Name: _____

Problem Set 5
SHOW ALL WORK FOR CREDIT
You may submit solutions on a separate sheet of paper if you prefer.

Assigned: 4/1/08 Due: 4/11/08 (2 pts off for each day late)

Part I. Special derivatives (check your classworks for help!)

1. Given the function $y = \arctan x$ ($y = \tan^{-1} x$ on your calculator).

a) Find the average slope over each interval. All degree measures should be in radians.

i. $x = 1$ and $x = 1.001$ ii. $x = 2$ and $x = 2.001$ iii. $x = 3$ and $x = 3.001$

b) Find the exact slope at each point.

i. $x = 1$ ii. $x = 2$ iii. $x = 3$

2. Find the derivative of each function.

a) $y = \arctan (3x+ 1)$ b) $y = \arcsin (e^x)$ c) $y = \arccos (\sin x)$

3. a) Show **using the chain rule** that the derivative of the function $y = \arcsin (\sin x)$ is 1.

b) Explain why this derivative must be correct by simplifying the function.

Part II. Implicit differentiation

4. Given the equation $y^5 = x$

a) Find the derivative of this function by solving for y.

b) Fill in the following chart showing the slope at each point of the equation.

x value	y value	SLOPE (as a fraction)	x value	y value	SLOPE (as a fraction)
(1	, 1)		(1024	, 4)	
(32	, 2)		(3125	, 5)	
(243	, 3)		(7776	, 6)	

c) Find the derivative with respect to **y** by using implicit differentiation.

d) Relate your answer in letter (c) to the chart in (b). (Show that c and b give the same answers.)

5. For each equation:
a) Find the derivative **normally** by first solving for y
b) Find the derivative implicitly in terms of y and x
c) Find a point that is on the graph of the equation
d) Find the slope at that point using both derivatives

Equation	Explicit (normal) derivative	Implicit derivative
$y^2 + 3 = x$		
Point on the graph:	Slope using normal derivative:	Slope using implicit derivative:

Equation	Explicit (normal) derivative	Implicit derivative
$y^2 + 6y = x$	hint: complete the square	
Point on the graph:	Slope using normal derivative:	Slope using implicit derivative:
$(x + 1)(y^2 - 2) = 24$		
Point on the graph:	Slope using normal derivative:	Slope using implicit derivative:
$xy - x = 12$		
Point on the graph:	Slope using normal derivative:	Slope using implicit derivative:

6. Consider the **general form of the equation of a line**, $Ax + By = C$, where A, B, and C are constants.

a) Determine the slope of the line by putting it in $y = mx + b$ form.

b) Take the derivative **implicitly**.

c) Explain the relationship between your answers for (a) and (b).

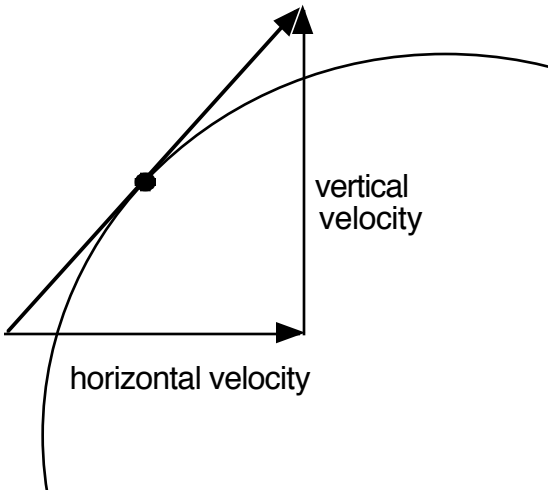
7. A point is travelling on the equation $x^2 + y^2 = 25$.

a) Find the x value of the graph when y = 4.

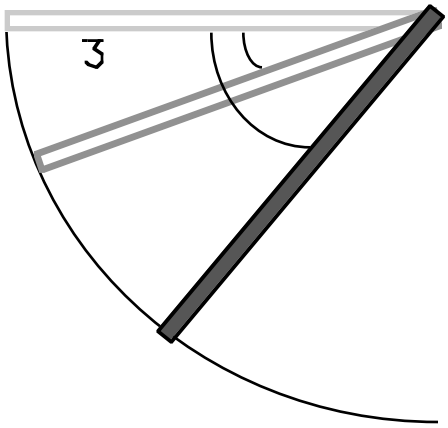
b) When y = 4, dx/dt is 2. Find dy/dt when y = 4 by **differentiating with respect to t.**

c) Use implicit differentiation **with respect to x** to find the slope at y = 4.

d) Use the diagram below to explain why your answer makes sense.



8. The bottom of a door gets a lot of paint on it. As the door swings, paint is getting on the floor. The door is 3 feet in length. The door is swinging through an angle of $\pi/8$ radians per second.



a) What angle has been covered after 1 second?

b) What angle has been covered after 1.01 seconds?

c) Find the area that has been painted after 1 second.

d) Find the area that has been painted after 1.01 seconds.

e) Calculate $\Delta A/\Delta t$ for the time interval between t = 1 second and t = 1.01 seconds.

d) Find dA/dt (A') for t = 1 second.

e) Imagine the door was somehow shrinking in length over time by .5 ft per second. After 1 second, the door is 2.5 feet in length. Find dA/dt under these new conditions.