

Name: \_\_\_\_\_

**Problem Set 2**  
**SHOW ALL WORK FOR CREDIT**

You may submit solutions on a separate sheet of paper if you prefer.

Assigned: 10/23/07

Due: 11/2/07 (2 pts off for each day late)

Work sessions: 10/25 & 11/1, room 421B

1. Given  $f(x) = x^3 - x^2$ , find the value of each expression below. **Simplify as much as possible.**

a)  $f(4)$

b)  $f(h)$

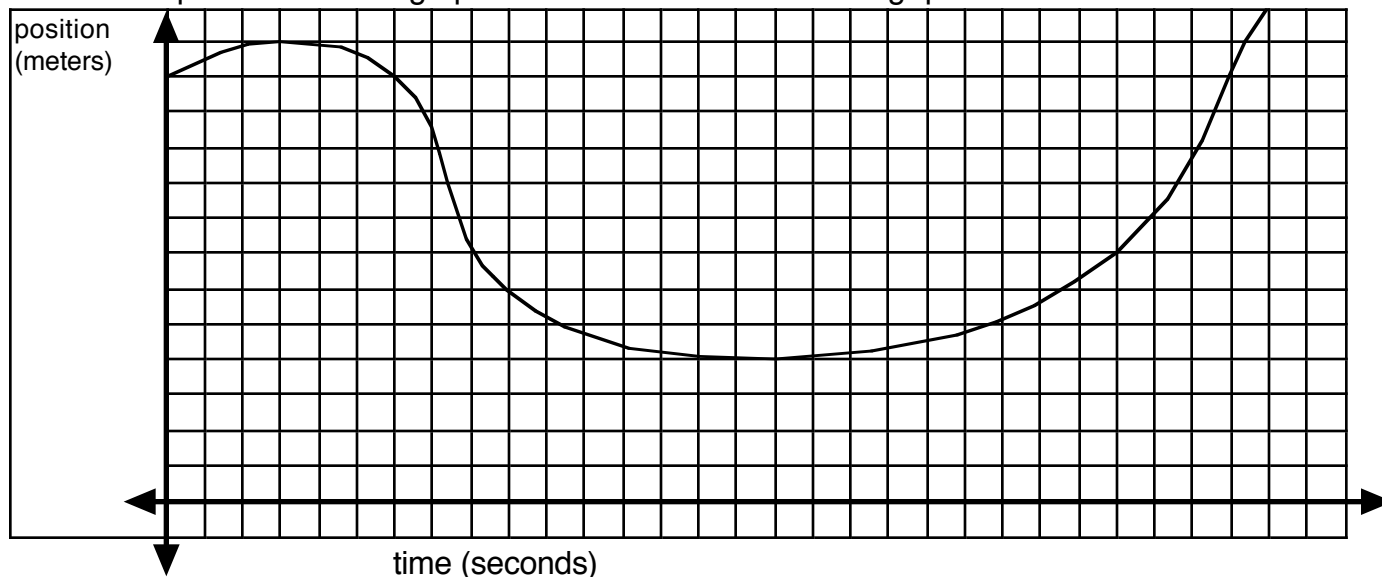
c)  $f(x + 1)$

d)  $f(x + h)$

e)  $f(a + b + c)$

2. Explain the relationship between slope and average speed. Use mathematical notation in your answer.

3. Use the position vs. time graph below to answer the following questions.



a). Calculate the average speed of the object over each interval.

i) 1 s to 3 s

ii) 3 s to 8 s

iii) 8 s to 14 s

b) Why can't you calculate an instantaneous speed at a single point (instant) using the usual formula of  $\Delta d / \Delta t$  ?

c) Draw a representation of the instantaneous speed of the object at 22 seconds on the graph. What is that speed?

d) When did the object stop moving?

e) How do you know the answer to letter (d) by looking at the slope of the graph?

4. The graph to the right shows the function

$$f(x) = \frac{1}{8}x^3 - x^2$$

a) Use the graph to approximate the slope at each of the following points. Be sure to actually find the slope (write the numerical answer).

i.  $x = 1$

ii.  $x = 6$

iii.  $x = -2$

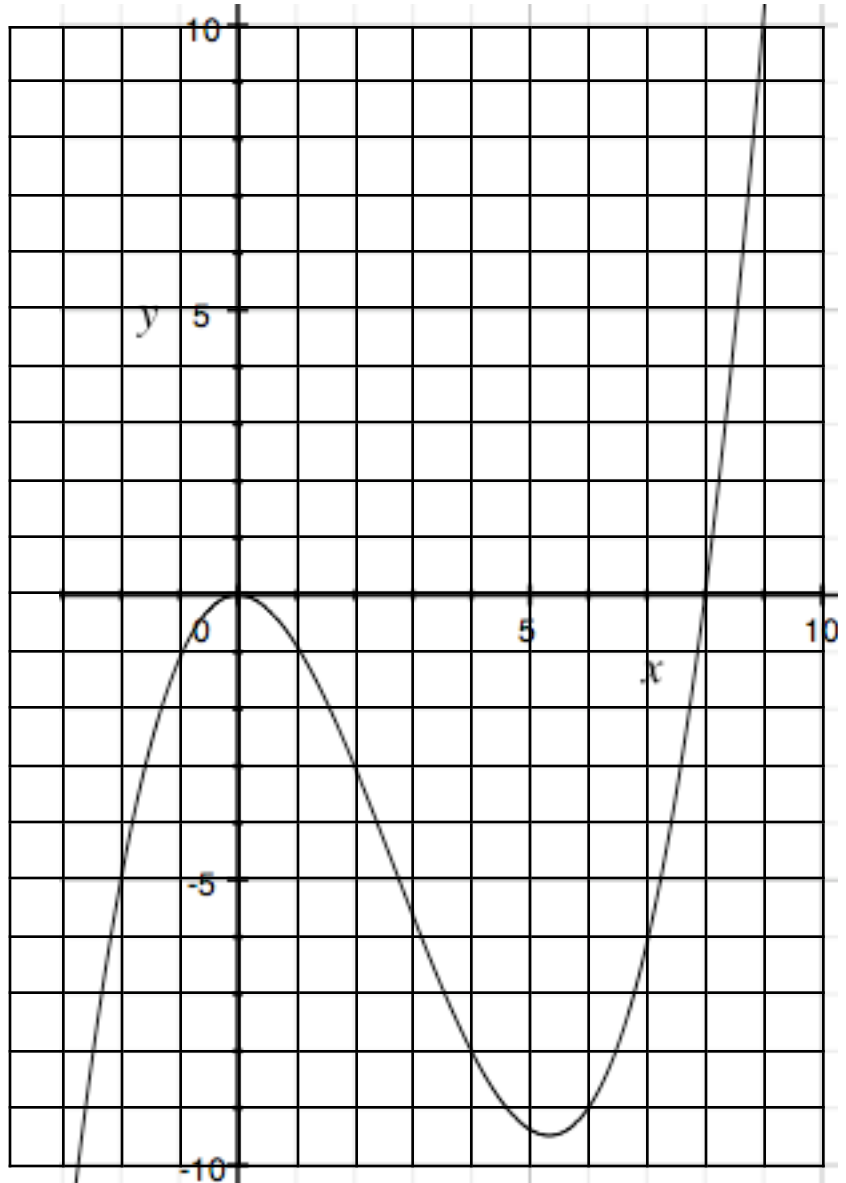
b) Why must we discuss the slope at only one point for a graph like this?

c) What is the formula for slope?

d) What happens to the slope formula if we consider only one point?

e) How can we get around this problem using a table?

f) Write your answer to (e) as a limit.



- g) Fill in the table below to approximate the instantaneous slopes at  $x = 1$ ,  $x = 6$ , and  $x = -2$ .  
For  $x = -2$ , fill in your own values to find the slope at exactly  $x = -2$ .

First point		Second point		$\Delta x$ “h”	$\Delta f(x)$	slope
5		5.1				
		5.05				
		5.01				
		5.001				
6				1		
				.5		
				.1		
				.01		
-2						

- h) Based on your table, predict the actual instantaneous slope at:

i.  $x = 5$

ii.  $x = 6$

iii.  $x = -2$

- i) Write a limit using **only**  $x$ ,  $f(x)$ , and  $h$  that will give you the exact slope at a point  $x$ .

- j) Plug in to your answer for (i) for  $f(x) = \frac{1}{8}x^3 - x^2$  and  $x = 5$  **and solve for the limit** to show that the slope you calculated for  $x = 5$  is correct.

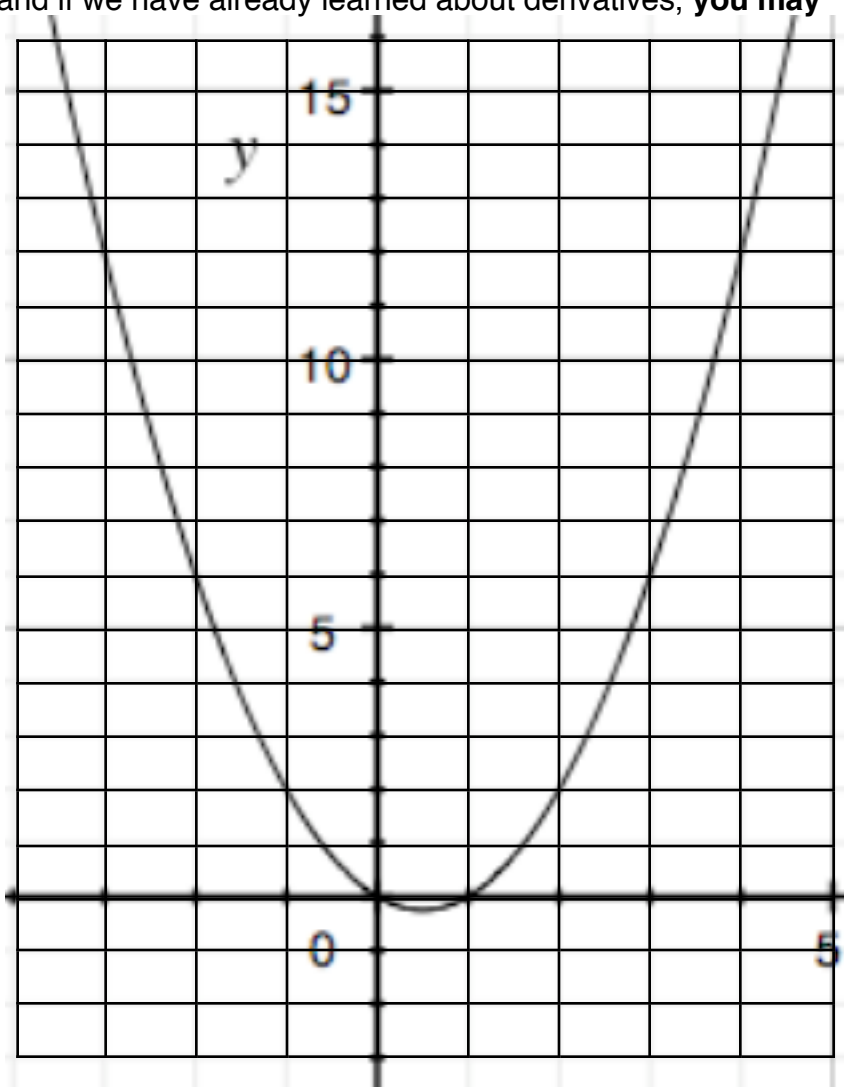
5. Use whatever techniques you would like to fill in the chart below as accurately as possible. However, you must **SHOW ALL OF YOUR WORK**, and if we have already learned about derivatives, **you may not** use the derivative.

$$f(x) = x^2 - x$$

a) CHART

$x = ?$	Slope of the graph at that $x$ value
-2	
-1	
0	
1	
2	
3	

WORK: (or attach an extra piece of paper)



b) Find a formula that will give you the slope at any  $x$ -value.

c) Write a limit for the slope at a point and use it to show your formula in (b) is correct.