Overshooting and Foreign Exchange Intervention in the Redux Model

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Abstract The paper discusses the role of foreign exchange intervention to remove overshooting in the model of Obstfeld and Rogoff (1995; 1996, Chapter 10) for a small economy with non-tradable goods. The result that foreign exchange intervention need not rule out overshooting is shown. JEL classification: F31, F41

1 Introduction

The model of the exchange rate developed recently by Obstfeld and Rogoff (OR) (1995; 1996, Chapter 10) is a dynamic perfect-foresight intertemporal two-country model that embeds monopolistic competition. The authors claim to provide the Dornbusch model with microfoundations. The model incorporates price stickiness in the so-called intertemporal approach to the current account. This allows for evaluating properly the impact of policies on output and the exchange rate in terms of an explicit welfare analysis. The OR model—or 'redux' model—focuses on a monopolistic supply sector. It extends to open-economy macroeconomics the new Keynesian research that stresses—along with trade theory—the importance of monopoly in explaining the progression from business cycles regularities to growth. Underpinned by the favorable evidence that nominal prices are sticky (a brief appreciation is found, for example, in OR, 1996, p. 676), monopoly is thought of as a rigorous justification for the assumption that output is demand-determined in the short run if prices are fixed. Indeed it is necessary that prices are set above marginal costs for an increase in demand to increase output if prices are fixed. (Lane, 1999 surveys the new wave of research—labeled 'new open economy macroeconomics'—launched by the model of Obstfeld and Rogoff.)

The benchmark redux model does not yield overshooting. The authors argue that, since evidence in support of it is thin, the fact that the phenomenon is not an essential property of their model is of no great concern. There is nothing intrinsic in the approach that precludes overshooting, however. Indeed, they show its possibility in a small-country version of their model, in which a non-tradable goods sector is the locus of monopoly and, therefore, of sticky-price problems.

The purpose of this paper is to show overshooting in the redux model for a small economy with non-tradables. A novelty in this presentation is to introduce foreign exchange intervention into the model to display the conditions for overshooting to occur in connection with a type of intervention.

2 The Model

The small-country case is a borderline case in which one country's relative size goes to zero. It yields overshooting but prevents the cross-country transmission effects fully discussed within the two-country framework. However, as argued below, the assumption that the country is small is not responsible by itself for overshooting.

The small-country model considers two sectors, tradable and non-tradable. In the tradable sector there is only one homogeneous good that is priced under perfect competition worldwide. In the non-tradable sector there are several goods produced under monopoly. Each individual monopolistic producer within the continuum $z \in [0,1]$ is endowed with a constant quantity of the tradable good Y_0^T each period, where the 0 subscript is used to indicate that it is a constant. Each producer has a monopoly over the production of one single differentiated perishable non-tradable good Y^N, also defined by z. Since the tradable and non-tradable goods are perishable there is no investment.

All individuals have identical preferences and produce motivated by the following intertemporal utility function U_t:

$$U_t \;=\; \sum_{s=t}^{\infty} \sigma^{s-t}\left[\nu \ln C_s^T \;+\; (1-\nu)\ln C_s^N \;+\; \frac{\delta}{\delta-1}\left(\frac{M_s}{P_s}\right)^{\frac{\delta-1}{\delta}} \;-\; \frac{\left(Y_s^N\right)^2}{2}\right], \tag{1}$$

where $\sigma \in (0,1)$ is a fixed preference parameter—called the subjective discount or time-preference factor—measuring the individual's impatience to consume; C^T stands for consumption of the tradable good; C^N is a composite consumption of non-tradable goods; parameter $\nu \in (0,1)$ denotes the share of the tradable good within the composite real consumption C (as defined below); variable M stands for the representative agent's holdings of nominal money balances entering period s; variable P is the consumption-based price index (as explained below); and $\delta \in (0,\infty)$ will turn out to be the income (consumption) elasticity of money demand, to which sensible values lie on the interval $\delta \in (0,1)$.

Since the country is small, it faces an exogenous world interest rate. By assumption, the subjective discount factor equals the market discount factor, i.e.

$$\sigma = \frac{1}{1+r_0}, \tag{2}$$

with $r_0 \in (0,\infty)$ denoting the constant world net real interest rate denominated in the tradable good. The nominal interest rate i_t between periods $t-1$ and t is also given in terms of the tradable and defined as

$$1+i_t \equiv \frac{P_{t+1}^T}{P_t^T}(1+r_0), \tag{3}$$

where P^T stands for the price of the tradable good.

According to (1), the representative consumer-producer maximizes a utility function that depends positively on consumption of both the tradable and non-tradable goods as well as on real balances, and negatively on work effort, which is positively related to the output of non-tradables. Following OR, the elasticity of intertemporal substitution is assumed to be equal to one. A more general formulation would be to allow for extra positive parameters—not necessarily equal to one—multiplying both C^T and C^N in (1).

The period utility function (1) is additively separable in the tradable and non-tradable goods. This feature is responsible for the result shown below that the representative consumer-producer smoothes consumption of the tradable regardless of production or consumption of non-tradables. It is also critical to producing long-run money neutrality in this model. If (1) were not additive in the tradable and non-tradables, consumption of non-tradables would affect the marginal utility of consumption of the tradable, and money shocks would affect the current account and net foreign assets, creating global spillover effects (OR, 1996, p. 694).

Real money balances also enter the isoelastic utility function (1) additively. Unlike OR, the consumption elasticity of money demand (parameter δ) is introduced here directly into (1). They take a parameter with no immediate economic meaning instead, which is the inverse of δ. As in the Dornbusch model (e.g. Da Silva, 2001), parameter δ will be a critical factor for overshooting to occur in this model. OR also use a positive parameter multiplying real balances. Here such a parameter is assumed to be equal to one. Since it does not enter the conditions for overshooting in the redux model, the latter assumption just simplifies this presentation.

The last term in (1) captures the disutility the individual experiences in having to produce more non-tradable output. As in OR, the elasticity of disutility from output of non-tradables equals two. The authors use, too, a parameter different from one (which falls as productivity rises) multiplying the last term of (1). Here such a parameter of productivity is assumed to be equal to one. Since this productivity parameter does not take part in the conditions for overshooting in the OR model, the latter assumption only simplifies matters.

The composite consumption of non-tradables in each period is defined as a generalization of a two-good constant-elasticity-of-substitution (CES) function taking the form

$$C_t^N = \left(\int_0^1 \left(c^N\right)^{\frac{\iota-1}{\iota}} dz \right)^{\frac{\iota}{\iota-1}}, \tag{4}$$

where c^N is the individual consumption of each non-tradable good z; and parameter $\iota \in (1,\infty)$ is the elasticity of substitution between different non-tradable goods; it will turn out to be, too, the price elasticity of demand faced by each monopolist. In addition, parameter ι may be considered as measuring the degree of competition in the economy. As the various non-tradable goods become close substitutes, ι becomes large and accordingly the distortions caused by monopoly are reduced. Thus, as $\iota \to \infty$ perfect competition holds; if $\iota \to 1$ pure monopoly obtains. Since the marginal revenue is negative when the price elasticity of demand is less than one, the requirement that $\iota > 1$ ensures an interior equilibrium with a positive level of non-tradable output.

The composite real consumption in each period C_t is then defined as

$$C_t \equiv \left(C_t^T\right)^{\nu} \left(C_t^N\right)^{1-\nu}. \tag{5}$$

Variable P is thought of as the minimum money cost of purchasing one unit of the composite real consumption C and is defined as

$$P_t \equiv \frac{\left(P_t^T\right)^{\nu}\left(P_t^N\right)^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}}, \tag{6}$$

where P_t^N stands for a non-tradable goods price index in each period given by

$$P_t^N = \left(\int_0^1 \left(p^N\right)^{1-\iota} dz\right)^{\frac{1}{1-\iota}}, \tag{7}$$

where p^N is the money price of the non-tradable good z. Price index (6) is associated with definition (4); formally, it solves the problem of minimizing the nominal budget constraint $Z = \int_0^1 p^N c^N dz$ (where Z is any fixed total nominal expenditure on non-tradable goods) subject to $C^N = 1$, as defined by (4).

Producers face the following period budget constraint in money terms:

$$P_t^T B_{t+1} + M_t = P_t^T(1+r)B_t + M_{t-1} + p_t^N Y_t^N + P_t^T Y_0^T - P_t^N C_t^N - P_t^T C_t^T - P_t^T T_t, \tag{8}$$

where bonds B and per capita non-distorting lump-sum taxes T are both denominated in the tradable good. Equation (8) is the same as the one appearing in the book version of the OR model (OR, 1996, p. 691), which differs slightly from that displayed in the earlier paper version (OR, 1995, p. 656), in which bonds are lagged one period.

The intertemporal government budget constraint in terms of the tradable good is, in turn, given by

$$\frac{M_t - M_{t-1}}{P_t^T} = -T_t. \tag{9}$$

There is no government spending in (9), and the government balances its budget each period so that all seignorage revenues are rebated to the producers in the form of transfers (negative per capita taxes).

As monopolists the producers of non-tradables face downward-sloping demand curves, i.e.

$$Y^N_{D_t} = \left(\frac{p^N_t}{P^N_t}\right)^{-\iota} C^N_{A_t}, \qquad (10)$$

where Y^N_D stands for the total world demand for non-tradable goods, and C^N_A is the aggregate per capita non-tradable consumption that producers take as given. Equation (10) can be derived by maximizing (4) subject to Z as defined above (a step-by-step presentation in given by OR, 1996, pp. 664-665).

Finally, since the country does not have any market power in the tradable good, the law of one price and therefore purchasing power parity (PPP) holds for the tradable every time period, i.e.

$$S_t = \frac{P^T_t}{P^{f^T}_0}, \qquad (11)$$

where S_t is the nominal exchange rate, which is defined as the price of foreign currency in units of domestic currency; and $P^{f^T}_0$ is the price of the tradable good in the foreign country, which is constant by assumption. This completes the description of the model. Table 1 below displays its relevant equations for which the endogenous variables are U_t, C^T_t, C^N_t, Y^N_t, P^T_t, P^N_t, and S_t.

Table 1. The redux model for a small economy with non-tradables

$$U_t = \sum_{s=t}^{\infty} \sigma^{s-t}\left[\nu \ln C^T_s + (1-\nu)\ln C^N_s + \frac{\delta}{\delta-1}\left(\frac{M_s}{P_s}\right)^{\frac{\delta-1}{\delta}} - \frac{\left(Y^N_s\right)^2}{2}\right] \qquad (1)$$

$$C^N_t = \left(\int_0^1 \left(c^N\right)^{\frac{\iota-1}{\iota}} dz\right)^{\frac{\iota}{\iota-1}} \qquad (4)$$

$$P_t^N = \left(\int_0^1 \left(p^N\right)^{1-\iota} dz \right)^{\frac{1}{1-\iota}} \tag{7}$$

$$P_t^T B_{t+1} + M_t = P_t^T (1 + r) B_t + M_{t-1} + p_t^N Y_t^N + P_t^T Y_0^T - P_t^N C_t^N - P_t^T C_t^T - P_t^T T_t \tag{8}$$

$$\frac{M_t - M_{t-1}}{P_t^T} = -T_t \tag{9}$$

$$Y_{D_t}^N = \left(\frac{p_t^N}{P_t^N} \right)^{-\iota} C_{A_t}^N \tag{10}$$

$$S_t = \frac{P_t^T}{P_0^{f^T}} \tag{11}$$

3 Solution

The first-order conditions for individual optimization are obtained by maximizing (1) subject to constraints (8) and (10) as follows. First, using (10) to substitute for p_t^N in (8); secondly, using the resulting expression to substitute for C^T in (1), after considering that, in the aggregate, world demand must equal world output of non-tradables, i.e. $Y_{D_t}^N = Y_t^N$; and, finally, differentiating (and setting equal to zero) the latter resulting expression with respect to B_{t+1}, M_t, C_t^N, and Y_t^N.

Differentiating with respect to B_{t+1} yields the first of the first-order conditions as

$$C_t^T = C_{t+1}^T . \tag{12}$$

Equation (12) is the well-known consumption Euler equation for the case in which the intertemporal elasticity of substitution is one. Due to assumption (2), the constant real interest rate implies that the consumer-producer desires a flat lifetime-consumption path. Equation (12) also shows that the producer smoothes the consumption of the tradable good regardless of the production and consumption of non-tradables, a result, however, that follows from the additive separability of the utility function (1).

Since there is neither investment in the tradable sector nor any government spending, making the assumption that there are no foreign assets—i.e. exports equal imports—in the starting time period yields the result that the tradable output equals the tradable consumption at the initial period only. As condition (12) is further considered, it turns out that

$$C_t^T = Y_0^T \tag{13}$$

and exports must equal imports in every time period. Thus, in the absence of productivity changes in the tradable production, the current account will be balanced regardless of shocks to money or non-tradable goods productivity (OR, 1995, p. 657; 1996, pp. 691-692).

Differentiating with respect to M_t and setting the result to zero produces the second first-order condition as

$$P_t^T = \frac{1}{\dfrac{C_t^T}{\nu P_t}\left(\dfrac{M_t}{P_t}\right)^{-\frac{1}{\delta}} + \dfrac{\sigma C_t^T}{P_{t+1}^T C_{t+1}^T}}, \tag{14}$$

which, after considering (2), (3), and (13), becomes the money demand

$$\frac{M_t}{P_t} = \left[\frac{P_t^T C_t^T}{\nu P_t}\left(\frac{1 + i_t}{i_t}\right)\right]^{\delta}, \tag{15}$$

where it is indeed apparent that parameter δ represents the consumption elasticity of money demand. Equation (15) is familiar from money-in-the-utility-function models. It arises from the equilibrium condition that the consumer is indifferent whether consuming a unit of the tradable good today or using the same funds to raise cash balances, enjoying the derived transactions utility today, and then converting the extra cash balances back to consumption tomorrow.

Differentiating with respect to C_t^N and setting the result to zero yields the third of the first-order conditions as

$$C_t^N = \frac{1 - \nu}{\nu}\left(\frac{P_t^T}{P_t^N}\right)C_t^T. \tag{16}$$

Equation (16) governs allocation between the tradable and non-tradable goods.

Finally, differentiating with respect to Y_t^N (taking $C_{A_t}^N$ as given) and setting the result to zero produces the last of the first-order conditions as

$$Y_t^N = \left[\frac{(\iota - 1)(1 - \nu)}{\iota}\left(\frac{1}{C_t^N}\right)\left(C_{A_t}^N\right)^{\frac{1}{\iota}}\right]^{\frac{\iota}{\iota+1}}. \tag{17}$$

The less familiar equation (17) is the labor-leisure trade-off condition ensuring that the marginal-utility cost of producing an extra unit of non-tradable output due to foregone leisure equals the marginal utility from consuming the added revenue that an extra unit of non-tradable output brings.

Except for the additional equation (16), equations (12), (15), and (17) are analogous to those of the two-country OR model where each good can be traded. Assuming no bubbles, the equilibrium is thus characterized by first-order conditions (12), (15), (16), and (17) along with period budget constraint (8).

To introduce foreign exchange intervention into this model, the following policy rule is considered:

$$\frac{M_t}{\overline{M}_t} = \left(\frac{S_t}{\overline{S}_t}\right)^{\phi}, \tag{18}$$

where $\overline{M}_t$ and $\overline{S}_t$ are targets of the money supply and exchange rate respectively. The central bank parameter ϕ captures the degree of intervention in the foreign exchange market. It is zero under free float and approaches plus or minus infinity under the regime of a fixed nominal exchange rate. Leaning-against-the-wind intervention is represented by $\phi \in (-\infty, 0)$, whereas leaning-into-the-wind intervention is given by $\phi \in (0, \infty)$. The natural-log version of such a policy rule is suggested by, for example, Marston (1985, p. 910).

Free float occurs when $\phi = 0$ because in that situation the central bank focuses exclusively on the target of the money supply $\overline{M}_t$, abstaining from any intervention in the foreign exchange market ($M_t = \overline{M}_t$ if $\phi = 0$). The fixed exchange-rate regime holds when $\phi \rightarrow \pm\infty$ because in that case the central bank focuses exclusively on its nominal exchange-rate target $\overline{S}_t$, without thinking about the money supply ($S_t = \overline{S}_t$ if $\phi \rightarrow \pm\infty$).

Leaning against the wind is the intervention operation that attempts to move the exchange rate in the opposite direction from its current trend, and leaning into the wind is motivated by the central bank's desire to support current exchange-rate trends. Here both leaning against the wind and leaning into the wind are carried out by changes in $\overline{M}_t$. It might be noted that whether such changes are sterilized is not discussed.

Thus, if $S > \overline{S}$ for any reason, the aim of leaning against the wind is to reduce the current nominal exchange rate S. That can be achieved by reducing $\overline{M}$ because $\phi < 0$. If $S < \overline{S}$, the aim of the leaning-against-the-wind intervention is to increase $\overline{M}$. Since leaning into the wind signifies supporting the current nominal exchange-rate trend, if $S > \overline{S}$ that sort of intervention means increasing $\overline{M}$ when $\phi > 0$. Finally, if $S < \overline{S}$, leaning into the wind implies reducing $\overline{M}$.

Policy rule (18) can be further inserted into the microfounded money demand function (15) to produce

$$\frac{\overline{M}_t}{P_t}\left(\frac{S_t}{\overline{S}_t}\right)^{\phi} = \left[\frac{P_t^T C_t^T}{\nu P_t}\left(\frac{1 + i_t}{i_t}\right)\right]^{\delta}. \tag{19}$$

4 Steady State Analysis

Due to monopoly pricing and endogenous output this model does not yield simple closed-form solutions for general paths of the exogenous variables. Instead of using numerical simulations to study the effects of exogenous shocks, OR adopt the following strategy. First, they define a steady state as a situation in which all prices are fully flexible and all exogenous variables—including the money supply—are constant. Even in such a steady state there is no simple closed-form solution; the authors then pick the special case in which initial net foreign assets are nil, i.e. $B_0 = 0$, where 0 subscript denotes the initial preshock steady state. Finally, they linearize the system around this particular well-defined steady state. Thus only natural-logarithm approximations to the solution are studied. (An illustration of how to implement the strategy above is provided in detail by OR, 1996, p. 245).

To analyze the short-run equilibrium response to an unanticipated money supply shock, it is useful first of all to divide the period budget constraint (8) by P_t^T. The government budget constraint (9) and the market-clearing condition of the tradable good (13) along with the assumption of zero initial net foreign assets, i.e. $B_t = B_{t+1} = B_0 = 0$, yield $P_t^N C_t^N = p_t^N Y_t^N$. If all producers are symmetric, they set the same price and output in equilibrium. Thus, imposing symmetry across the various domestic producers—i.e. $p_0^N = P_0^N$—gives $Y_t^N = C_t^N$. After considering (10) this also implies $Y_{D_t}^N = C_{A_t}^N$. But in the steady state, flexible prices ensure that $Y_{D_t}^N = Y_t^N$ and, accordingly, $Y_t^N = C_t^N = C_{A_t}^N$ for all z. Inserting the latter result into (17) gives the steady-state output of non-tradables as

$$Y_0^N = C_0^N = C_{A_0}^N = \left[\frac{(\iota - 1)(1 - \nu)}{\iota}\right]^{\frac{1}{2}}, \tag{20}$$

where the expression on the right hand side of (20) is a constant. Monetary shocks will not affect Y^N, C^N, or C_A^N, as discussed below.

As long as the price of the tradable good is constant in the steady state—i.e. $P_t^T = P_{t+1}^T = P_0^T$—long-run monetary neutrality obtains. Constancy of P^T can be derived from a no-speculative-bubbles condition (OR, 1995, p. 657; 1996, p. 692). Money neutrality can be seen by looking for an expression to P_0 using (3), (6), (13), (16), (19), and (20) along with $P_t^T = P_{t+1}^T$ and the assumption that in the steady state $S_0 = \overline{S}_0$. This gives an expression showing that $\overline{M}_0$ equals P_0 times a constant term, which indicates that the money supply target can only affect the consumption-based price index. (In the benchmark two-country model money is non-neutral even in the long run, however, because money shocks affect wealth (the current account)).

The initial steady state has been defined as time period 0. Now the short run is defined as time period 1, whereas the final steady state refers to time period 2. Thus, it is implied that the economy reaches its long-run equilibrium in just one period, by period 2. Since money is neutral in the long run, only nominal variables change across steady states. An unanticipated and permanent money increase occurring in the short run in this model means that the shock at time 1 is not expected at time 0 and it lasts until time 2.

Price stickiness in the monopolistic non-tradable goods sector means in this model that prices are set a period in advance and adjust to the shock only by period 2, i.e. $P_1^N = P_0^N \neq P_2^N$. Price flexibility of the tradable good means, in turn, that $P_1^T \neq P_0^T \neq P_2^T$. Since $P_1^N = P_0^N$ and producers are symmetric—i.e. $p_0^N = P_0^N$ and therefore $p_1^N = P_1^N$—it turns out that the non-tradable output is demand-determined in the short run. Indeed taking (10) into account gives $Y_{D_1}^N = C_{A_1}^N$. Equation (20) in turn implies that $Y_0^N = C_0^N = C_{A_0}^N = Y_1^N = C_1^N = C_{A_1}^N$ because these variables are independent of money shocks. As a result, one obtains

$$Y_{D_1}^N = Y_1^N = C_1^N. \tag{21}$$

Inserting (13) into (16), after considering (21) and the fact that $P_1^N = P_0^N$ in the short run, one derives

$$Y_1^N = C_1^N = \frac{1 - \nu}{\nu}\left(\frac{P_1^T}{P_0^N}\right)Y_0^T. \tag{22}$$

Since P_0^N and Y_0^T are constant, equation (22) gives output and consumption of non-tradables in the short run as functions of the price of the tradable good, which is flexible and related to the nominal exchange rate through PPP equation (11). Thus, to focus analysis on the exchange rate, the remaining task is to find an expression for P_1^T.

5 Log-linearization

We can proceed by log-linearizing the key equations allowing us to demonstrate the possibility of overshooting in this model. The first equation to begin with may be PPP. Taking the natural logarithm of (11) and totally differentiating the result yields (after recalling that $P_0^{f^T}$ is a constant)

$$\hat{S} = \hat{P}^T, \tag{23}$$

where $\hat{S}$ denotes the short-run percentage deviation of the nominal exchange rate from the initial steady state; and $\hat{P}^T$ stands for the short-run percentage deviation of the price of the tradable good from the initial steady state.

Taking (11) to period $t + 1$, log-linearization produces

$$\tilde{S} = \tilde{P}^T, \tag{24}$$

where $\tilde{S}$ stands for the long-run percentage deviation of the nominal exchange rate from the initial steady state; and $\tilde{P}^T$ represents the long-run percentage deviation of the price of the tradable good from the initial steady state. Here and below, short-run variations of a given variable X are defined as $dX_t \equiv X_1 - X_0$, and short-run percentage deviations of X from the initial steady state (period 0) are denoted by $\hat{X} \equiv (X_1 - X_0) / X_0$, whereas long-run percentage deviations from the initial steady state are denoted by $\tilde{X} \equiv (X_2 - X_0) / X_0$.

The money demand (19) is log-linearized as follows. Firstly, its natural logarithm is taken and the differential of the resulting expression is calculated, remembering that C_t^T is a constant (equation (13)). Secondly, i_t is removed from the obtained expression using (2), (3), and the assumption that $P_{t+1}^T = P_t^T = P_1^T = P_0^T$ in the steady-state (period 0) before the money-supply shock had taken place. It turns out that $i_t = (1 - \sigma) / \sigma$ and $di_t = (1 / \sigma)(dP^T_{t+1} / P^T - dP^T_t / P^T_t)$. Thirdly, the values of (period 0) initial steady state for the variables of the remaining equation are taken, and then the definitions of short- and long-run percentage variations from the initial steady state are used. As a result, one obtains (after recalling that $P^T_{t+1} = P^T_t = P^T_1 = P^T_0$)

$$\frac{1}{\delta}(\hat{M} - \hat{P}) + \frac{\phi}{\delta}(\hat{S} - \hat{\bar{S}}) = \hat{P}^T - \hat{P} + \frac{\sigma}{1 - \sigma}(\hat{P}^T - \tilde{P}^T). \tag{25}$$

Considering that P^N is fixed in the short run, log-linearization of price-index (6) gives

$$\hat{P} = \nu\hat{P}^T. \tag{26}$$

Due to long-run monetary neutrality, only prices change across steady states (periods 0 and 2), which means $d\bar{M}_{t+1} / \bar{M}_0 = dP_{t+1} / P_0 = (\bar{M}_2 - \bar{M}_0) / \bar{M}_0 = (P_2 - P_0) / P_0$. If a money-supply increase is permanent—i.e. $(\bar{M}_1 - \bar{M}_0) / \bar{M}_0 = (\bar{M}_2 - \bar{M}_0) / \bar{M}_0$—one obtains $\hat{M} = \tilde{M} = \tilde{P}$. In the steady state, $P^T_{t+2} = P^T_{t+1} = P^T_t = P^T_1 = P^T_0$ is valid, and its log-linearized form is given by $dP^T_{t+2} / P^T_0 = dP^T_{t+1} / P^T_0 = dP^T_t / P^T_0$, i.e. $\tilde{P}^T = \hat{P}^T$. Using these two results along with the log-linearized money-demand (25) rewritten to the steady state $(t + 1)$, one has (after remembering that $S_2 = \bar{S}_2$)

$$\tilde{P}^T = \tilde{M} = \hat{M} = \tilde{P}. \tag{27}$$

Plugging (26) and (27) into (25) yields

$$\hat{P}^T = \frac{1 - \sigma(1 - \delta)}{\delta\sigma + (1 - \sigma)[\delta(1 - \nu) + \nu]}\hat{M} + \frac{\phi(1 - \sigma)}{\delta\sigma + (1 - \sigma)[\delta(1 - \nu) + \nu]}(\hat{S} - \hat{\bar{S}}). \tag{28}$$

Equation (28) is similar to the one appearing in OR (1995, p. 658; 1996, p. 693), apart from the second term on the right hand side, because this treatment generalizes that of OR for considering foreign exchange intervention. Indeed if $\phi = 0$, equation (28) collapses to the authors' equation.

6 Overshooting

Assuming for convenience that percentage changes of the target to the nominal exchange rate equal percentage changes of the long-run nominal exchange rate, and that both are normalized to unity—i.e. $\hat{\bar{S}} = \tilde{S} = 1$—equations (23), (24), and (27) are inserted into (28) to find the possibility of overshooting—i.e. $\hat{S} > \tilde{S} = 1$—as given by $\delta < 1$ with both free float ($\phi = 0$) and leaning-against-the-wind intervention ($\phi \in (-\infty, 0)$). Such a condition is the same as the one by OR (1995, p. 658; 1996, p. 693). If $\delta < 1$, overshooting still emerges with policies of leaning into the wind ($\phi \in (0, \infty)$) when $\phi \leq \delta\sigma / (1 - \sigma) + \delta(1 - \nu) + \nu$. If $\delta > 1$, overshooting occurs with leaning into the wind when $\phi > \delta\sigma / (1 - \sigma) + \delta(1 - \nu) + \nu$. If $\delta = 1$, there is no overshooting with leaning-into-the-wind intervention. In short, foreign exchange intervention need not rule out overshooting in the context of the redux model.

Condition $\delta \in (0,1)$ falls into the probable empirically relevant case. To understand the reason why it provokes overshooting in this model, let us imagine first that $\delta = 1$ and accordingly overshooting is not possible whatever the type of intervention. In that case, equation (28) along with the assumptions made above related to it imply that $\hat{S} = \tilde{S} = \hat{\bar{S}} = \tilde{P}^T = \hat{P}^T = \hat{M} = 1$. Replacing $\hat{P}^T$ with $\hat{M}$ in (26)—in which P^N is assumed to be fixed in the short run—after multiplying by -1 and adding up $\hat{M}$ yields $\hat{M} - \hat{P} = (1 - \nu)\hat{M}$. Since $\nu \in (0,1)$, a one percent increase in the money supply target causes the supply of real balances to grow by less than one percent of that amount, i.e. $1 - \nu$, simply because prices of non-tradables are fixed in the short run. But a one percent rise in $\hat{M}$ also means a one percent increase in $\hat{P}^T$. Taking into account the log-linearized version of a more general expression corresponding to (22) (also assuming P^N fixed in the short run), one obtains $\hat{Y}^N = \hat{C}^N = \hat{P}^T$. Thus, a one percent rise in $\hat{P}^T$ causes an equiproportionate rise in both $\hat{C}^N$ and $\hat{Y}^N$. But a one percent rise in $\hat{C}^N$ leads to a less than one percent increase in real consumption C, because $\hat{C} = (1 - \nu)\hat{C}^N$. (The latter expression can be obtained after plugging (13) into (5) and log-linearizing the resulting equation).

As far as the demand for real balances is concerned, after substituting (26) and condition $\hat{S} = \tilde{S} = \hat{\bar{S}} = \tilde{P}^T = \hat{P}^T = \hat{M} = 1$ into the log-linearized money demand (25), one has $\hat{M} - \hat{P} = \delta(1 - \nu)\hat{M}$, where the consumption elasticity of money demand is left explicit for convenience. If $\delta = 1$ a one percent rise in $\hat{M}$ causes real balances to increase by $1 - \nu$ as earlier. However, if $\delta < 1$ a one percent rise in the money supply target provokes real balances to grow by less than before, i.e. by $\delta(1 - \nu)$. Therefore, if the demand for real balances rises by less than the supply, the price of the tradable good will have to rise further to reach equilibrium, thereby overshooting its long-run level. Since the nominal exchange rate is tied to the price of the tradable good by PPP, it must overshoot as well. Thus, price stickiness of non-tradable goods causes both the price of the tradable good and the nominal exchange rate to overshoot in response to a monetary shock. This demonstrates the possibility of overshooting in a sticky price model of the exchange rate with microfoundations.

Finally, it is worth noticing that since a rise in the money supply target increases real balances as well as non-tradable consumption and output, then the utility of the representative individual is increased in (1). Thus, monetary policy is unambiguously welfare-improving. An unanticipated rise in the money supply target improves welfare by coordinating an increase in output across consumers-producers in the monopolistic non-tradable goods sector (OR, 1995, p. 658; 1996, p. 694).

The authors argue that the overshooting result would still be achieved if the country were large; and this suggests that the assumption of a small country facing an exogenous world interest rate is not crucial at all. What seems to be critical is the presumption that a money shock produces no current-account imbalance, which is embodied in the model through the assumption that the utility function is additive in the tradable and non-tradable goods (OR, 1996, p. 694). This appears to be the reason why overshooting is not possible in their benchmark two-country model featuring the property of long-run non-neutrality of money. Strikingly, in the latter model preset prices actually *reduce* the nominal exchange rate volatility due to monetary shocks, because the inflating country experiences an improvement in its long-run terms of trade that tempers the need for initial depreciation (OR, 1995, p. 644). Roughly, the rationale runs as follows. A short-run depreciation temporarily raises the domestic real income relative to the foreign one, so that the domestic country runs a current account surplus. This higher long-run wealth leads to substitution into leisure, a fall in the supply of domestic goods, and therefore an improvement of the domestic terms of trade. Since real income and consumption of the domestic country rise in the long run, the nominal exchange rate does not need to depreciate as much as it would otherwise. A comprehensive appreciation of this point is presented by OR (1995, pp. 639-646; 1996, pp. 677-682).

7 Concluding Remarks

The Obstfeld-Rogoff model updates the Dornbusch model as regards microfoundations and shows that overshooting is not an essential property of a benchmark two-country framework. However, there is nothing intrinsic in the approach that precludes the possibility of the phenomenon. In a small country version of their model allowing for non-tradable goods, the authors show that sticky prices of non-tradable consumption goods may be a critical factor for overshooting of both the price of the tradable good and the nominal exchange rate. The condition for that is the consumption elasticity of money demand to fall into its likely empirically relevant case. Extending this result to consider foreign exchange intervention, this paper shows that such a condition for overshooting with free float also holds with leaning-against-the-wind intervention. Also, overshooting still emerges with some policies of leaning into the wind. Thus foreign exchange intervention need not rule out overshooting as far as the redux model is concerned.

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