Example of a Power Law

To illustrate what a power law is about, let us take some data from the experiment of dropping an object from different levels of a tall tower (e.g. Peitgen *et al.* 1992, pp. 195-197). The law governing free fall can be thought of as a power law. The first column in Table 1 lists the height of free fall in meters. The second column lists the drop time of the object in seconds. The last two columns display their respective logarithms to the base 10. Figures 1 and 2 show the data graphically. The relation between height and drop time is not linear (Figure 1), but there is a power law (Figure 2) because the drop time (t) changes as if it were a power of the height (h).

Table 1

Height	time	log (height)	log (time)
4	0.89	0.602059991	-0.050609993
8	1.26	0.903089987	0.100370545
12	1.55	1.079181246	0.190331698
16	1.79	1.204119983	0.252853031
20	2	1.301029996	0.301029996
24	2.19	1.380211242	0.340444115
28	2.37	1.447158031	0.374748346
32	2.53	1.505149978	0.403120521

The power law takes the form

$t = c \cdot h^d$.

The problem is to verify such a conjecture and determine c and d. Taking the logarithm on both sides obtains

$\log t = d \cdot \log h + \log c$.

Figure 2 shows that a log-log plot of t versus h generates a straight line with slope d and y-intercept $b = \log c$. So $c = 10^b$. From Figure 2 we can get $d = 0.5$ and $\log c = -0.34$. Then $c = 10^{-0.34} = 0.45$.

The power law is thus

$t = 0.45 \cdot h^{0.5}$.

This agrees with the Newtonian law of motion, which implies that the distance fallen is proportional to the square of the drop time:

$$h = \frac{g}{2} t^2$$

(where g . 9.81 m/sec^2 is the gravitational acceleration). Solving for t yields

$$t = \sqrt{\frac{2h}{g}} \approx 0.452 \cdot h^{0.5}.$$

Figure 1. Drop time versus height of free fall

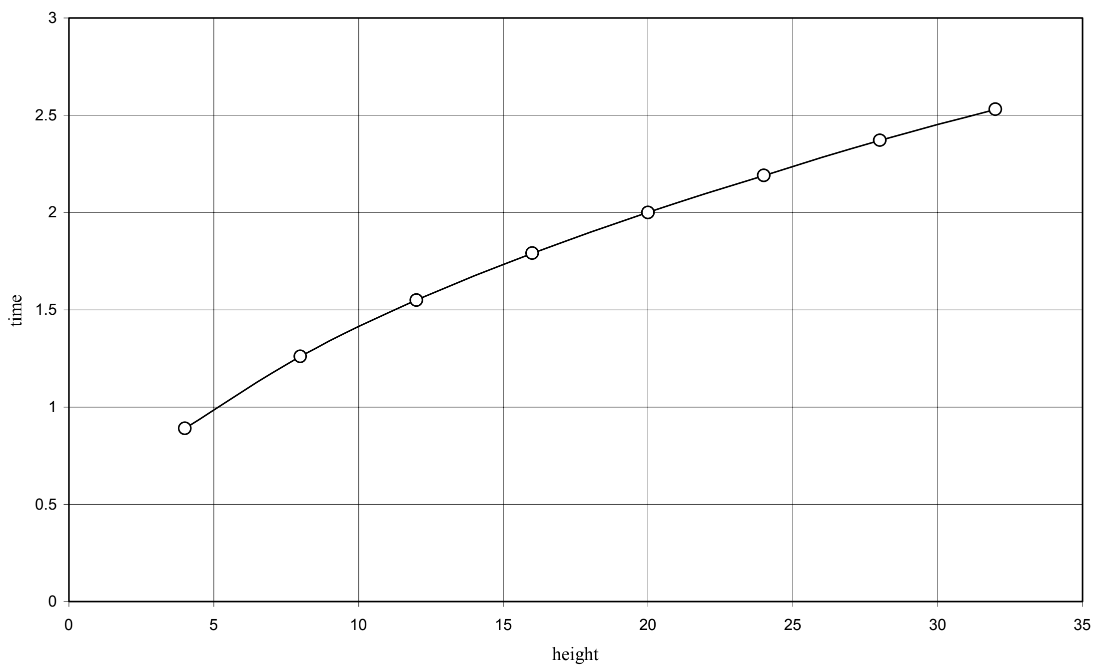

Figure 2. Log of drop time versus log of height of fall