1} = \beta(1 + r_{t+1})C^{\frac{1}{\sigma}}_t, \tag{36}$$

$$\frac{M_t}{P_t} = \left[C^{\frac{1}{\sigma}}_t\left(\frac{1 + i_{t+1}}{i_{t+1}}\right)\right]^{\delta}, \tag{37}$$

and

$$y_t^{\frac{\theta+1}{\theta}} = \frac{\theta - 1}{\theta} C_t^{-\frac{1}{\sigma}} \left(C_t^{\mathrm{W}}\right)^{\frac{1}{\theta}}. \tag{38}$$

Equation (36) is the standard first-order consumption Euler equation. Equation (37) is familiar from money-in-the-utility-function models and was obtained using additionally (23) and (36). Finally, (38), which was obtained considering C^{W} as given, is the labor–leisure trade-off condition. First-order conditions (36)–(38), along with the period budget constraint (16) and a no-bubble transversality condition (which can be derived by iterating (16)), fully characterize equilibrium. Analogous equations hold for the foreign country.

Owing to monopoly pricing and endogenous output, this type of model does not yield simple closed-form solutions for general paths of exogenous variables. Rather than using numerical simulations to study the effects of exogenous shocks, Obstfeld and Rogoff adopt the following strategy: (1) they define the steady state as a situation in which all prices are fully flexible and all exogenous variables—including the nominal money supply—are constant; (2) even to this steady state there is no simple closed-form solution, so the authors pick the special case where there are no initial net foreign assets; (3) finally, they linearize the system around this particular well-defined steady state. Thus, only natural-logarithm approximations to the model solutions are studied. Here, analysis adopts steps 1 and 2 above but discards step 3. Numerical simulations are used instead. A reason for such a procedure is that a linear version of the model would wash out from the start the possibility of chaos, which is the focus of analysis here.

The entire particular strategy adopted in this article can be described as follows. The first-order conditions are rewritten to provide a single expression for the nominal exchange rate in which other endogenous variables are present. The extra endogenous variables then receive further rationale following steps 1 and 2 of Obstfeld and Rogoff, as described above.

Using (23) and (24), first-order conditions (36)–(38) can be reduced to a single expression as follows. First, we insert (24) into (23) and then substitute in (36); second, we plug (24) into (37); third, we substitute the first resulting expression in both the second one and (38); and finally, we combine the two remaining expressions. This produces

$$E_t = \frac{(1 + i_{t+1}^{*})E_{t+1}^{\mathrm{e}}}{1 + R_t + \frac{\theta-1}{\theta}\frac{(1+i_{t+1}^{*})E_{t+1}^{\mathrm{e}}}{E_t}\left(\frac{P_t}{M_t}\right)^{\frac{1}{\delta}}\left(C_t^{\mathrm{W}}\right)^{\frac{1}{\theta}} y_t^{-\frac{\theta+1}{\theta}}}. \tag{39}$$

Obviously, (39) is not an equation describing the behavior of the nominal exchange rate because, apart from E_{t+1}^{e}, the price index P_t and output y_t are not exogenous (remember, too, that R_t is given by the speculative dynamics of the previous time period).

1.9. *Closing the model*

To obtain expressions for P_t and y_t, the model is closed for the short run, when prices are fixed in local currency terms and output is demand determined. Following Obstfeld and Rogoff, period 0 is considered to be the initial steady state, in which prices are flexible and output is determined independently of monetary factors; period 1 is the short run, where output is determined entirely by the demand equation (21); and the final steady state is period 2.

Since an assumption of symmetry holds, all producers in a country determine the same price and output in equilibrium. However, this does not mean that $\bar{p}_t(z)/\bar{P}_t = 1$ in a steady state (steady states are marked by overbars). Since countries may have different levels of wealth, their marginal utilities of leisure differ. Even though individuals in the two countries face the same relative price for any good z (equation (15)), the relative price of home and foreign goods—the terms of trade—can vary. Even the steady-state terms of trade vary as relative wealth changes, because the marginal benefit from production is declining in wealth. Thus, in general, there is no simple closed-form solution for this model even in the steady state.

However, a solution does exist when initial net foreign assets are zero and the countries have the same per capita outputs and real money holdings. Given that global net foreign assets must be zero—i.e., $nB + (1-n)B^* = 0$—one particular steady state is defined by $\bar{B}_0 = \bar{B}_0^* = 0$. Such a bonds market-clearing condition, along with a money market-clearing condition, allows one to derive a global goods market-clearing condition. Equilibrium is completely symmetrical across the two countries in the special case where initial foreign assets are zero. In the globally symmetrical equilibrium, any two goods produced anywhere in the world have the same price when prices are measured in the same currency. This situation implies that $\bar{p}_0/\bar{P}_0 = \bar{p}_0^*/\bar{P}_0^* = 1$, where 0 subscripts on barred variables denote the initial preshock symmetrical steady state in which $\bar{B}_0 = \bar{B}_0^* = 0$. This is the rationale given by Obstfeld and Rogoff (1996, p. 668).

By denoting a variable without a time subscript as a short-run (period 1) variable, it might be noted that $p(z) = P = \bar{p}_0(z) = \bar{P}_0$ is implied in the short run (period 1) because prices continue to be the same as in period 0. In the short run, the world demand (21) thus becomes

$$y^{\mathrm{d}}(z) = y(z) = C^{\mathrm{W}}. \tag{40}$$

As in Da Silva (2000), central bank intervention can be in turn introduced by replacing M_t using the following feedback policy rule:

$$\frac{M_t}{M_t^T} = \left(\frac{E_t}{E_t^T}\right)^{\phi} \tag{41}$$

where M_t^T and E_t^T are domestic central-bank targets at time period t for the nominal money supply and the nominal exchange rate respectively; and ϕ is a policy parameter that is zero for free float and approaches either plus or minus infinity for a fixed exchange rate. "Leaning-against-the-wind" intervention is represented by $\phi \in (-\infty, 0)$, whereas "leaning into the wind" is given by $\phi \in (0, \infty)$. Although a feedback rule similar to (41) could apply to the foreign country, foreign exchange intervention is here assumed (for simplicity) to take place in the home country only, in order to counteract domestic chartist activity.

When (40) and (41) are substituted in (39), the result is that

$$E_t = \frac{(1 + i^*_{t+1})E^{\mathrm{e}}_{t+1}}{1 + R_t + \frac{\theta - 1}{\theta}\frac{(1+i^*_{t+1})E^{\mathrm{e}}_{t+1}}{E_t}\left(\frac{E_t^T}{E_t}\right)^{\frac{\phi}{\delta}}\left(\frac{P}{M_t^T}\right)^{\frac{1}{\delta}}\frac{1}{C^{\mathrm{W}}}}. \tag{42}$$

Apart from E^{e}_{t+1}, all variables explaining E_t in (42) are now exogenous. Without loss of generality, every exogenous variable is assumed to be constant and normalized to unity, i.e.,

$$M_t^T = P = E_t^T = C^{\mathrm{W}} = R_t = 1 + i^*_{t+1} = 1. \tag{43}$$

Insertion of (32) and (43) into (42) then gives the final equation for the nominal exchange rate:

$$2E_t + \frac{\theta - 1}{\theta}E_t^{-\frac{\phi}{\delta}}E_{t-1}^{f_1}E_{t-2}^{f_2}E_{t-3}^{f_3} - E_{t-1}^{f_1}E_{t-2}^{f_2}E_{t-3}^{f_3} = 0. \tag{44}$$

Equation (44) is a nonlinear difference equation for which an analytical solution is not available. To solve it numerically, initial conditions—i.e., values for $E_{t-1},\ E_{t-2}$, and E_{t-3}—are required. Even given such values, (44) has as many solutions as there are parameter combinations. To perform numerical simulations, the nominal exchange rate is assumed to be at its equilibrium-PPP value at the starting point, i.e., $E_{t-3} = E_{t-1}^{\mathrm{PPP}} = 1$. In the two subsequent periods, small deviations from this equilibrium are allowed. In particular, $E_{t-2} = 0.99$ and $E_{t-1} = 1.02$ are assumed. This set of initial conditions suffice to generate very complex dynamics in this model.

Since the endogenous variable (E_t) cannot be isolated on the left-hand side of (44), we need to employ Newton's algorithm to solve the equation. Newton's algorithm calculates the value of the nominal exchange rate at the next step of iteration as given by its current value minus the ratio between the function given by the left-hand side of (44) and its derivative. To carry out simulations using Newton's algorithm, there is also a technical need for an extra guess as to the fourth value of the nominal exchange-rate time series. This fourth value is assumed to be 0.99.

Table 1. Solutions to the model in the $(\nu,\ \lambda)$ space: degree of past extrapolation in charting (ν) versus expected speed of return of the current nominal exchange rate toward its equilibrium-PPP value (λ).

ν λ	0.1	0.5	1	3	5	10
900	U	U	U	U	U	U
800	U	U	U	U	U	U
275	U	ST	ST	CH	U	U
225	ST	ST	ST	CH	U	U
200	ST	ST	ST	CH	U	U
100	ST	ST	ST	CH	CH	U
10	ST	ST	ST	CH	CH	U
0.1	ST	ST	ST	CH	CH	U

Note: ST = stable solution, CH = chaotic solution, U = unstable solution. The greater ν is, the more chartists extrapolate the past into the future in exchange rate forecasts. The greater λ is, the faster fundamentalists expect the nominal exchange rate to go back toward its equilibrium value; values of λ greater than one mean that fundamentalists expect convergence after a transitional period of volatility. Other values are $E_{t-3} = 1.000000000,\ E_{t-2} = 0.990000000,\ E_{t-1} = 1.020000000,\ \phi = -10^2,\ \theta = 1.5,\ \iota = 10^4$, and $\delta = 0.5$.

1.10. Simulation results

Table 1 selects some of the solutions to (44) by focusing on the behavior of chartists and fundamentalists through parameters ν and λ. The simulations range up 15,000 datapoints, each one with nine decimal places. The other parameter values that are taken as given in Table 1 are $\phi = -10^2,\ \theta = 1.5,\ \iota = 10^4$, and $\delta = 0.5$. This sets up an environment in which the consumption elasticity of money demand assumes a sensible value $(\delta = 0.5)$; the speed at which forecasts based on charts switch to those based on fundamentals is relatively high $(\iota = 10^4)$; the market structure is one of strong monopolistic competition $(\theta = 1.5)$; and there is a leaning-against-the-wind foreign exchange intervention $(\phi = -10^2)$. In such a real-world-like scenario, Table 1 shows that chaotic behavior for the nominal exchange rate is possible.

Table 1 suggests that stability is associated with low values of ν together with values of λ that are not greater than one. Thus, the nominal exchange rate is more likely to be stable (1) the less that chartists extrapolate the past into the future in their forecasts and (2) when fundamentalists do not expect a transitional period of volatility prior to convergence toward PPP.

Instability is, by contrast, associated with high values of ν; the two rows at the top of Table 1 exemplify this. Instability is the norm, too, for values of ν greater than 900 (not shown in Table 1). However, even if the values of ν are low, instability can also emerge for values of λ greater than one. In particular, the nominal exchange rate diverges to infinity for values of λ equal to 10 (the last column in Table 1). Values of λ that are greater than 10 also generate solutions that are mostly unstable (not shown in Table 1).

The nominal exchange rate can go chaotic when the presence of charting seems to induce fundamentalists to expect a transitional period of volatility. This is illustrated in the columns for λ equal to 3 and 5. In particular, for high values of ν—such as those for the rows for 100, 200, 225, and 275 in Table 1—the fundamentalists' beliefs are in line with actual activity in charting. However, the possibility of chaos for low values of ν—such as those displayed in the rows for 0.1 and 10—suggests that self-fulfilling elements can also play a role when fundamentalists expect a transitional period of volatility prior to convergence, even if actual charting is negligible. Formal tests for the presence of chaos in these solutions are provided in the appendix.

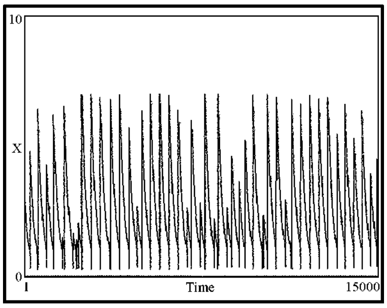
2. Discussion

As with the De Grauwe, Dewachter, and Embrechts model, our model is consistent with a number of stylized facts. The fact that actual nominal exchange rates seem to exhibit a random-like movement is replicated in the chaotic solutions to the model. The good news is that such "random" motions are generated by a deterministic equation and, accordingly, accurate short-run predictions are in theory possible. The bad news, however, is that meaningful long-run predictions are not possible due to the butterfly effect of chaotic series. However, even if real-world exchange rates are not chaotic, chaos (fake randomness) can be thought of as a proxy for genuine randomness.

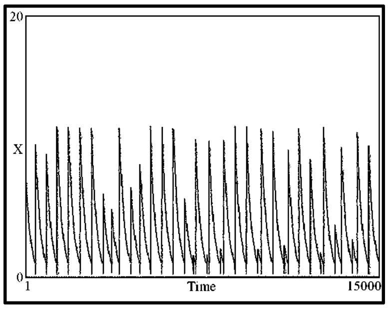
Figure 2 gives two examples of chaos accompanied by currency crashes for the solutions with $(\nu, \lambda) = (100, 3)$ and $(\nu, \lambda) = (10, 3)$. Not only does the nominal exchange rate exhibit a random-like behavior but also heteroskedasticity is present. Therefore, the stylized fact that actual behavior of exchange rates can be described as a martingale process is repeated in this chaotic model.

Contrary to the so-called *news approach*—which explains every nominal exchange rate movement by a given unexpected shock—chaotic exchange rate models such as this one do not need to rely on random shocks to explain swings in the nominal exchange rate, because crashes may occur with no random external influences. "Endogenous" crashes, such as the two spikes emerging in Figure 2 (mentioned above), occur with no change in any of the fundamental exogenous variables in the model.

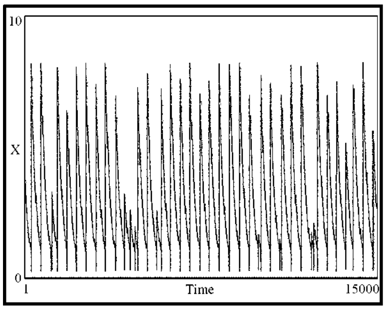
This model also provides a case for the importance of macromodels—in which fundamentals play a role—in explaining nominal exchange rate behavior. This is in line with recent attempts to revive explanations based on fundamentals to beat the simple random-walk hypothesis. In this model, fundamentals are captured by the interaction between exchange rate policy and speculative private behavior together with the existing market structure and the decision to hold money. Here, fundamentals matter because all these factors influence the



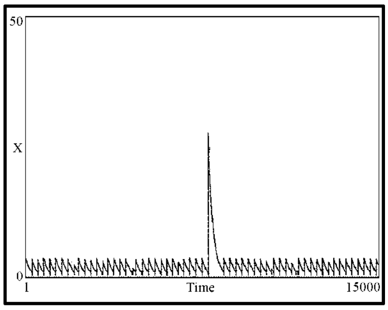
$(\nu, \lambda) = (200, 3)$



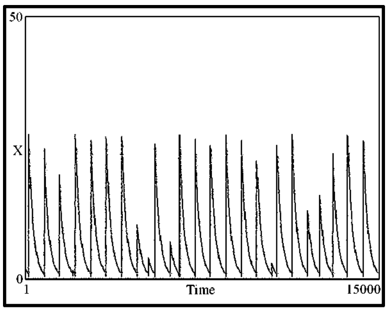
$(\nu, \lambda) = (275, 3)$



$(\nu, \lambda) = (225, 3)$



$(\nu, \lambda) = (100, 3)$



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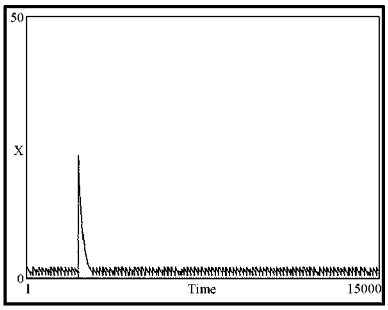

$(\nu, \lambda) = (10, 3)$

Figure 2. Chaotic solutions to the model displayed in Table 1. Range: 100–15,100. The first 100 datapoints were skipped to allow for a time series to settle into its final behavior. Other values are: $E_{t-3} = 1.000000000$, $E_{t-2} = 0.990000000$, $E_{t-1} = 1.020000000$, $\phi = -10^2$, $\theta = 1.5$, $\iota = 10^4$, and $\delta = 0.5$.

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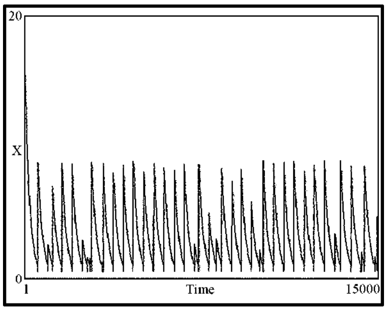
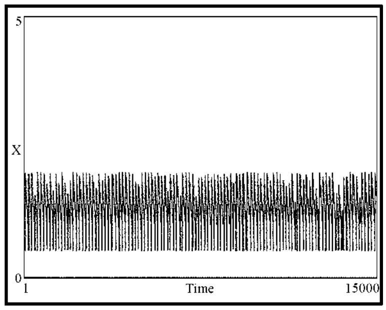
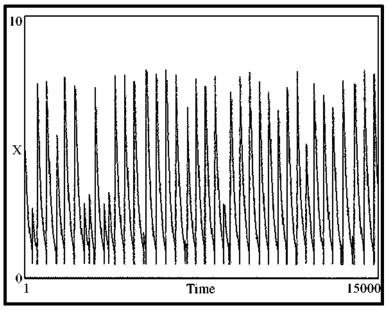


(ν, λ) = (10, 5)



(ν, λ) = (0.1, 3)



(ν, λ) = (0.1, 5)

Figure 2. (*Continued*).

dynamics of the model. Indeed, particular combinations of these fundamentals can give rise to chaos and thus mimic a random walk.

The results in this article are not incompatible with the fact that little or no evidence for chaos has been found in foreign exchange data, although a surprising amount of nonlinear structure remains unexplained (LeBaron, 1994, p. 397). The results imply that when looking for evidence of chaos in actual data, we should consider the interference of foreign exchange intervention. LeBaron (1996) shows that after removing periods in which the Federal Reserve is active, the ability to predict future nominal exchange rates using technical trading rules is dramatically reduced. This finding matches with the result that chaotic behavior of the nominal exchange rate is associated with charting in the presence of a nonzero amount of foreign exchange intervention. Silber (1994) also shows in a cross-sectional context that technical trading rules have value whenever governments are present as major players. Szpiro (1994), too, argues that an intervening central bank may induce chaos in nominal exchange rates.

As far as the chaotic solutions in Table 1 are concerned, one might wish to verify that stability does not obtain with free float. Indeed, taking the same initial conditions and parameter values, except now making $\phi = 0$, the previous chaotic solutions do not necessarily become stable. This finding suggests that chaos (and instability) can only give way to stability by some sort of appropriate intervention $\phi^{S} \neq 0$. Elsewhere (Da Silva, 1998, Section IV.3), I tackled the problem of how a nonzero equilibrium amount of intervention obtains as foreign exchange intervention is endogenized in the model presented here. The conditions under which there may be an equilibrium intervention that is compatible with stability in a chaotic foreign exchange rate market were discussed within a game-theoretic framework.

Newton's algorithm was used to solve (44), and this approach raises the question of whether some of the chaotic solutions are due to the approximation errors inherent to the algorithm. This possibility cannot be discarded, although the ability of Newton's algorithm to produce chaos in otherwise nonchaotic solutions remains to be proved.

3. Conclusion

This article generalizes the results shown in De Grauwe, Dewachter, and Embrechts (1993) in a more sophisticated, new open economy macroeconomics framework. The model of De Grauwe, Dewachter, and Embrechts is blended with the redux model of Obstfeld and Rogoff (1995, 1996, Chapter 10) to show the possibility of a chaotic nominal exchange rate for sensible parameter values. The redux model is modified to consider speculative behavior in the domestic country, where consumer-producers are assumed to behave like chartists and fundamentalists at the previous time period.

The model generates multiple equilibria. As far as stable equilibria are concerned, the solutions to the model show that the exchange rate is more likely to be stable (1) the less that chartists extrapolate the past into the future in their forecasts and (2) when fundamentalists do not expect a transitional period of volatility prior to convergence toward PPP. Unstable equilibria are, by contrast, associated with either (1) actual massive charting or (2) the possibility of charting, which induces fundamentalists to expect a volatility that turns out to be self-fulfilling. Regarding chaotic equilibria, the model shows that the exchange rate can go chaotic when the presence of charting induces fundamentalists to expect volatility. Chaos emerges whether or not the fundamentalists' beliefs are consistent with actual activity in charting; in the case where actual charting is negligible, a role for self-fulfilling elements is suggested.

The solutions to the model are evaluated for arguably sensible values of its parameters. Indeed, a real-world-like environment is set up to allow for speculation based on charts and fundamentals to take place; to consider the speed at which forecasts based on charts switch to those based on fundamentals as relatively high; to take into account the presence of leaning-against-the-wind foreign exchange intervention; to think of the market structure as one of strong monopolistic competition; and to allow for the consumption elasticity of money demand to assume a sensible value.

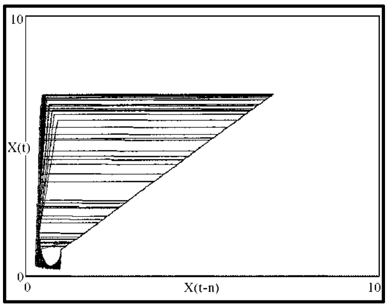
4. Appendix

The data sets were obtained from the time series generated from the chaotic solutions to the model displayed in Table 1, where the initial values are 1.000000000, 0.990000000, and 1.020000000 and the given parameters values are $\phi = -10^2,\ \theta = 1.5,\ \iota = 10^4$, and $\delta = 0.5$. In the record of 15,000 datapoints, each point has nine decimal places. Such time series were built up after skipping the first 100 points of an original series to allow for the nominal exchange rate to settle into its final behavior. The program employed for data analysis was *Chaos Data Analyzer: The Professional Version 2.1*® by J.C. Sprott, copyright © 1995 by the American Institute of Physics. Figures 2 and 3 were obtained using such software. Information regarding the description of statistics as well as suggestions of analysis strategy were taken from the PC user's manual of the program by Sprott and Rowlands (1995).

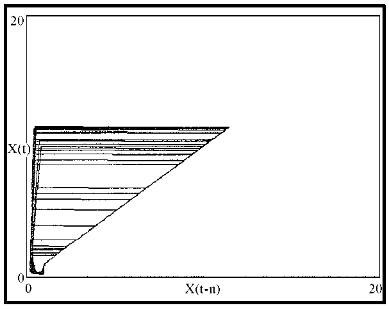
The pictures displayed in Figure 2 show that the data in these series are aperiodic. However, they are not genuinely random. There is an underlying structure that is revealed by "strange attractors" (Figure 3).Strange attractors are suggestive pictures that can be plotted from chaotic series showing some order in fake randomness. Since the data are aperiodic but not random, they are chaotic. Indeed, chaos is defined as apparently stochastic behavior occurring in deterministic systems (Stewart, 1997, p. 12). The probability distributions of the data sets (not shown) also show fractal shapes associated with chaos. If these data were genuinely random, bell-shaped Gaussian distributions would have emerged; if the data were periodic, simple histograms with sharp edges would have appeared.

Chaotic attractors can be quantified by measures of their dimension and their largest Lyapunov exponents. The dimension evaluates complexity, whereas the Lyapunov exponent measures sensitivity to change in initial conditions, i.e., the famous butterfly effect of chaotic series. Extreme sensitivity to tiny changes in initial conditions and therefore evidence of chaos is obtained as long as the largest Lyapunov exponent is positive. A zero exponent occurs near a bifurcation, periodicity is associated with a negative Lyapunov exponent, and white (uncorrelated) noise is related to an exponent approaching infinity. Table 2 shows that the largest Lyapunov exponents calculated from the data sets are positive; the solution to $(\nu, \lambda) = (100, 5)$ might also be a bifurcation. That outcome gives evidence of chaos, although colored (correlated) noise can have a positive exponent, too.

To test whether the evidence of hidden determinism in the data sets is robust, it is prudent to repeat the calculations of the Lyapunov exponents using surrogate data that resemble the original data but with the determinism



(ν, λ) = (200, 3)



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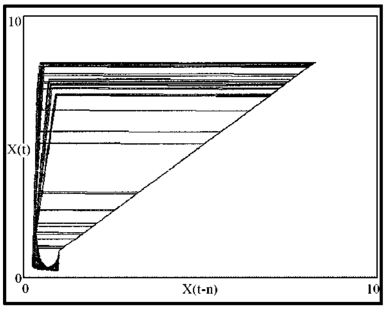

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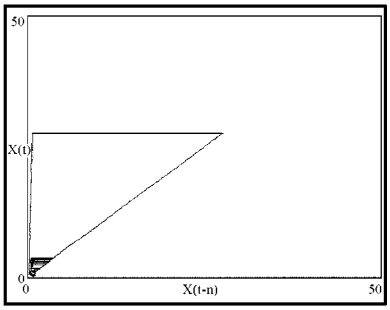

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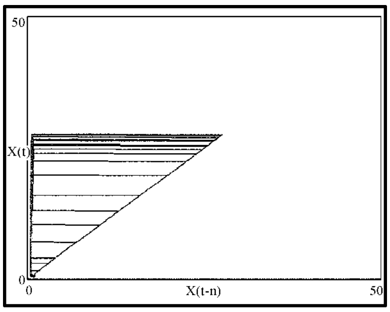

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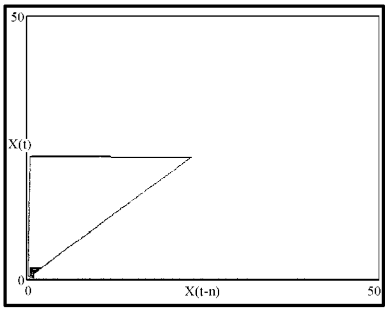

(ν, λ) = (10, 3)

Figure 3. Strange attractors of the chaotic solutions displayed in Table 1. Graph of data in two-dimensional plots. These plots reveal structure in apparently random data. If the data were genuinely random, "random dusts" would have emerged. Other values are: $E_{t-3} = 1.000000000$, $E_{t-2} = 0.990000000$, $E_{t-1} = 1.020000000$, $\phi = -10^2$, $\theta = 1.5$, $\iota = 10^4$, and $\delta = 0.5$.

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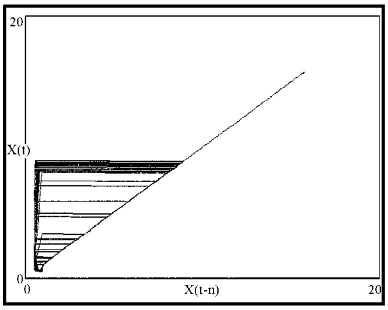
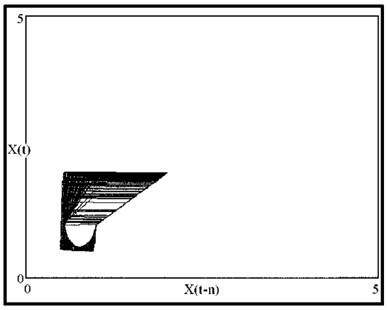
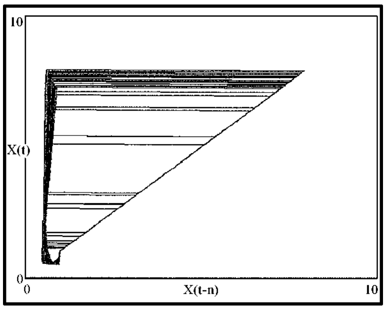


$(\nu, \lambda) = (10, 5)$



$(\nu, \lambda) = (0.1, 3)$



$(\nu, \lambda) = (0.1, 5)$

Figure 3. (*Continued*).

removed. Robustness implies that analysis of these surrogate data should provide values that are statistically distinct from those calculated from the original data. Three tests consecutively Fourier-transformed the data sets, randomized the phases, and then inverse Fourier-transformed the results, and a fourth test was carried out after simply shuffling the original data values. The Lyapunov exponents were then calculated, and the results (not displayed in this article) showed that they lay out of the range of values calculated from the original data sets. Differences are thus statistically significant, and one can conclude that the data sets are really chaotic and distinguishable from colored noise.

Capacity dimension and correlation dimension are major measures of dimension of a chaotic attractor. Values greater than about five for these measures give an indication of randomness, whereas values less than five provide further evidence of chaos. Table 2 shows that the capacity and correlation dimensions calculated from the data sets fall short of 5, which gives an additional piece of evidence for the presence of chaos in the data sets.

The measures of dimension assumed a proper embedding of 3 and time delay of 1. This assumption is generous, since the correlation dimensions saturate

Table 2. Summary of statistics for the chaotic solutions displayed in Table 1.

$(\nu,\ \lambda)$	Largest Lyapunov exponent[a]	Largest Lyapunov exponent to the base e[a]	Capacity dimension[b]	Correlation dimension[b]
(200, 3)	0.056 $\pm$ 0.010	0.039 $\pm$ 0.007	1.175 $\pm$ 0.067	1.041 $\pm$ 0.027
(275, 3)	0.007 $\pm$ 0.005	0.005 $\pm$ 0.003	1.018 $\pm$ 0.058	1.016 $\pm$ 0.038
(225, 3)	0.052 $\pm$ 0.010	0.036 $\pm$ 0.007	1.101 $\pm$ 0.063	1.009 $\pm$ 0.026
(100, 3)	0.072 $\pm$ 0.012	0.050 $\pm$ 0.008	1.196 $\pm$ 0.068	1.000 $\pm$ 0.043
(100, 5)	0.005 $\pm$ 0.005	0.003 $\pm$ 0.003	0.963 $\pm$ 0.055	1.031 $\pm$ 0.026
(10, 3)	0.022 $\pm$ 0.006	0.015 $\pm$ 0.004	1.085 $\pm$ 0.062	1.033 $\pm$ 0.019
(10, 5)	0.063 $\pm$ 0.010	0.044 $\pm$ 0.007	0.937 $\pm$ 0.054	1.027 $\pm$ 0.005
(0.1, 3)	0.084 $\pm$ 0.012	0.058 $\pm$ 0.008	1.012 $\pm$ 0.058	1.049 $\pm$ 0.029
(0.1, 5)	0.070 $\pm$ 0.011	0.049 $\pm$ 0.008	0.944 $\pm$ 0.054	1.043 $\pm$ 0.025

[a]Calculations considered the proper embedding dimension as given by 3, using three time steps and an accuracy of 10^{-4}.
[b]Calculations considered the proper embedding dimension equal to 3 and time delay equal to 1.
Note: Evidence of chaos is here associated with the largest Lyapunov exponents that are positive along with capacity and correlation dimensions that fall short of about 5. Other values are $E_{t-3} = 1.000000000$, $E_{t-2} = 0.990000000$, $E_{t-1} = 1.020000000$, $\phi = -10^2$, $\theta = 1.5$, $\iota = 10^4$, and $\delta = 0.5$.

at around 1. The proper embedding dimension of 3 was also assumed in the calculation of the Lyapunov exponents, using three time steps and an accuracy of 10^{-4}. Further discussion on these technical methods is provided by Sprott and Rowlands (1995).

Acknowledgments

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