A. Title Integers as Infinite Sequences.
B. Subject Area Mathematics
C. Task Level Calculus
D. Key Cognitive Strategies (Habits of Mind) Assessed
  Problem solving
  Reasoning
  Precision

E. Task Narrative
Consider this infinite expression:

$$\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+...\sqrt{a+\sqrt{a+...}}}}}}}$$

For specific values of a the limit of the expression, as the number of iterations approaches infinity, is equal to a natural number. The goal of this task is to first rewrite the expression above in order to manipulate it mathematically, find values of a for which the limit of the infinite expression equals a *natural number* (which is a counting number like 1, 2, 3, 4,…), and then try to determine if all natural numbers can be written this way.

A. Exploration:
  1) The sequence $\{r_n\}$ is defined recursively as follows.

  $r_1 = \sqrt{2} \quad r_{n+1} = \sqrt{2+r_n}$

  a. Carefully write the first four terms of the sequence $\{r_n\}$ and use a calculator to approximate the first 10 terms of the sequence.

  b. Make a conjecture for $\lim_{n \to \infty} r_n$.

  2) Repeat the steps of part 1 above for each of the following sequences.

  a. $s_1 = 6 \quad s_{n+1} = \sqrt{6+s_n}$

  b. $t_1 = 12 \quad t_{n+1} = \sqrt{12+t_n}$

B. Generalization:
  1) Investigate other numbers besides 6 and 12 to see if they satisfy the above statement. What are the next two numbers that will work (give you natural number limits)?

  2) Based on your investigation, do you think any non-negative integer can be written as the limit of an infinite sequence of the type defined? Explain your reasoning.

  3) Find a generalization or formula that can be used to generate a list of natural numbers that can be written as the limit of an infinite expression like the one above. Hint: List the numbers that **can** be written in this way and look for a pattern.

4) Derive your generalization from (3) using algebra and your understanding of limits. Hint: Since we are talking about limits, what happens to the difference between t_n and t_{n+1} as $n \to \infty$?

Key Cognitive Strategies (Habits of Mind)
Each of the parts below outlines a Key Cognitive Strategy (Habit of Mind) you will use to reach a solution. The prompts that follow are designed to lead you through the processes necessary to reach a solution and effectively communicate your results.

Part I: Problem Solving
    A. Do you understand the questions being asked? Restate them in your own words.
    B. What do you think your answer will be? Which numbers do you think will produce a natural number?
    C. How will you approach the problem? How will you start?

Part II: Reasoning
    A. Describe how you developed your generalizations. What system did you use to find counter-examples? Were you able to validate your conjectures?
    B. Is your work organized in a logical order? Were any charts or visual aids useful to you and are they included? Did you identify your final conclusions?
    C. Reflect on your answers, then re-examine the questions to make sure you answered them all correctly. For your final draft, include a reflection on your strategies and their effectiveness. Did you respond to feedback from your teacher and/or your class discussions? Did you change your strategy after feedback and/or discussions? Are you convinced that your chosen methods were the most efficient and useful possible? Explain how you know.

Part III: Precision (This Habit of Mind is assessed primarily on the final draft.)
    A. Check all of your calculations and proofs to make sure they are clear and correct. What methods did you use to check for possible errors? Are all solutions precise? Are the visuals you included helpful, error free, and appropriately located in the work?
    B. Are all components of the task included? Did you follow all directions and use all prompts? Did you thoroughly edit your work, removing extra information that does not enhance your final results?
    C. Are all terms, notations, and symbols used properly and appropriately for this level of math? Are all visuals relevant to the conclusion? Is your work clear and legible? (An unreadable paper will not be graded.)

Work Products
First Draft:
The first draft of your findings and results **will be due on**_____ and must include information and responses to parts A and B and to the prompts for Parts I, II and III as outlined above. Your work must be legible, though not in its final form.

Final Draft:

The final draft of your findings and results **will be due on**_____. Review your report and make improvements based on feedback from your teacher. The final report will have the following sections in addition to what was in your rough draft:

- A section at the end of your paper should critique the strategy you used in your rough draft to solve the problem in light of the other strategies that were discussed in class. You may change the strategy you use to solve the problem from your rough draft to your final draft, if you believe another strategy is better or more efficient.
- Regardless of whether you change your strategy, you should follow your strategy critique with a written reflection justifying the strategy you used in your final report. This reflection should include clear details about why your chosen strategy is the most efficient and best way of solving the problem.

The final report should be written clearly and concisely using language, symbols, and terms appropriate to this level of math. It should be a synthesis of the information gathered for the first draft, presented in a complete, clear, and organized way. You will be assessed on each of the three Key Cognitive Strategies (Habits of Mind) described above: Problem Solving/Strategizing, Reasoning, and Precision. Make sure your final draft communicates how each of these habits was used in your work. Attach your first draft to the final draft before submitting it.