**Name: ANSWER KEY**

## Problem Set 2
## SHOW ALL WORK FOR CREDIT

You may submit solutions on a separate sheet of paper if you prefer.

Assigned: 10/23/07 Due: 11/2/07 (2 pts off for each day late)

1. Given $f(x) = x^3 - x^2$ , find the value of each expression below. **Simplify as much as possible.**

a) f(4)

**$4^3 - 4^2 = 64 - 16 = 48$**

b) f(h)

**$h^3 - h^2$**

c) f(x + 1)

**$(x^3 + 3x^2 + 3x + 1) - (x^2 + 2x + 1) =$**
**$x^3 + 3x^2 + 3x + 1 - x^2 - 2x - 1 =$**
**$x^3 + 2x^2 + x$**

d) f(x + h)

**$(x^3 + 3x^2h + 3xh^2 + h^3) - (x^2 + 2xh + h^2) =$**
**$x^3 + 3x^2h + 3xh^2 + h^3 - x^2 + 2xh + h^2$**

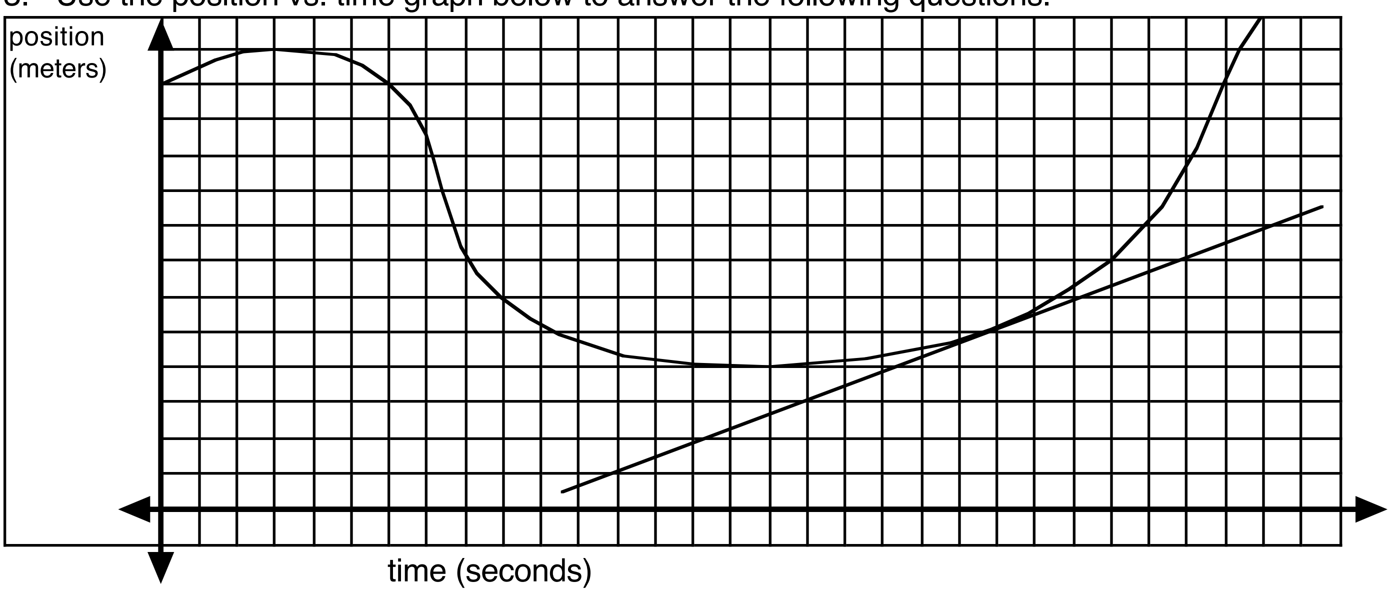
e) f(a + b + c)

***$(a + b + c)(a + b + c) =$***
**$a^2 + ab + ac + ba + b^2 + bc + ca + cb + c^2$**
**$= a^2 + 2ab + 2ac + b^2 + 2bc + c^2$**
***$(a + b + c)(a + b + c)(a + b + c)$*** **$= (a + b + c)(a^2 + 2ab + 2ac + b^2 + 2bc + c^2) =$**
**$a^3 + 2a^2b + 2a^2c + ab^2 + 2abc + ac^2 + ba^2 + 2ab^2 + 2abc + b^3 + 2b^2c + bc^2 + ca^2 + 2abc + 2ac^2 + cb^2 + 2bc^2 + c^3$**
**$= a^3 + b^3 + c^3 + 6abc + 3a^2b + 3a^2c + 3ab^2 + 3ac^2 + 3b^2c + 3bc^2$**
**$a^3 + b^3 + c^3 + 6abc + 3a^2b + 3a^2c + 3ab^2 + 3ac^2 + 3b^2c + 3bc^2 - a^2 + 2ab + 2ac + b^2 + 2bc + c^2$**

2. Explain the relationship between slope and average speed. Use mathematical notation in your answer.

**Slope is the same thing as average speed if we graph distance vs. time, because we graph distance on the y-axis and time, the independent variable, on the x-axis. Then $\Delta y/\Delta x = \Delta d/\Delta x$**

3. Use the position vs. time graph below to answer the following questions.




a). Calculate the average speed of the object over each interval.

i) 1 s to 3 s
**from 12.5 m to 13 m**
**=> .5 m/ 2 s = .25 m/s**

ii) 3 s to 8 s
**from 13 m to 7 m**
**=> -6 m/5 s = -1.2 m/s**

iii) 8 s to 14 s
**from 7 m to 4 m**
**=> -3 m/6 s = - .5 m/s**

b) Why can't you calculate an instantaneous speed at a single point (instant) using the usual formula of $\Delta d/\Delta t$ ? **Because at a single point, the $\Delta t = 0$ and the $\Delta d = 0$ also because the object has no time to move, so we get 0/0 which is an indeterminate form.**

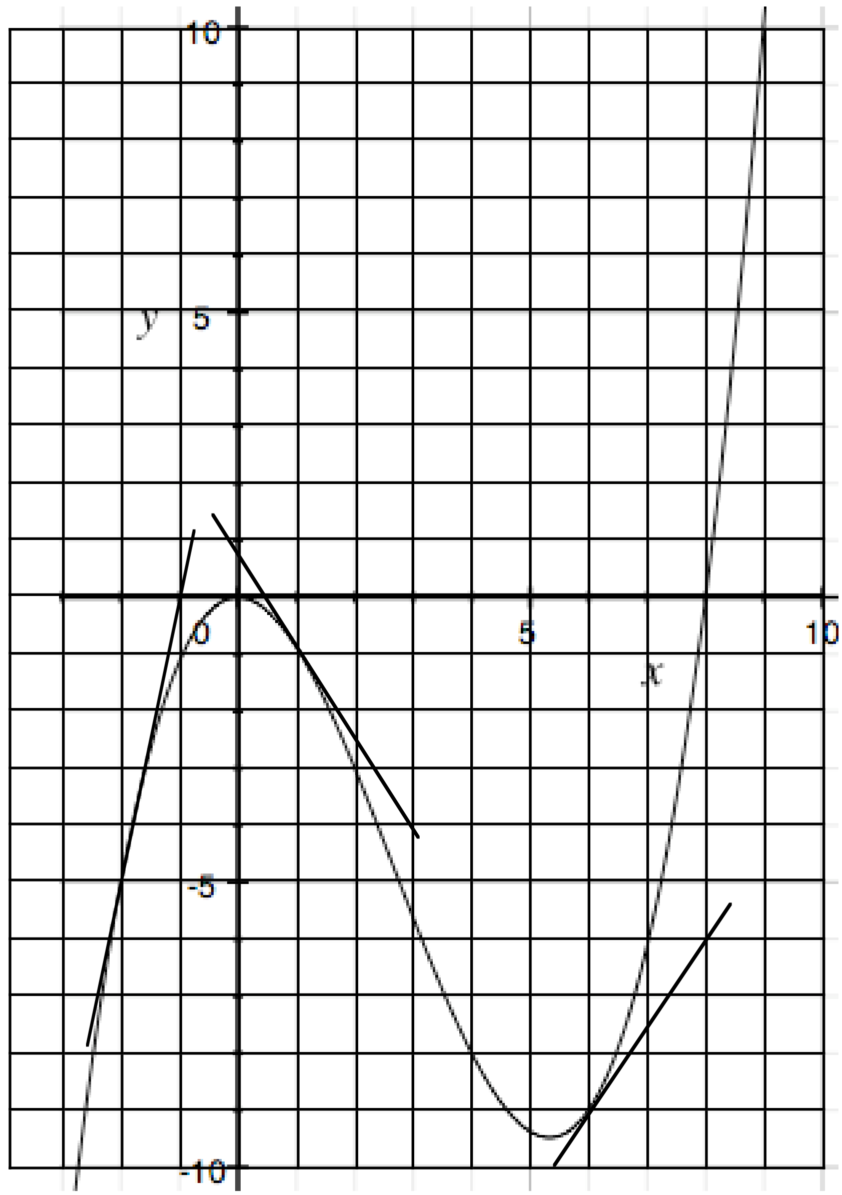
c) Draw a representation of the instantaneous speed of the object at 22 seconds on the graph. What is that speed? **Based on my tangent, it is about 2/5 = .4 m/s**

d) When did the object stop moving?
**At approximately t = 3 seconds and t = 16 seconds.**

e) How do you know the answer to letter (d) by looking at the slope of the graph?
**We look for when the slope, also known as the speed, is 0, or when the graph is flat, because when the speed is 0 the object is stopped for an instant at least.**

4. The graph to the right shows the function

$f(x) = 1/8x^3 - x^2$




a) Use the graph to approximate the slope at each of the following points. Be sure to actually find the slope (write the numerical answer).

i. x = 1
**-3/2 based on my tangent line**

ii. x = 6
**3/2 based on my tangent line**

iii. x = -2
**5 based on my tangent line**

b) Why must we discuss the slope at only one point for a graph like this?
**Because the slope is constantly changing.**

c) What is the formula for slope?
**Δy/Δx**

d) What happens to the slope formula if we consider only one point?
**We get 0/0, an indeterminate form, because there is no Δy or Δx.**

e) How can we get around this problem using a table?
**By taking very very small changes in x.**

f) Write your answer to (e) as a limit.

$$\lim_{\Delta x \to 0} \Delta y/\Delta x$$

g) Fill in the table below to approximate the instantaneous slopes at $x = 1$, $x = 6$, and $x = -2$.
For $x = -2$, fill in your own values to find the slope at exactly $x = -2$.

| First point | | Second point | | Δx "h" | Δ f(x) | slope |
|---|---|---|---|---|---|---|
| 5 | **-9.375** | 5.1 | **-9.428625** | **.1** | **-.053625** | **-.5363** |
| **5** | **-9.375** | 5.05 | **-9.404046875** | **.05** | **-.029046875** | **-.5809** |
| **5** | **-9.375** | 5.01 | **-9.381162375** | **.01** | **-.006162375** | **-.6162** |
| **5** | **-9.375** | 5.001 | **-9.3756241249** | **.001** | **-.000624125** | **-.6241** |
| 6 | **-9** | **7** | **-6.125** | 1 | **2.875** | **2.875** |
| **6** | **-9** | **6.5** | **-7.921875** | .5 | **1.078125** | **2.15625** |
| **6** | **-9** | **6.1** | **-8.837375** | .1 | **.162625** | **1.62625** |
| **6** | **-9** | **6.01** | **-8.984874875** | .01 | **.015125** | **1.5125** |
| -2 | **-5** | **e.g. -3** | **-12.375** | **-1** | **-7.375** | **7.375** |
| **-2** | **-5** | **-2.5** | **-8.203125** | **-.5** | **-3.203125** | **6.40625** |
| **-2** | **-5** | **-2.1** | **-5.567625** | **-.1** | **-.567625** | **5.67625** |
| **-2** | **-5** | **-2.01** | **-5.055175125** | **-.01** | **-.055175** | **5.5175** |

h) Based on your table, predict the actual instantaneous slope at:

i. $x = 5$
**about -.62**

ii. $x = 6$
**about 1.5**

iii. $x = -2$
**about 5.5**

i) Write a limit using **only** x, f(x), and h that will give you the exact slope at a point **x.**

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

j) Plug in to your answer for (i) for $f(x) = 1/8x^3 - x^2$ and $x = 5$ **and solve for the limit** to show that the slope you calculated for $x = 5$ is correct.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{1/8(5+h)^3 - (5+h)^2 - (1/8(5)^3 - 5^2)}{h} =$$

$$\lim_{h \to 0} \frac{1/8(5^3 + 3(5^2)h + 3(5)h^2 + h^3) - (5^2 + 10h + h^2) - (-9.375)}{h} =$$

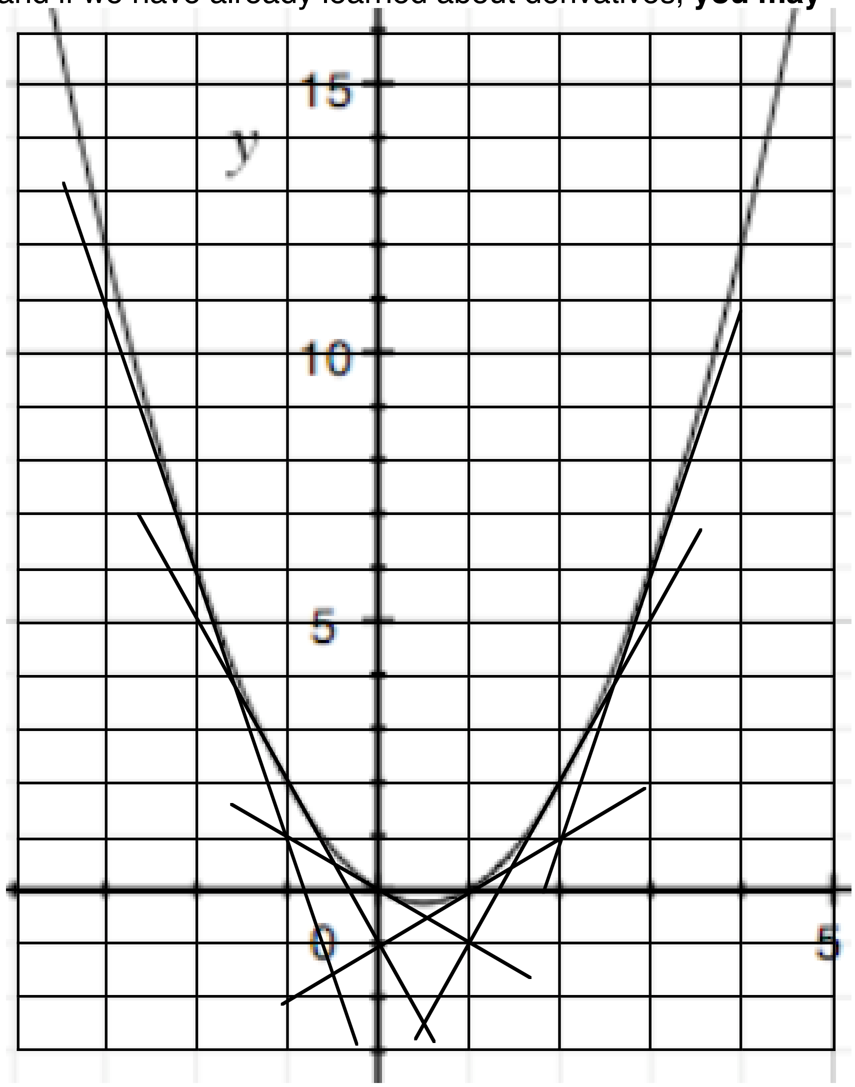
$$\lim_{h \to 0} \frac{1/8(125 + 75h + 15h^2 + h^3) - 25 - 10h - h^2 + 9.375}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{15.625} + 9.375h + 1.875h^2 + .125h^3 - \cancel{25} - 10h - h^2 + \cancel{9.375}}{h} =$$

$$\lim_{h \to 0} \frac{-0.625h + 0.875h^2 + .125h^3}{h} = -0.625 + 0.875h + .125h^2 = -0.625 \text{ as h goes to 0}$$

5. Use whatever techniques you would like to fill in the chart below as accurately as possible. However, you must **SHOW ALL OF YOUR WORK**, and if we have already learned about derivatives, **you may not** use the derivative.

$f(x) = x^2 - x$




a) CHART

| x = ? | Slope of the graph at that x value |
|---|---|
| -2 | **-5** |
| -1 | **-3** |
| 0 | **-1** |
| 1 | **1** |
| 2 | **3** |
| 3 | **5** |

WORK: (or attach an extra piece of paper)

**The two main techniques you could have used would be tangent lines or a slope formula with a small change in x. Both are shown.**

| **Points** | **Slope (about)** |
|---|---|
| **(-2, 6) & (-1.999, 5.995...)** | **- 5** |
| **(-1, 2) & (-.999, 1.997...)** | **- 3** |
| **(0, 0) & (.001, -.001...)** | **-1** |
| **(1, 0) & (1.001, .001...)** | **1** |
| **(2, 2) & (2.001, 2.003...)** | **3** |
| **(3, 6) & (3.001, 6.005...)** | **5** |

b) Find a formula that will give you the slope at any x-value.

**Slope = 2x - 1 (the slopes are going up by 2 every time x goes up by 1, & at 0 the slope is -1)**

c) Write a limit for the slope at a point and use it to show your formula in (b) is correct.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h} = \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} =$$

$$\lim_{h \to 0} \frac{2xh + h^2 - h}{h} = 2x + h - 1 = 2x - 1$$ **as h goes to 0.**