Does Foreign Exchange Intervention Remove Overshooting?

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The paper discusses exchange rate overshooting in connection with foreign exchange intervention using standard models. The result that overshooting may emerge out of free float and cannot be removed by unqualified foreign exchange intervention is shown to hold. JEL Classification: F31, F41

1 Introduction

Overshooting has been until now the standard explanation for the volatility of free-float exchange rates, although it is far from being "the only game in town". The exchange rate overshoots its new equilibrium level when, in response to a disturbance, it first moves beyond the equilibrium it ultimately will reach and then gradually returns to the long-run equilibrium position (Dornbusch and Fischer, 1994, p. 620). The phenomenon is commonly associated with the Dornbusch (1976) model, which explains it by goods price stickiness.

Those who believe that exchange rate overshooting introduces an undesirable instability into the economy argue that the authorities should intervene to avoid exchange rate volatility (Dornbusch and Fischer, 1994, p. 620). Throughout this paper such a policy implication will be evaluated by explicitly incorporating foreign exchange intervention into standard models. It will be seen that intervention must be qualified in terms of the parameters of a model to be effective.

The paper is organized as follows. Section 2 sets out a Dornbusch-style model. Section 3 extends the model to account for foreign exchange intervention. Using a quantitative analysis, Section 4 generalizes further the Dornbusch model with intervention to consider a more general expectations rule. Section 5 concludes.

2 The Dornbusch model

2.1 The model

The model is defined by equations (1)C(4) displayed in Table 1.

Table 1. A benchmark Dornbusch model

$y = \beta(s + p^f - p) - \alpha r$	(1)
$\dot{p} = \pi(y - \overline{y}) + \dot{p}^e$	(2)
$m = p + \delta y - \theta i$	(3)
$\dot{s}^e = i - i^f$	(4)

All variables are in natural logarithms, exceptions being the domestic nominal interest rate i, the foreign nominal interest rate i^f, and the domestic real interest rate r. Both i and i^f are the nominal interest rates available on similar domestic and foreign securities respectively, with the same periods to maturity.

Variable s is the nominal exchange rate, which is defined as the price of foreign currency in units of domestic currency; p stands for the domestic price level; p^f is the foreign price level; y gives the aggregate demand for domestic output; $\overline{y}$ is the domestic natural output; $\dot{p}$ stands for domestic price level changes; $\dot{p}^e$ represents expected changes in the domestic price level; m is the nominal money supply; and $\dot{s}^e$ stands for expected depreciations of the nominal exchange rate (or expected appreciations with the minus sign) referring to forecasts made at this time period for the next-period exchange rate.

Parameter α gives the semi-interest elasticity of investment demand; β represents the real exchange-rate elasticity of the demand for exports; δ stands for the income elasticity of the demand for money; θ is the semi-interest elasticity of the demand for money; and π signifies the steepness of the short-run Phillips curve. These parameters are set in absolute values so they are all positive.

The relative price of foreign goods in terms of domestic goods is defined as the real exchange rate, i.e.

$$e = s + p^f - p, \tag{5}$$

where e stands for the natural log of the real exchange rate. The version of the model given in Table 1 is standard.

Equations (1) and (2) describe the goods market. Equation (1) is the IS relation, in which the current aggregate domestic demand depends positively on the real exchange rate, and negatively on the real interest rate. In response to increases (depreciations) in the nominal exchange rate, exports are stimulated and the aggregate demand rises; if the foreign (domestic) price level increases (falls), the aggregate demand for domestic output moves upward. In turn, if the real interest rate is reduced, the investment demand increases and the aggregate demand rises. Equation (2) is the expectations-augmented Phillips curve, in which changes occurring in the price level are procyclical in relation to the output gap as well as dependent on expected changes in the price level.

Equations (3) and (4) refer to the asset markets. Equation (3) defines the LM relation, where the real money supply is set equal to the money demand, which in turn depends positively on the current income, and negatively on the nominal interest rate. In particular, the money demand in (3) results from taking the natural logarithm of a conventional function of the form $Y^{\delta}\exp(-\theta i)$, where Y is the aggregate demand for domestic output in levels. Equation (4) is the uncovered interest rate parity (UIP) hypothesis, which ensures that expected rates of return, expressed in a common currency, are equalized; in other words, the expected foreign-exchange gain from holding one currency rather than another (the expected exchange-rate change) must be just offset by the opportunity cost of holding funds in this currency rather than the other (the interest rate differential).

Since goods prices are assumed to be sticky in the short run, agents expect the price level not to vary; in turn, the future exchange rate is forecast by rational expectations (perfect foresight), i.e.

$$\dot{p}^e = 0 \tag{6}$$

and

$$\dot{s}^e = \dot{s}, \tag{7}$$

where $\dot{s}$ stands for exchange rate depreciations in natural log.

Plugging (6) into (2) yields

$$\dot{p} = \pi(y - \overline{y}). \tag{8}$$

Definition $r \equiv i - \dot{p}^e$ along with (6) produce $r = i$. When this result is inserted into (3) one obtains

$$m = p + \delta y - \theta r. \tag{9}$$

Using the analogous definition $r^f \equiv i^f - \dot{p}^{fe}$ for foreign variables (throughout in this paper, superscript f denotes foreign variables) together with the foreign analogous of (6)Ci.e. $\dot{p}^{fe} = 0$ Cyield $r^f = i^f$. Introducing (7), $r = i$, and $r^f = i^f$ into (4) produces

$$\dot{s} = r - r^f. \tag{10}$$

The model turns out to be made up of the four equations (1), (8), (9), and (10) (Table 2), where the five endogenous variables are y, r, $\dot{p}$, $\dot{s}$, and s. The indeterminacy is removed by assuming that s is free to jump at a point in time. This procedure is quite standard in ad hoc models such as this one, though it cannot be considered satisfactory. However, consideration of bubbles is beyond the scope of this paper.

Table 2. The basic Dornbusch model

$y = \beta(s + p^f - p) - \alpha r$	(1)
$\dot{p} = \pi(y - \overline{y})$	(8)
$m = p + \delta y - \theta r$	(9)
$\dot{s} = r - r^f$	(10)

2.2 Solution

Substituting (10) in (1) and (9), and making the resulting expressions equal, one obtains

$$\dot{s} = \frac{\beta\delta}{\alpha\delta + \theta} s + \frac{1 - \beta\delta}{\alpha\delta + \theta} p + \frac{\beta\delta}{\alpha\delta + \theta} p^f - r^f - \frac{1}{\alpha\delta + \theta} m . \tag{11}$$

Inserting (8) into (1) and (9), and equating the resulting expressions yields

$$\dot{p} = \frac{\beta\theta\pi}{\alpha\delta + \theta} s - \frac{\pi(\alpha + \beta\theta)}{\alpha\delta + \theta} p + \frac{\beta\theta\pi}{\alpha\delta + \theta} p^f - \pi\overline{y} + \frac{\alpha\pi}{\alpha\delta + \theta} m . \tag{12}$$

The model is now reduced to equations (11) and (12). Taking their total differentials, the steepness of the $\dot{s} = 0$ locus in Figure 1 (at the end of this paper) is found to be positive if $\delta > 1/\beta$, and negative if $\delta < 1/\beta$. The $\dot{p} = 0$ schedule, however, is always positively sloped because $ds/dp = (\alpha + \beta\theta)/\beta\theta > 0$.

2.3 Overshooting

Overshooting is possible in the case $\delta < 1/\beta$ displayed in Figure 1. In such a situation, if the central bank pushes up the money supply m unexpectedly and once and for all, this shifts both the $\dot{s} = 0$ schedule upwardCbecause $ds/dm = 1/\beta\delta > 0$ in (11)Cand the $\dot{p} = 0$ locus downwardCbecause

$ds/dm = -\alpha/\beta\theta < 0$ in (12). The initial full equilibrium is represented by point A, whereas the final full equilibrium is given by point C.

Long-run neutrality of money implies that the monetary expansion should cause an equiproportionate increase in the domestic price level and the nominal exchange rate. As purchasing power parity (PPP) also holds in the long run, the nominal exchange rate should rise by precisely the right amount to offset the effects of the domestic price-level increase on the real exchange rate. The necessary condition for an exchange depreciation between points A and C to keep the purchasing power of domestic goods constant in terms of foreign goods is for point C to be above point A, which implies that the $\dot{s} = 0$ schedule should shift by more than the $\dot{p} = 0$ locus, and this requires $\delta < \theta/\alpha$.

If any given point in Figure 1 falls above (below) the $\dot{s} = 0$ locus, the nominal exchange rate must rise (fall) because, *ceteris paribus*, $d\dot{s}/ds = \beta\delta/(\alpha\delta + \theta) > 0$, and $d\dot{p}/dp = -\pi(\alpha + \beta\theta)/(\alpha\delta + \theta) < 0$. These forces pushing the observation point away from full equilibrium represent an unstable characteristic root. Similarly, if a point is to the right (left) of the $\dot{p} = 0$ schedule, the domestic price level falls (rises). These forces pushing the observation point toward full equilibrium represent a stable characteristic root. As a consequence, the saddle path SS slopes down from left to right in the $s - p$ space. Since one of the characteristic roots is unstable and the nominal exchange rate is free to jump at a point in time, the initial value of s can be set at just what is required to preclude the unstable root from operating; this setting constitutes placing the system on the saddle path SS.

The nominal exchange rate depreciates beyond the final full equilibrium because the economy jumps from point A to B instantly. The initial value s_0 jumps toward the particular level s_1 to ensure the economy is on the saddle path SS. Since p cannot jump in the short run C defined as the period of time that elapses before the domestic price level starts to react C the nominal exchange rate moves down (appreciates) gradually from point B to C. In the impact period when p is fixed, s_1 overshoots the final full-equilibrium exchange rate s^*. (In Section 4, overshooting is shown in an algebraic, rather than graphical, manner.)

The rationale for overshooting given by Dornbusch (1976, p. 1168) runs as follows. At the initial price level, the monetary expansion instantly reduces the domestic (nominal and real) interest rate, which leads to the anticipation of a depreciation in the long run and, at the current exchange rate, to the expectation of a depreciating exchange rate. This reduces the attractiveness of domestic assets, leads to an incipient capital outflow, and causes the spot rate to depreciate. The extent of that depreciation has to be sufficient to give rise to the anticipation of appreciation at just sufficient a rate to offset the reduced domestic interest rate. The short-run effect of a money supply increase is, therefore, to induce an immediate depreciation in the spot rate that exceeds the long-run depreciation, since only under these circumstances will agents anticipate an appreciating exchange rate and thus be compensated for the reduced interest on domestic assets.

In terms of the version of the model of exogenous output presented here, if initially the domestic and foreign interest rates are equal, UIP (4) ensures that the expected depreciation of the exchange rate must also be zero. Since the domestic price level is fixed in the impact period, the money supply increase is translated into an initial rise in the real money supply and into a consequent instantaneous fall in the domestic nominal interest rate to clear the money market; as a result, $\dot{s}^e$

must become negative in (4); the exchange rate is expected to appreciate. As expectations are rational, the exchange rate actually appreciates after the money supply increase. But since the long run effect (point C in Figure 1) is a depreciation, the spot rate must depreciate instantly by more (point B) than its steady-state depreciation, and then appreciate steadily toward its full equilibrium value (from B to C). As the domestic price level begins to rise in response to the increase in the nominal money supply, the real money supply begins to decline, and the domestic interest rate begins to rise. The nominal exchange rate then appreciates slowly toward long-run PPP, at the same pace as that of the domestic price level (as shown by Dornbusch, 1976, p. 1165). Once the long-run equilibrium at point C is attained, interest rates are equal internationally, the goods market clear, prices are constant, and expected exchange-rate changes are zero.

It might be noted that in such an adjustment process the (nominal and real) exchange rate and the domestic (nominal and real) interest rate overshoot in tandem, although the domestic interest rate takes the opposite direction, as is apparent from the above description. Also, if $\delta < 1/\beta$, overshooting can still occur whenever the foreign price level decreases (both the $\dot{s} = 0$ and $\dot{p} = 0$ schedules shift up to the same extent), the foreign interest rate rises (the $\dot{s} = 0$ locus shifts up), or the domestic natural output increases (the $\dot{p} = 0$ locus shifts up).

3 Foreign exchange intervention

3.1 The policy rule

Here it is investigated to what extent central bank intervention removes overshooting. To focus on the relationship between overshooting and intervention, the following policy rule is now embodied in the model in Table 2:

$$m = \overline{m} + \phi(s - \overline{s}), \tag{13}$$

where $\overline{m}$ represents the central bank target to the nominal money supply given in natural logarithm; and $\overline{s}$ is its target to the nominal exchange rate in natural logarithm. The central bank parameter ϕ captures the degree of intervention in the foreign exchange market. It is zero under free float and approaches plus or minus infinity under the regime of a fixed exchange rate. Leaning-against-the-wind intervention is represented by $\phi \in (-\infty, 0)$, whereas leaning-into-the-wind intervention is given by $\phi \in (0, \infty)$.

The economy is under free float when $\phi = 0$ because in such a situation the central bank focuses exclusively on the target of the domestic money supply $\overline{m}$, abstaining from any intervention in the foreign exchange market. The fixed exchange-rate regime holds when $\phi \to \pm\infty$ because in this case the authorities focus exclusively on the exchange rate target $\overline{s}$, without thinking about the domestic money supply.

Leaning against the wind is the intervention operation that attempts to move the exchange rate in the opposite direction from its current trend, whereas leaning into the wind is motivated by the central bank's desire to support the current exchange-rate trend. Here both leaning against the wind and leaning into the wind are carried out by changes in $\overline{m}$. It might be noted that whether such changes are sterilized is not discussed.

Thus, if $s > \overline{s}$ for any reason, the aim of leaning against the wind is to reduce the current exchange rate s. That can be achieved by reducing $\overline{m}$ as far as (13) is concerned to $\phi < 0$. If $s < \overline{s}$, the aim of the leaning-against-the-wind intervention is to increase $\overline{m}$. Since leaning into the wind signifies supporting the current exchange-rate trend, if $s > \overline{s}$ this sort of intervention means increasing $\overline{m}$ in this model (this can be checked in (13) to $\phi > 0$). Finally, if $s < \overline{s}$, leaning into the wind implies reducing $\overline{m}$. Throughout, the borderline case of a fixed exchange rate is not discussed, since it is not meaningful to talk about overshooting with respect to a fixed nominal exchange rate. Emphasis is thus on leaning-against-the-wind and leaning-into-the-wind foreign exchange interventions.

3.2 The model

A new LM expression can be obtained by plugging policy rule (13) into the m appearing in equation (9). This yields

$$\overline{m} = p + \delta y - \theta r - \phi(s - \overline{s}). \quad (14)$$

The model of this section thus consists of equations (1), (8), (14), and (10) (Table 3).

Table 3. The Dornbusch model with foreign exchange intervention

$y = \beta(s + p^f - p) - \alpha r$	(1)
$\dot{p} = \pi(y - \overline{y})$	(8)
$\overline{m} = p + \delta y - \theta r - \phi(s - \overline{s})$	(14)
$\dot{s} = r - r^f$	(10)

It might be noted that considering $m = \overline{m}$ and $\phi = 0$ in (14), this equation collapses to the LM given by (9). For this reason, it may be argued that the Dornbusch model implicitly assumes free float. Indeed, in the original presentation by Dornbusch free float is implicit (e.g. Dornbusch, 1976, pp. 1161 n1, 1173). In particular, the model in Table 2 turns out to be a special case of the model in Table 3 in that the former presupposes $\phi = 0$.

3.3 Solution

Substituting (10) in (1) and (14), and then equating the resulting expressions yields

$$\dot{s} = \frac{\beta\delta - \phi}{\alpha\delta + \theta} s + \frac{1 - \beta\delta}{\alpha\delta + \theta} p + \frac{\beta\delta}{\alpha\delta + \theta} p^f - r^f + \frac{\phi}{\alpha\delta + \theta} \overline{s} - \frac{1}{\alpha\delta + \theta} \overline{m}. \qquad (15)$$

Inserting (8) into (1) and (14), and then making the resulting expressions equal gives

$$\dot{p} = \frac{\pi(\alpha\phi + \beta\theta)}{\alpha\delta + \theta} s - \frac{\pi(\alpha + \beta\theta)}{\alpha\delta + \theta} p + \frac{\beta\theta\pi}{\alpha\delta + \theta} p^f - \pi\overline{y} - \frac{\alpha\pi\phi}{\alpha\delta + \theta} \overline{s} + \frac{\alpha\pi}{\alpha\delta + \theta} \overline{m}. \qquad (16)$$

The model is thus reduced to the two differential equations (15) and (16).

By totally differentiating (15) and (16) it can be clearly recognized that most of the expressions for the multipliers now depend on ϕ. In particular, steepness of both the $\dot{s} = 0$ schedule C given by $(\beta\delta - 1)/(\beta\delta - \phi)$ C and the $\dot{p} = 0$ locus C given by $(\alpha + \beta\theta)/(\alpha\phi + \beta\theta)$ C can now assume any value, because they depend not only on positive private parameters as before but also on the policy parameter, which may be positive, negative, or nil.

3.4 Overshooting

Overshooting emerges under free float ($\phi = 0$) as long as $\delta < 1/\beta$. It may be noted that this is the same condition presented in the model in Table 2 (and displayed in Figure 1), which implicitly assumes a flexible exchange rate. The nominal exchange rate obviously cannot overshoot if it is tied by the authorities. This fact arises in this model as the steepness of both the $\dot{s} = 0$ and $\dot{p} = 0$ schedules are horizontal when $\phi \to \pm\infty$.

Since the $\dot{p} = 0$ locus is always positively sloped with leaning into the wind ($\phi > 0$), the conditions required for the $\dot{s} = 0$ schedule to be negative imply that overshooting may emerge if either $\phi < \beta\delta < 1$ or $\phi > \beta\delta > 1$. As far as leaning against the wind ($\phi < 0$) is concerned, checking all the expressions of steepness that are able to produce saddle paths that are sloped down from the left to the right (as the one in Figure 1) allows us to find out that the condition for overshooting is given by $\delta < 1/\beta$. Since the steepness of the $\dot{p} = 0$ locus does not interfere with the slope of the saddle path, policy parameter ϕ plays no role. Once $\delta < 1/\beta$ is the same condition holding for free float, one must conclude that leaning-against-the-wind intervention cannot prevent overshooting of the exchange rate. (In Section 4, a quantitative condition for overshooting in connection with foreign exchange intervention is derived.)

To make a counterpoint to the above results it is interesting to note that the model in this section still provides a case for foreign exchange intervention as its dynamic properties are explored. Without relying on saddle paths and overshooting, intervention can be here associated with the conditions required for the system to be stable. Indeed, taking the matrix format of the system given by (15) and (16), i.e.

$$\begin{bmatrix} \dot{s} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} \dfrac{\beta\delta - \phi}{\alpha\delta + \theta} & \dfrac{1 - \beta\delta}{\alpha\delta + \theta} \\ \dfrac{\pi(\alpha\phi + \beta\theta)}{\alpha\delta + \theta} & -\dfrac{\pi(\alpha + \beta\theta)}{\alpha\delta + \theta} \end{bmatrix} \begin{bmatrix} s \\ p \end{bmatrix} + \Delta, \tag{17}$$

where Δ stands for the other exogenous terms, a necessary and sufficient condition for stability is for the determinant of the coefficient matrix in (17) to be positive and its trace to be negative. This implies $\phi > 1$, and $\phi > \beta\delta - \beta\theta\pi - \alpha\pi$, so that there are leaning-into-the-wind interventions generating stability.

4 Quantitative analysis

4.1 Expectations formation

Employing a quantitative analysis in place of the qualitative analysis adopted so far, the result that unqualified foreign exchange intervention cannot remove overshooting is now shown to hold for an extension of the Dornbusch model with intervention which considers a more general expectations formation rule.

Starting with the model in Section 3, a more general expectations formation rule will comprise rational expectations as a particular case. (Although a different model is employed in this section, analysis draws on Gärtner, 1993, Chapter 2).

While maintaining the notion that goods and assets markets actually adjust at different speeds, the expectations formation rule used in the original Dornbusch presentation (1976, p. 1163) is now employed; thus, agents are assumed to make forecasts in the foreign exchange market using the following rule:

$$\dot{s}^e = \lambda(s^* - s), \tag{18}$$

where s^* gives the equilibrium value of the nominal exchange rate in natural log; and $\lambda \in (0, \infty)$ is the expected speed of adjustment of the nominal exchange rate toward its equilibrium value. Equation (18) states that the expected rate of depreciation of the nominal exchange rate is proportional to the discrepancy between the long-run rate and the current spot rate. As in Dornbusch (1976, p. 1163), the long-run equilibrium exchange rate s^* is assumed to be known.

While expectations formation according to (18) may appear ad hoc, it is consistent with perfect foresight (Dornbusch, 1976, p. 1163); actually, perfect foresight (given by equation (7)) holds as a special case to which the expected speed of adjustment λ equals the actual rate of convergence of the nominal exchange rate (and of prices as well, as shown by Dornbusch, 1976, p. 1165). Since agents are assumed to believe that following a shock the nominal exchange rate s will return asymptotically toward its equilibrium value s^* with the expected speed of adjustment λ, the restriction $\lambda > 0$ imposes stability (Dornbusch, 1976, p. 1167 n9), so that bubbles are not considered. Here expectations formation rule (18) will prove to be more suitable than (7).

4.2 The model

Taking equation (18) in place of (7) implies that (10) can no longer take part in the model here. Repetition of the same rounds of substitution adopted in Sections 2 and 3 yields a substitute for (10) as given by

$$s = \frac{1}{\lambda}(r^f - r) + s^*. \tag{19}$$

The resulting model is thus made up of equations (1), (8), (14), and (19) (Table 4).

Table 4. The Dornbusch model with foreign exchange intervention and a more general expectations formation rule for the exchange rate

$y = \beta(s + p^f - p) - \alpha r$	(1)
$\dot{p} = \pi(y - \overline{y})$	(8)
$\overline{m} = p + \delta y - \theta r - \phi(s - \overline{s})$	(14)
$s = \frac{1}{\lambda}(r^f - r) + s^*$	(19)

Insertion of (19) into (1), and substitution of (8) in the resulting expression yields

$$\dot{p} = \pi(\alpha\lambda + \beta)\, s - \beta\pi p + \beta\pi p^f - \pi\overline{y} - \alpha\pi r^f - \alpha\lambda\pi s^*. \tag{20}$$

Substitution of (19) in (14), and insertion of (8) into the obtained expression gives

$$\dot{p} = \frac{\pi(\phi - \theta\lambda)}{\delta} s - \frac{\pi}{\delta} p - \pi\overline{y} + \frac{\theta\pi}{\delta} r^f - \frac{\pi\phi}{\delta}\overline{s} + \frac{\theta\lambda\pi}{\delta} s^* + \frac{\pi}{\delta}\overline{m}. \tag{21}$$

The model is thus compacted into equations (20) and (21).

4.3 Steady state analysis

The goods market will be in equilibrium when

$$\dot{p} = 0, \tag{22}$$

as $y = \overline{y}$ in (8). Condition (22) can be thought of as an algebraic counterpart of the $\dot{p} = 0$ schedule in Sections 2 and 3. Plugging (22) into (20) and (21) gives respectively

$$p = \frac{\alpha\lambda + \beta}{\beta} s + p^f - \frac{1}{\beta}\overline{y} - \frac{\alpha}{\beta} r^f - \frac{\alpha\lambda}{\beta} s^* \tag{23}$$

and

$$p = (\phi - \theta\lambda)\, s - \delta\overline{y} + \theta r^f - \phi\overline{s} + \theta\lambda s^* + \overline{m}. \tag{24}$$

In full equilibrium $p = p^*$ and $s = s^*$ are implied (where p^* is the natural log of the domestic price level of steady state), so that (23) and (24) should be rewritten as

$$p^* = s^* + p^f - \frac{1}{\beta}\overline{y} - \frac{\alpha}{\beta}r^f \tag{25}$$

and

$$p^* = \phi s^* - \delta\overline{y} + \theta r^f - \phi\overline{s} + \overline{m}. \tag{26}$$

Thus, equations (25) and (26) describe the long-run setting of the model. To know how the nominal exchange rate behaves in the long run, they are equated to produce

$$s^* = -\frac{1}{1-\phi}p^f + \frac{1-\beta\delta}{\beta(1-\phi)}\overline{y} + \frac{\alpha+\beta\theta}{\beta(1-\phi)}r^f - \frac{\phi}{1-\phi}\overline{s} + \frac{1}{1-\phi}\overline{m}, \tag{27}$$

which implies

$$\frac{ds^*}{d\overline{m}} = \frac{1}{1-\phi}. \tag{28}$$

From (26) it can be seen that the ultimate effect of an increase in the money supply target is an equiproportionate rise in the domestic price level. From (28), however, that will also give rise to an equiproportionate increase in the nominal exchange rate only if the economy is under pure float ($\phi = 0$). In other words, PPP holds for free float only.

4.4 Overshooting

Whether the nominal exchange rate overshoots in response to a monetary shock can be appreciated in the above model focusing exclusively on equation (24), because p is constant in the impact period and, accordingly, $\dot{p} = 0$ (equation (22)). For given values of $\overline{y}$, r^f, and $\overline{s}$, the monetary impact on the nominal exchange rate in the short run can be found by differentiating (24) with respect to $\overline{m}$ to obtain

$$\frac{ds}{d\overline{m}} = \frac{\phi - (\theta\lambda + 1)}{\phi - \theta\lambda}\left(\frac{ds^*}{d\overline{m}}\right). \tag{29}$$

Equation (29) generalizes the one presented by Dornbusch (1976, p. 1169), which assumes $\phi = 0$. The general condition for overshooting to occur in this more general formulation is given by $\left|ds / d\overline{m}\right| > \left|ds^* / d\overline{m}\right|$. Considering (29) this implies $[\phi - (\theta\lambda + 1)]/(\phi - \theta\lambda) > 1$ in the model in this section. It is thus possible to investigate whether such a condition is met for different types of foreign exchange intervention.

Overshooting should occur with a floating exchange rate ($\phi = 0$), provided that the economy is not under the liquidity trap ($\theta \to \infty$) and/or the expected speed of convergence of the nominal exchange rate is not instantaneous ($\lambda \to \infty$); this gives the same condition presented by Dornbusch (1976, p. 1169). Moreover, the phenomenon should also be seen even if there is leaning-against-the-wind intervention ($\phi \in (0, -\infty)$). With leaning into the wind ($\phi \in (0, \infty)$), overshooting can also emerge if $\phi < \theta\lambda$. Thus, the result in Section 3 that overshooting may appear out of free float and cannot be removed by unqualified foreign exchange intervention is replicated here.

5 Conclusions

Does foreign exchange intervention remove overshooting? When a policy rule is embodied in standard models, this paper shows that overshooting is possible out of free float and cannot be removed by unqualified intervention.

Section 2 presents the well-known result of overshooting. In this treatment, the condition for overshooting to occur is the income elasticity of money demand to fall short of both the inverse of the real exchange-rate elasticity of the demand for exports ($\delta < 1/\beta$) and the ratio of the semi-interest elasticity of money demand to the semi-interest elasticity of investment demand ($\delta < \theta/\alpha$). It is made clear that the (nominal and real) exchange rate and the domestic (nominal and real) interest rate overshoot in tandem.

Extending the model in Section 2 to consider foreign exchange intervention, Section 3 illustrates that overshooting emerges with free float if $\delta < 1/\beta$, which is the same condition as the one in the model of Dornbusch (Table 2). Thus, the Dornbusch model becomes a special case to which $\phi = 0$. The foreign exchange intervention necessary to remove overshooting must be qualified in terms of the model parameters to be effective. Leaning-into-the-wind intervention cannot prevent overshooting if either $\phi < \beta\delta < 1$ or $\phi > \beta\delta > 1$. In turn, leaning-against-the-wind intervention turns out to be completely ineffective in removing overshooting.

When a more general expectations formation rule replaces rational expectations in the model with foreign exchange intervention, the results in Section 3 still hold. Overshooting may appear out of free float without it being possible to remove it by unqualified intervention. The phenomenon should thus emerge with both leaning-against-the-wind and leaning-into-the-wind interventions. In the latter case, the nominal exchange rate overshoots if $\phi < \theta\lambda$.

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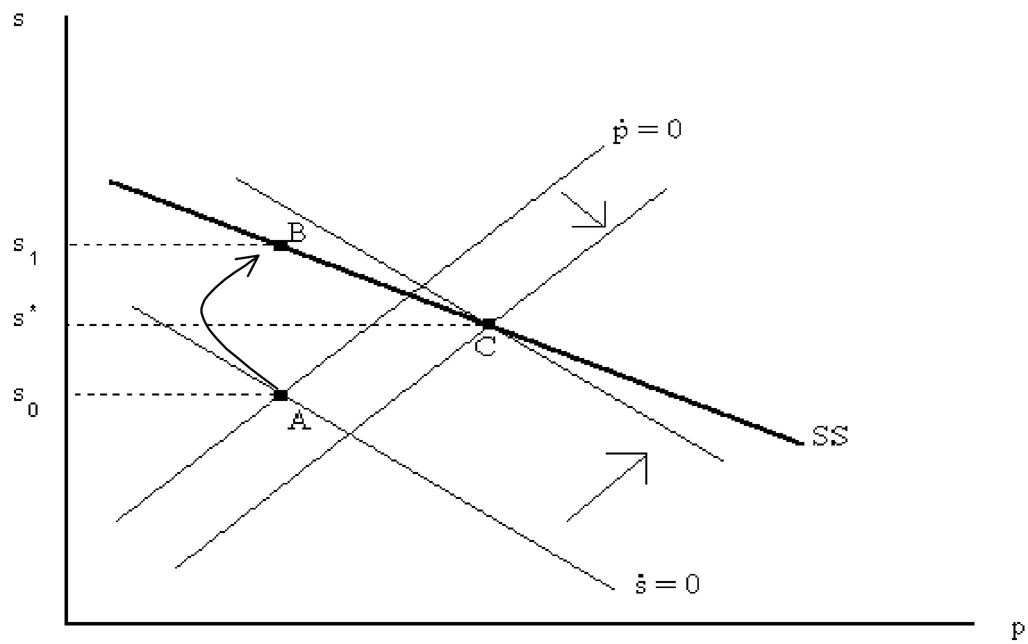
Figure 1



Overshooting of the exchange rate in the Dornbusch model ($\delta < 1/\beta$, and $\delta < \theta/\alpha$).

If the central bank pushes up the money supply m unexpectedly and once and for all, this shifts both the $\dot{s} = 0$ schedule upwardCbecause $ds/dm = 1/\beta\delta > 0$ in equation (11)Cand the $\dot{p} = 0$ locus downwardCbecause $ds/dm = -\alpha/\beta\theta < 0$ in (12).
The initial full equilibrium is represented by point A, whereas the final full equilibrium is given by point C.
The necessary condition for an exchange depreciation between points A and C to keep the purchasing power of domestic goods constant in terms of foreign goods is for point C to be above point A, which implies that the $\dot{s} = 0$ schedule should shift by more than the $\dot{p} = 0$ locus, and this requires $\delta < \theta/\alpha$.
The nominal exchange rate depreciates beyond the final full equilibrium because the economy jumps from point A to B instantly.