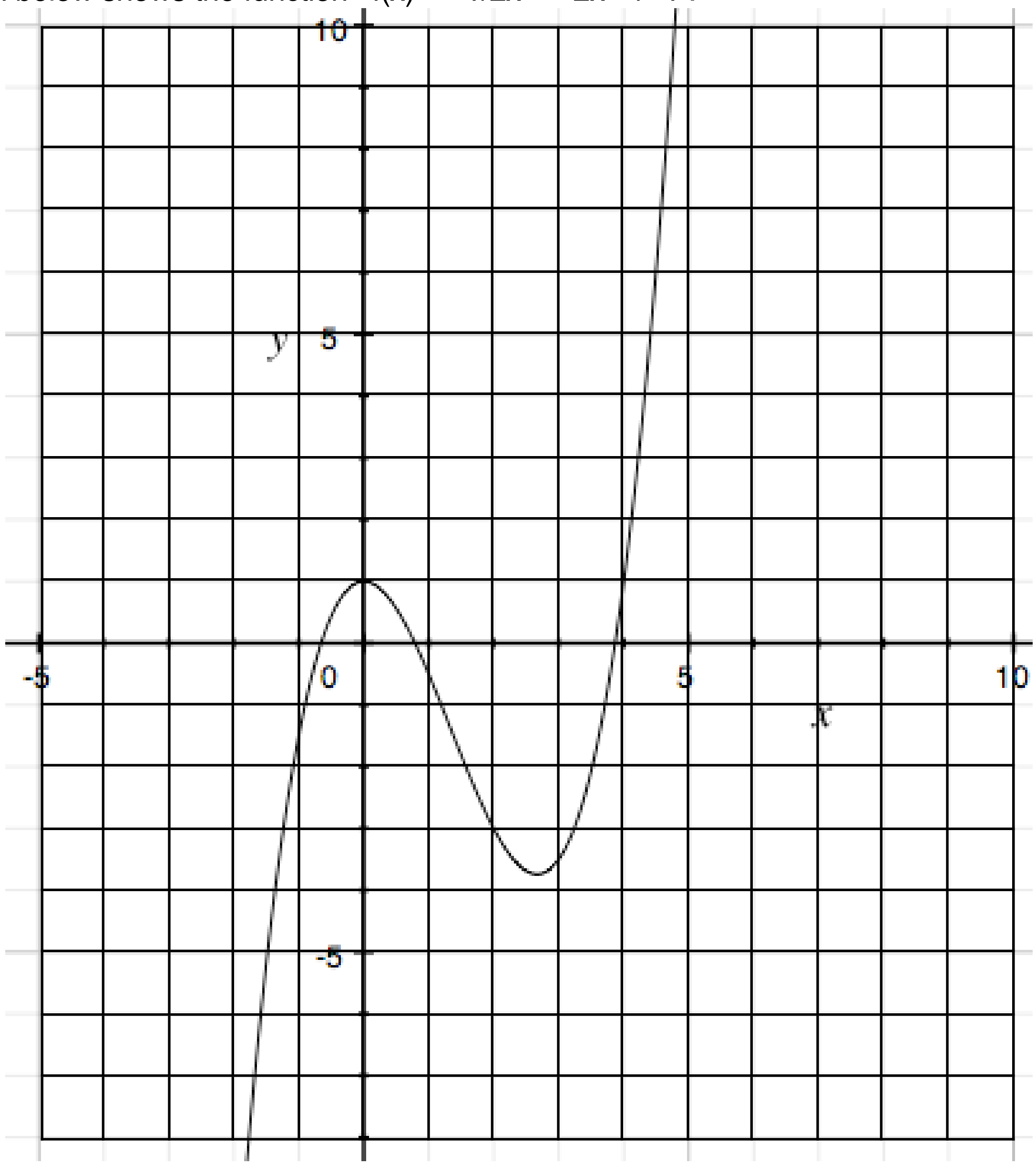
Name: ___________________________

UNIT 2 EXAM

Show ALL of your work for full credit

1. The graph below shows the function $f(x) = 1/2x^3 - 2x^2 + 1$.



a. Why must we talk about slope for this graph in terms of a single point instead of 2 points?

b. The slope formula has 2 points, and yet we want to talk about slope at a single point. How do we get around this problem?

c. Let y represent position in meters and x represent time in seconds. Find the average speed over each interval.

i. 3 s to 3.5 s   ii. 3 s to 3.1 s   iii. 3 s to 3.01 s   iv. 3 s to 3.001 s

d. Based on your answers to (c), what do you predict for the **instantaneous** speed at 3 seconds?

e. What is the relationship between speed and slope? Explain.

f. Draw a representation of the instantaneous speed at $t = 3$ on the graph (on page 1). What is its slope?

g. Use the **derivative** of the function to find the exact answer for instantaneous speed at 3 seconds. Show all of your work.

h. When does this graph have a slope of zero? Use calculus to find the exact answer(s).

i. Why are your answer(s) to (h) reasonable based on the graph? (What is happening in the graph at the points you've identified?)

3. a. Use the calculator to approximate the derivative of $y = (\log x)(\sin x)$ at $x = 7$. Show all your work.

b. Set up, **but do not solve**, a limit that would give you the answer to (a) “the long way”.

4. a. What is the derivative of $f(x) = x^3 + x$?

b. Prove that your answer is correct using the **definition of the derivative (the limit of.../"the long way")**

c. When does f(x) have a slope of 49?

d. What is the slope of f(x) at $x = -2$?

5. Find the derivative of each function.

a. $y = x^{10}$

b. $y = 3x^5 - 2x^2 + 5$

c. $y = \sqrt[4]{x}$

d. $y = \frac{1}{x^4}$

e. $y = 4x^{2/5}$

f. $y = \frac{3}{\sqrt[5]{x}} + 4x - \frac{x^2}{2}$

g. $y = 8\sqrt{x} \cdot x$

6. Prove using the limit definition that the derivative of (c) in the previous problem actually fits the rule. (Hint: Use the conjugate twice)

7. Explain why the derivative of a constant term (a number without a variable) is always 0.

8. a. Find the derivative of $y = mx + b$ using the limit definition (the long way).

b. Explain why you should know the answer to this **without** even doing calculus (rule or definition).

9. Find the tangent line to the curve $y = 1/x$ at $x = 2$.

10. A line goes through the point $(0, -9)$. It is tangent to the curve $y = x^2$. Where does the line hit the curve? (hint: draw a picture)

11. A parabola has a slope of -2 at $x = 2$ and a slope of 3 at $x = 12$. It goes through the point $(0,6)$. Find the equation of the parabola.