Endogenous Foreign Exchange Intervention

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Abstract
The paper shows that there is an equilibrium foreign exchange intervention bias when the exchange rate regime is endogenous in the sense that it is the outcome of an exchange rate policy game. What is more, the intervention bias holds regardless of the status of the players. (JEL classification: F31, F41)

1 Introduction

This paper shows how a non-zero equilibrium amount of intervention obtains as foreign exchange intervention is endogenized. Loss functions of a central bank and a representative speculator are specified to yield a solution which is not free float, regardless of the status of the players in the exchange rate policy game; so an 'intervention bias' is likely to occur. Here analysis draws on Almekinders (1995, Chapter 7), although a different model is employed.

2 The Model

2.1 Loss functions of the central bank and the representative speculator

A domestic central bank is assumed to intervene in the foreign exchange market at the end of time period s to react to shocks which materialized between the end of $s-1$ and the end of s. The domestic central bank's loss function L_t^{CB} is postulated as

$$(1) \qquad L_t^{CB} = \sum_{s=t}^{\infty} \rho^{s-t} \left[\frac{(\psi\phi_s)^2}{2} + \frac{\mu(\phi_s - \phi_s^e)^2}{2} + \frac{\gamma(E_s - E_s^T)^2}{2} \right] - U_t ,$$

where parameter $\rho \in (0,1)$ is a discount factor applied to future losses of the central bank regarding exchange rate management; $\psi \in (0,\infty)$ is a factor of proportionality for the costs of intervention through the portfolio balance channel; $\mu \in (0,\infty)$ is a parameter capturing the costs of the expectations channel of intervention; $\gamma \in (0,\infty)$ weights the central bank aversion to deviations from its exchange rate target; ϕ_s indicates the type of foreign exchange intervention at time period s (as discussed in detail below); $\phi_s^e \in (-\infty,\infty)$ represents the central bank intervention expected by the representative speculator at the end of time period s; E_s is the nominal exchange rate (home-currency price of foreign currency) at time period s; E_s^T stands for the domestic central-bank target at time period s for the nominal exchange rate; and U_t represents a generic utility function at time period t.

A domestic representative speculator's loss function L_t^S is assumed to be given by

(2) $$L_t^S = \sum_{s=t}^{\infty} \sigma^{s-t} \left[\frac{(\phi_s^e - \phi_s)^2}{2} \right] - U_t ,$$

where $\sigma \in (0,1)$ is a discount factor relative to future losses of the speculator.

The first terms in brackets on the right hand side in both loss functions (1) and (2) are similar to the ones proposed by Almekinders (1995, pp. 173-174, 186), while the idea of taking the utility function in the second terms on the right hand side in (1) and (2) is borrowed from Obstfeld and Rogoff (1996, p. 714). Time periods are assumed not to be connected, and thus minimization of (1) and (2) is equivalent to minimizing such loss functions for each time period separately. Accordingly, analysis collapses to a study of a repeated play of one-shot games.

The first and second terms in the brackets on the right-hand side in the central bank's loss function (1) proxy the costs of intervention through the portfolio balance and expectations channels respectively. Central bank losses are assumed to increase more than proportionately with the volume of these interventions. Since intervention volumes are very tiny compared to the daily turnover on the foreign exchange market, interventions through the portfolio channel may be negligible implying $\psi = 0$. Owing to this, equation (1) leaves open the possibility that central bank intervention works through the expectations channel. So the second term in the brackets on the right-hand side in (1) allows the central bank to exploit the divergence between actual and expected interventions to influence the nominal exchange rate by catching the representative speculator off balance. If the expectations channel were not operative—i.e. $\mu = 0$—there would be no costs for this sort of intervention.

The third term in the brackets on the right-hand side in (1) reflects the fact that central bank losses increase more than proportionately with changes in the nominal exchange rate which are not caused by the relative state of the macroeconomic 'fundamentals' in the domestic country. The nominal exchange-rate target is assumed to be given by its previous purchasing power parity (PPP) value which is normalized to unity, i.e. $E_t^T = E_{t-1}^{PPP} = 1$. The last term on the right-hand side in (1) shows that the central bank not only pays attention to exchange rate policy but also considers the utility of domestic residents.

The first term in brackets on the right-hand side in (2) reflects the domestic representative speculator's aversion to being fooled by the central bank. When this is the case, the speculator's losses increase more than proportionately. The second term on the right-hand side in (2) shows that the speculator considers these losses on exchange-rate policy grounds after discounting the gains given by utility. In loss function (2), the strategic interaction between the central bank and speculators is assumed to come into play only after a shock to the nominal exchange rate has materialized. Since such a shock is probably the outcome of the interaction between the various sorts of decision made by a number of market participants, that assumption implies that issues of intra-speculators' behavior can be prevented and it is thus possible to treat distinct foreign currency traders as a unique representative speculator.

The following policy rule is considered to model foreign exchange intervention:

$$(3) \qquad \frac{M_t}{M_t^T} = \left(\frac{E_t}{E_t^T}\right)^{\phi},$$

where M_t is the domestic money supply at t; and M_t^T and E_t^T are targets of the money supply and exchange rate respectively. The central bank parameter ϕ captures the degree of intervention in the foreign exchange market. It is zero under free float and approaches plus or minus infinity under the regime of a fixed nominal exchange rate. Leaning-against-the-wind intervention is represented by $\phi \in (-\infty, 0)$, whereas leaning-into-the-wind intervention is given by $\phi \in (0, \infty)$. The natural-log version of such a policy rule is suggested by, for example, Marston (1985, p. 910).

Free float occurs when $\phi = 0$ because in that situation the central bank focuses exclusively on the target of the money supply M_t^T, abstaining from any intervention in the foreign exchange market ($M_t = M_t^T$ if $\phi = 0$). The fixed exchange-rate regime holds when $\phi \to \pm\infty$ because in that case the central bank focuses exclusively on its nominal exchange-rate target E_t^T, without thinking about the money supply ($E_t = E_t^T$ if $\phi \to \pm\infty$).

Leaning against the wind is the intervention operation that attempts to move the exchange rate in the opposite direction from its current trend, and leaning into the wind is motivated by the central bank's desire to support current exchange-rate trends. Here both leaning against the wind and leaning into the wind are carried out by changes in M_t^T. It might be noted that whether such changes are sterilized is not discussed.

Thus, if $E > E^T$ for any reason, the aim of leaning against the wind is to reduce the current nominal exchange rate E. That can be achieved by reducing M^T because $\phi < 0$. If $E < E^T$, the aim of the leaning-against-the-wind intervention is to increase M^T. Since leaning into the wind signifies supporting the current nominal exchange-rate trend, if $E > E^T$ that sort of intervention means increasing M^T when $\phi > 0$. Finally, if $E < E^T$, leaning into the wind implies reducing M^T.

2.2 Nash equilibrium of the exchange rate policy game

Since the strategic interaction between the central bank and the representative speculator in the domestic country takes place after a shock to the nominal exchange rate has occurred, the shock can be observed by both the central bank and the speculator. The central bank and the speculator then simultaneously set ϕ_t and ϕ_t^e respectively. As in Barro/Gordon-type models, such an equal status of the players leads to a Nash equilibrium.

The function of the central bank's reaction to the expectation of the representative speculator can be obtained from the minimization of its loss at time period t, taking ϕ_t^e as given. So partially differentiating (1) relative to ϕ_t and setting the result to zero yields

$$(4) \qquad \phi_t = \frac{1}{\mu + \psi^2}\left[\frac{\partial U_t}{\partial \phi_t} - \gamma(E_t - 1)\frac{\partial E_t}{\partial \phi_t} + \mu\phi_t^e\right].$$

In reaction function (4), it may be noted that ϕ affects both utility and the nominal exchange rate in particular ways that are left implicit in $\partial U_t/\partial\phi_t$ and $\partial E_t/\partial\phi_t$ respectively.

The representative speculator's reaction function is in turn obtained by partially differentiating (2) relative to ϕ_t^e (taking ϕ_t as given) and setting the result to zero. This produces

(5) $$\phi_t^e = \phi_t .$$

The intervention that corresponds to the Nash equilibrium is found by plugging (5) into (4) to give

(6) $$\phi_t = \frac{1}{\psi^2}\left[\frac{\partial U_t}{\partial \phi_t} - \gamma(E_t - 1)\frac{\partial E_t}{\partial \phi_t}\right]$$

In equation (6), the greater the costs of intervention through the portfolio channel (the greater ψ), equilibrium intervention is more than proportionately lower. Besides that, except if by chance $\partial U_t/\partial \phi_t = \gamma(E_t - 1)\partial E_t/\partial \phi_t$, there is a non-zero equilibrium 'intervention bias' ($\phi_t \neq 0$).

Such an intervention bias of discretionary exchange rate policy has been put forward in literature already (e.g. Almekinders, 1995, pp. 175-177) and it is analogous to the well-known 'inflationary bias' of discretionary monetary policy in Barro/Gordon-type models.

Equation (6) also implies that equilibrium intervention will be leaning against the wind ($\phi_t < 0$) if exchange-rate management factors—i.e. $|\gamma(E_t - 1)\partial E_t/\partial \phi_t|$—are given greater importance than utility considerations—$|\partial U_t/\partial \phi_t|$—from the part of the central bank. Leaning-into-the-wind intervention ($\phi_t > 0$) will be optimal if the reverse applies.

It can be argued that the intervention bias is inefficient because the costs involved are not offset by any benefits. Also, as the non-zero amount of intervention is fully anticipated by the representative speculator, it is not successful at limiting the impact of the shock to the exchange rate (Almekinders, 1995, p. 177). As shown below, such an intervention bias holds regardless of the status of the players in the exchange rate policy game.

2.3 Stackelberg equilibrium

The central bank acts as a leader in a Stackelberg game when it takes action before the representative speculator and knows the exact shape of the reaction function of the latter, so that it is able to choose the point on the speculator's reaction function that minimizes its own loss function.

The domestic central bank thus sets ϕ_t, and thereby ϕ_t^e is determined endogenously. The Stackelberg equilibrium is obtained by first substituting the speculator's reaction function (5) into the central bank's loss function (1), then by partially differentiating relative to ϕ_t, and finally by setting the result to zero. As one may wish to verify, equation (6) again obtains.

The representative speculator acts as a Stackelberg leader before the central bank when he sets ϕ_t^e, and thereby ϕ_t is determined endogenously. Now the Stackelberg equilibrium is obtained as follows. First, by inserting the central bank's reaction function (4) into the speculator's loss function (2), then by partially differentiating relative to ϕ_t^e and setting the result to zero, and finally by plugging the resulting expression for the expected volume of intervention into (4). The result is again equation (6), as one may wish to verify.

3 Concluding Remarks

The paper shows that there is an equilibrium foreign exchange intervention bias when the exchange rate regime is endogenous, i.e. when it is the outcome of an exchange rate policy game. What is more, such an intervention bias holds regardless of the status of the players.

It might be noted that this result depends critically on the shape of the postulated (ad hoc) loss functions (1) and (2). However, the result that it is possible for a non-zero amount of foreign exchange intervention be an equilibrium regardless of the status of the players in the exchange rate policy game remains of interest.

References

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