Name: ANSWER KEY

Problem Set 1
SHOW ALL WORK FOR CREDIT
You may submit solutions on a separate sheet of paper if you prefer.

Assigned: 9/11/07 Due: 9/25/07 (2 pts off for each day late)

1. a) Expand (a + b)(c + d)(e + f)(g + h)

= (ac + ad + bc + bd) (eg + eh + fg + fh)
= aceg + aceh + acfg + acfh + adeg + adeh + adfg + adfh +
bceg + bceh + bcfg + bcfh + bdeg + bdeh + bdfg + bdfh

Or, use the fact that you will choose one letter from each parentheses and generate all 16 unique combinations.

b) Expand $(x + 1)^{11}$

$$x^{11} + 11x^{10} + 55x^9 + 165x^8 + 330x^7 + 462x^6 + 462x^5 + 330x^4 + 165x^3 + 59x^2 + 11x + 1$$

c) Expand $(2x + 2)^9$

$$= [\,(2)(x + 1)\,]^9 = (2)^9(x + 1)^9 = 2^9\,(x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1)$$
$$= 512\,(x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1)$$
$$= 512x^9 + 4608x^8 + 18432x^7 + 43008x^6 + 64512x^5 + 64512x^4 + 43008x^3 + 18432x^2 + 4608x + 512$$

2. Pascal's triangle is partly based on the principle that one row of combinations generates another level of combinations when you add them. In other words, it is based on the principle that ${}_5C_2 + {}_5C_3 = {}_6C_3$, ${}_6C_4 + {}_6C_5 = {}_7C_5$, ${}_{10}C_4 + {}_{10}C_5 = {}_{11}C_5$, etc.

a) Generalize this addition of combinations rule using variables to represent the numbers. Hint: you should only need two variables.

$${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$$

b) Prove that this rule works (no matter what numbers are substituted for the variables), by using the formula for "choose". Hint: ${}_nC_r = \frac{n!}{r!\,(n-r)!}$

We have to use the principle that since 4! = 4·3·2·1 and 3! = 3·2·1, 4! = 4·3!, or in general, n! = n(n-1)!

$${}_nC_r + {}_nC_{r+1} \overset{?}{=} {}_{n+1}C_{r+1}$$

$$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(r+1)!(n-(r+1))!} = \frac{(n+1)!}{(r+1)!\,((n+1)-(r+1))!}$$ *Let's simplify a bit*

$$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!} = \frac{(n+1)!}{(r+1)!\,(n+1-r-1)!}$$ *Break it down with that principle...* **<------ just = (n - r)**

$$\frac{n!}{r!\,(n-r)(n-r-1)!} + \frac{n!}{(r+1)r!\,(n-r-1)!} = \frac{(n+1)!}{(r+1)!\,(n-r)!}$$ *Get a common denominator of* **(r + 1)r!(n - r)(n - r - 1)!**

$$\frac{n!}{r!\,(n-r)(n-r-1)!}\frac{(r+1)}{(r+1)} + \frac{n!}{(r+1)r!\,(n-r-1)!}\frac{(n-r)}{(n-r)} = \frac{(n+1)!}{(r+1)!\,(n-r)!}$$ *Combine the fractions*

$$\frac{n!\,(r+1) + n!\,(n-r)}{(r+1)r!\,(n-r)(n-r-1)!}$$ *= Factor n! on the top. Simplify the bottom.* $$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!} = \frac{n!(n+1)}{(r+1)!(n-r)!} = \frac{(n+1)!}{(r+1)!\,(n-r)!}$$

Q.E.D. (that means we proved it, the proof is done)

3. Explain what is wrong with the following solution. (If you propose an alternate solution, explain why yours is necessary.)

$$x^2 + x = 0$$
$$\quad - x \quad - x$$
$$x^2 = -x$$
$$\frac{x^2}{x} = \frac{-x}{x}$$
$$x = -1$$

We are ignoring the fact that x might = 0 when we divide both sides by x. Hence, nothing illegal is done, but we miss a second answer. **$x = -1$ OR $x = 0$ (clearly** $0^2 + 0 = 0$**).**

To find both roots we should have factored, $x(x + 1) = 0$

$x = 0$ OR $x = -1$

4. Find the limits of the following functions. (Calculator solution: 2 pts, non-Calculator solution: 3 pts)

a) $\lim_{x \to 0} \frac{3x}{x^3} = \frac{3 \cdot x}{x^2 \cdot x} = \frac{3}{x^2}$

Since x^2 is always positive, $3/{+0} \Rightarrow \infty$

b) $\lim_{x \to \infty} \frac{3x}{x^3} = \frac{3 \cdot x}{x^2 \cdot x} = \frac{3}{x^2}$

$\Rightarrow 3/\infty \Rightarrow 0$

c) $\lim_{x \to 3} \frac{x^2 + 6x - 7}{x - 1}$

Plug in. $\frac{9 + 18 - 7}{2} = 20/2 = 10$

d) $\lim_{x \to 1} \frac{x^2 + 6x - 7}{x - 1}$

$= \frac{(x + 7)(x - 1)}{(x - 1)} \Rightarrow 1 + 7 \Rightarrow 8$

e) $\lim_{x \to 4} \frac{\sqrt{x} - 2(\sqrt{x} + 2)}{x - 4(\sqrt{x} + 2)}$

Use the conjugate or factor.

$= \frac{(\sqrt{x})^2 - 2^2}{(x - 4)(\sqrt{x} + 2)} = \frac{x - 4}{(x - 4)(\sqrt{x} + 2)}$

$\Rightarrow 1/(\sqrt{x} + 2) = 1/(\sqrt{4} + 2)$

$= 1/(2 + 2) = 1/4$

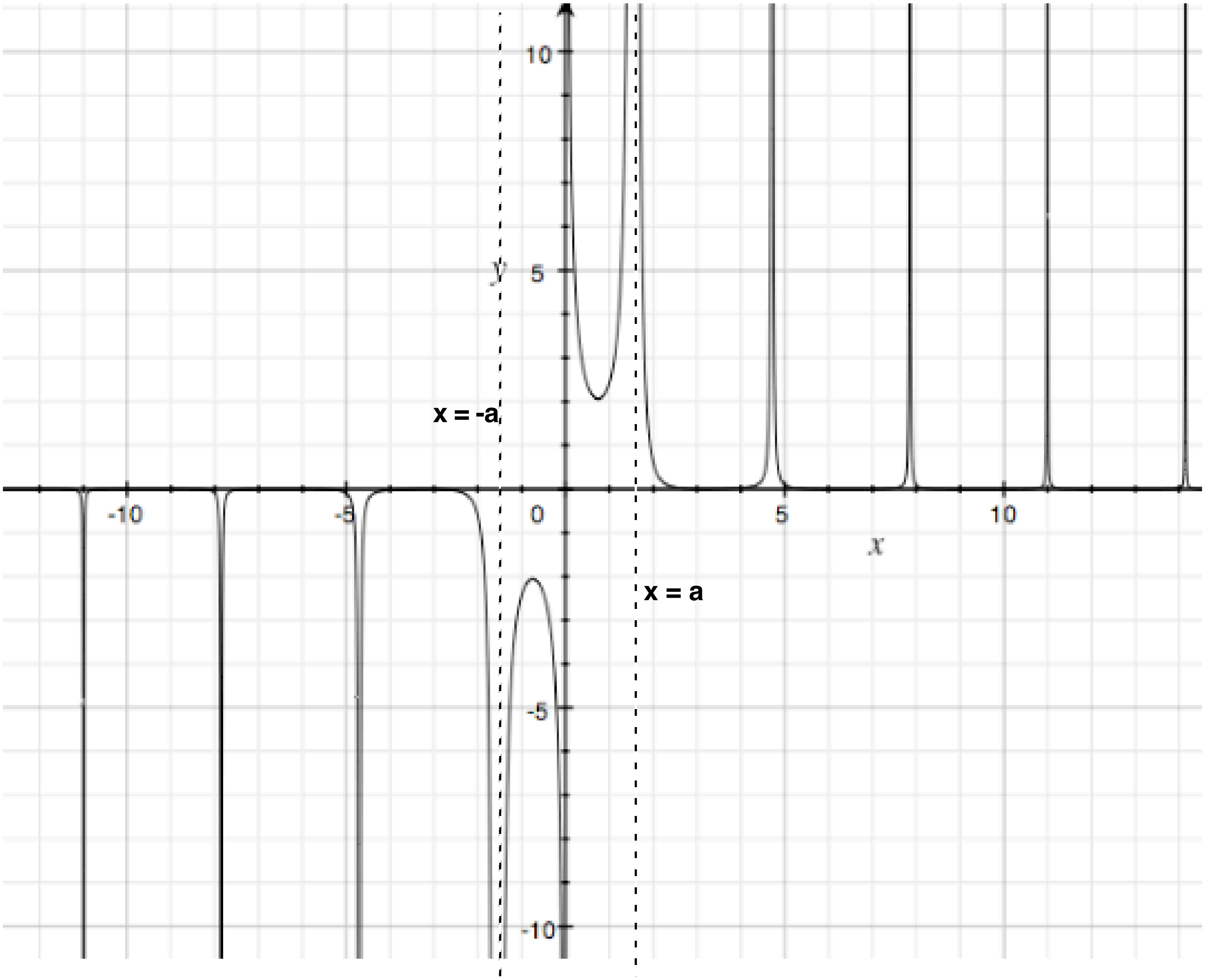
f) $\lim_{x \to -1} \frac{x^6 + 5x^5 + 10x^4 + 10x^3 + 5x^2 + x}{x + 1}$

The top is $(x + 1)^5$ {Pascal's triangle}

$\Rightarrow \frac{(x + 1)^5}{x + 1} \Rightarrow (x + 1)^4$

$\Rightarrow (-1 + 1)^4 \Rightarrow 0$

5. Find the limits of the function requested using the graph of f(x) below, which has asymptotes at **x = a** and **x = -a.**



a) $\lim_{x \to -a} f(x)$

- ∞

b) $\lim_{x \to a} f(x)$

+∞

c) $\lim_{x \to 0^+} f(x)$

+∞

d) $\lim_{x \to 0^-} f(x)$

-∞

e) $\lim_{x \to 0} f(x)$

D.N.E.

f) $\lim_{x \to \infty} f(x)$

D.N.E.

Hint: this function involves a trigonometric function.

g) Explain your answer to (f) in words.

The function appears to be periodic, meaning it will never stop going up asymptotically and then approaching 0 and then going up again. Accordingly, there is no "wall" or "stopping point" the function is hitting as x => ∞. The function value is unpredictable for a number that we only know to be a huge number.