# Topics in Calculus:  Syllabus and Curriculum Map

Materials:   Teacher-created packets
Textbook reference:  Stewart, James. *Calculus.* Pacific Grove, Ca: Brooks/Cole, 2002.  (5th edition)
Technology:  All students will have TI-83 Graphing calculators assigned to them for the duration of the course

| Unit name/scope | Main ideas/Essential questions | Skills/Topics |
|---|---|---|
| Unit 1: Introduction to Calculus, Limits, and Derivatives | How are different branches of mathematics connected? What are the patterns in binomial expansion? What questions can't arithmetic answer? How does Calculus seek to answer them? What is a limit? How can we find limits given indeterminate forms? How does slope relate to average speed? How can we find the instantaneous speed or slope at a point? What patterns exist in the way slopes change for given functions? How can we express the relationship of the slope at a point to its x-value? | •Constructing and detecting regularities in Pascal's triangle<br>•Connecting Pascal's triangle to binomial expansion<br>•Connecting Pascal's triangle and binomial expansion to combinatorix<br>•Understanding the concept of a limit<br>•Finding a limit using a chart or the calculator's table function<br>•Finding a limit using a graph, including of those of piecemeal functions and functions with holes in them<br>•Finding a limit through algebraic manipulation (splitting up a fraction, factoring and cancelling, using the conjugate, dividing by the highest degree monomial...)<br>•Calculating average speed over intervals (graphically and from a table)<br>•Predicting instantaneous speed by using smaller and smaller intervals of average speed<br>•Representing instantaneous speed as a tangent to a graph at a point<br>•Relationship of speed to slope and thus the derivative<br>•The derivative as an instantaneous rate of change of other quantities<br>•Calculating the derivative using the limit definition<br>•Investigating patterns in the (numerical values of) derivatives of polynomials using tangent lines and local linear approximation<br>•Generalizing "The Power rule" for derivatives of polynomials<br>•Applying the power rule to roots and negative exponents |
| Unit 2: Higher derivatives and their applications | When do maxima and minima occur? What is the connection between extrema and slope? How can we apply the derivative to find maxima and minima? How are extrema useful in solving real-world problems? What is the second derivative? What does it tell us about graphs? How do the first and second derivatives relate to motion in the physical world? How do we find the derivatives of more complex functions? | •Finding extrema from graphs<br>•Locating extrema using derivatives<br>•Maximizing and minimizing real-world situations using the derivative<br>•Finding the second derivative and higher derivatives<br>•Describing curvature and points of inflection using the second derivative<br>•Applying first and second derivatives to physics equations of motion including distance, velocity, & acceleration<br>•Investigating the derivative of a function that is the product of two functions<br>•Applying the product rule & deriving/applying the quotient rule<br>•Investigating the derivative of a function that is the function of another function<br>•Applying the chain rule |

| Unit name/scope | Main ideas/Essential questions | Skills/Topics |
|---|---|---|
| Unit 3:  Special derivatives, implicit differentiation, and related rates | What is the derivative of sin x ?<br>What is the derivative of cos x ?<br>What are exponential and logarithmic functions and what are their derivatives?<br>How can we use special derivatives to find the derivatives of more complex functions?<br>How can we use the calculator to approximate derivatives?<br>How can we find derivatives for implicitly defined functions?<br>How can we use the rules of calculus to solve real-world related rate problems? | •Graphing the derivative of sin x by hand<br>•Graphing the derivative of cos x by hand<br>•Using the calculator to show the derivatives of trigonometric functions<br>•Investigating exponential functions<br>•Investigating logarithms<br>•Using the calculator to suggest the derivatives of exponential, power, inverse trigonometric and logarithmic functions<br>•Using the product rule and chain rule to find the derivatives of more complex functions by hand<br>•Using the limit definition of a derivative to prove the derivatives of exponential and logarithmic functions<br>•Using the product rule and chain rule to take the derivative of implicitly defined functions and functions where $x = f(y)$<br>•Isolating y, when possible, to show that implicit and explicit differentiation yield the same result<br>•Using implicit differentiation to show the derivatives of inverse trigonometric functions<br>•Expressing word problems as equations involving derivatives<br>•Solving real-world related rate problems |
| Unit 4: Introduction to integration | What is the anti-derivative of a function and how can we calculate it?<br>What is the meaning of an area function that gives the area of a curve from 0 to x?<br>What patterns emerge in the area function of a horizontal line?  In that of a line of slope m?<br>What does the area under a velocity-time curve represent?<br>What patterns emerge in the area function of a parabola?<br>What happens when we take the derivative of the area function?<br>What is the relationship between the antiderivative and the integral?<br>What are some ways to find integrals of complex functions? | •Finding f given f' by inspection<br>•Finding areas under linear curves by hand<br>•Writing equations to describe the area function of a linear curve<br>•Using calculus to find the area of a triangle<br>•Using area to find displacement given a speed-time curve<br>•Approximating the area under a parabola or other polynomial function using Riemann sums<br>•Graphing the area function on the same axes as the original function/ Comparing the graphs of functions and their antiderivatives<br>•Using fnINT on the calculator to suggest the relationship between area and antiderivative<br>•Proving the relationship between the area under a curve and its antiderivative (Fundamental Theorem of Calculus)<br>•Exploring "negative" area (or accumulated negative change)<br>•Manipulating constant terms in definite integrals<br>•Other properties of integrals<br>•Using substitution to solve integrals<br>•Recognizing special integrals like 1/x and those yielding trigonometric functions<br>•Functions with no analytically- found antiderivative function<br>•Using the calculator to evaluate definite integrals |

| Unit name/scope | Main ideas/Essential questions | Skills/Topics |
|---|---|---|
| Unit 5:<br>Applications of integration & Differential equations | How is the integral used in a real-world context?<br>What problems does integration allow us to solve?<br>How does the integral relate to physics?<br>How can integration find a volume defined by a function or functions?<br>How can we visualize the family of functions represented by a differential equation?<br>How can we find a solution function given a differential equation? | •Applying integration to speed, velocity, and acceleration (summing distance x time compared to final position - initial position)<br>•The integral as the total given a rate of change (and thus the difference in end states)<br>•Average value, especially average speed, as related to integration<br>•Volumes created by rotation<br>•Constructing a table of values and then a slope field from a differential equation<br>•Predicting the solution curve from a slope field<br>•Solving separable differential equations |
| Unit 6:  Review and Enrichment | How do we explain mathematics?<br>How do we summarize and synthesize what we have done all year?<br>How do we express mathematics symbolically, visually, orally, and in writing?<br>What constitutes good notation?<br>What are the implications of good notation?<br>How can we "mathematize" a problem that is not necessarily framed mathematically? | •Creating a "dictionary" of calculus vocabulary and symbols<br>•Creating review materials for each unit covered<br><br>Culminating Project 1 (ongoing through the final unit):<br>Each student in the class will pick a topic we have covered in the previous units and teach a review lesson about it to the class.  Student presentations will include a written lesson plan (to turn in), an oral presentation, a mini-quiz for the end of class, and an answer key to that quiz with detailed explanations, in complete sentences, of each problem solved.<br>Culminating Project 2- The Sudoku Challenge:<br>Students work in small groups to develop their own system of notation for describing, solving, and writing theorems about the Sudoku puzzle.  Students get the opportunity to see how difficult it can be to come up with a concise, clear, intuitive, and suggestive way of symbolizing a problem.  Students also get to do their own "mathematical work", which is presented to the teacher as a written project for a grade and to their classmates as oral presentations. |