Name: ANSWER KEY

UNIT 1 EXAM

Show ALL of your work for full credit

1. Expand $(a + b)(c + d + e + g)$

$=a(c + d + e + g) + b(c + d + e + g) = ac + ad + ae + ag + bc + bd + be + bg$

2. a) Expand $(x + 1)^{12}$

$= {}_{12}C_0x^{12} + {}_{12}C_1x^{11} + {}_{12}C_2x^{10} + {}_{12}C_3x^9 + {}_{12}C_4x^8 + {}_{12}C_5x^7 + {}_{12}C_6x^6 + {}_{12}C_7x^5 + {}_{12}C_8x^4 + {}_{12}C_9x^3 + {}_{12}C_{10}x^2 + {}_{12}C_{11}x^1 + {}_{12}C_{12}x^0$

$= x^{12} + 12x^{11} + 66x^{10} + 220x^9 + 495x^8 + 792x^7 + 924x^6 + 792x^5 + 495x^4 + 220x^3 + 66x^2 + 12x + 1$

b) Explain how you got your answer.

Use Pascal's triangle or combinations on the calculator.

3. a) Find $\lim_{x \to 0+} \frac{4}{x} = \infty$

b) Explain why this happens and why it makes sense using words.

As we divide 4 by a tiny positive number, that positive number goes into 4 more and more times. It's like saying how many times can you take a molecule out of 4 pizzas– a lot!

c) Find $\lim_{x \to \infty} \frac{4}{x} = 0$

d) Explain why this happens and why it makes sense using words.

As we divide 4 by a huge number, the pieces get really really small. It's like having 4 pizzas that you are splitting up among an unlimited number of students– each student will get practically nothing.

4. a) Use the calculator to find the following limit, or say the limit does not exist:

$$\lim_{x \to \infty} \frac{(\sin x)(\log x)}{x}$$

The limit is 0.

b) Explain in words what you did on your calculator to get this answer.

Put $(\sin x)(\log x)/x$ into Y1 on your calculator and then either look at the right side of the graph, or use the table function. By inputting large values for x, you can see that the y values become closer and closer to zero.

5. Use the chart below to find each limit. (**Warning: these are made-up functions. Don't try to identify them!)**

a)

x	f(x)
3.2	8.5
3.1	8.15
3.01	8.09
3.001	8.01
2.999	7.99
2.99	7.92
2.9	7.8
2.8	7.6
2.7	7.5

$\lim_{x \to 3} f(x) = 8$

b)

x	f(x)
80	1
100	-1
1200	1
14000	-1
300000	1
5000000	-1
60000000	1
700000000	-1
9000000000	1

$\lim_{x \to \infty} f(x)$ = D.N.E.

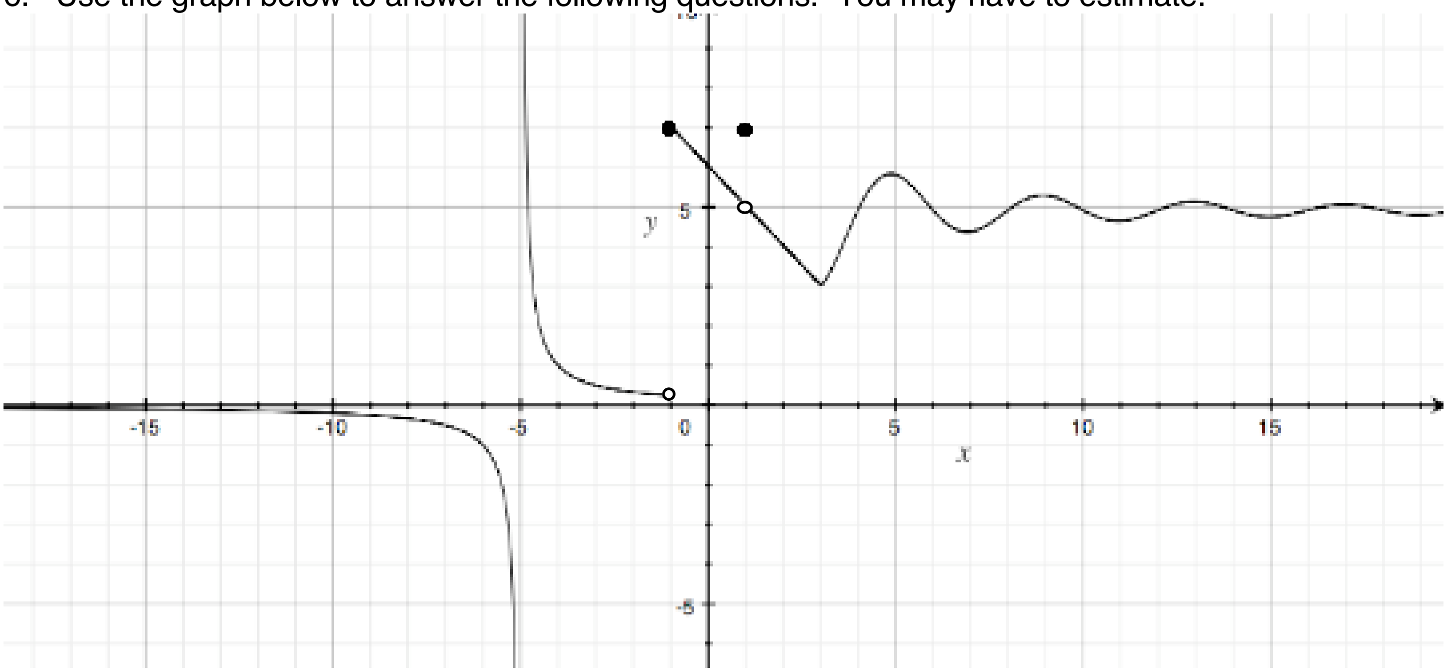
c)

x	f(x)
6.8	14.75
6.9	14.89
6.99	14.95
6.999	14.99
7.2	-9.5
7.1	-9.7
7.01	-9.9
7.001	-9.99
7.0001	-9.999

$\lim_{x \to 7} f(x)$ = D.N.E.

*D.N. E. = Does not exist

6. Use the graph below to answer the following questions. You may have to estimate.



a) $\lim_{x \to -\infty} f(x) = 0$

b) $\lim_{x \to \infty} f(x) = 5$

c) $\lim_{x \to -5^-} f(x) = -\infty$

d) $\lim_{x \to -5^+} f(x) = \infty$

e) $\lim_{x \to 5} f(x) = 6$

f) $\lim_{x \to 1} f(x) = 5$

g) $f(1) = 7$

h) $\lim_{x \to -1^+} f(x) = 7$

i) $\lim_{x \to -1^-} f(x) =$ 0.2 (ish)

j) $\lim_{x \to -1} f(x)$ = D. N. E.

k) $\lim_{x \to 3} f(x) = 3$

7. Find each limit. You must show pen & paper work for full credit.

a) $\lim_{x \to 3+} \dfrac{x+3}{x^2 - 9}$

Plug in and get $6/+0 \Rightarrow +\infty$

b) $\lim_{x > 0+} \dfrac{\sqrt{2 - x} - 2}{x}$

I screwed this problem up. But, as written, you can just plug in and get $(\sqrt{2} - 2)/+0 = -.6/+0 = -\infty$

c) $\lim_{x \to 4} \dfrac{x^2 - 7x + 12}{x - 4}$

Factor. $\dfrac{(x-3)\cancel{(x-4)}}{\cancel{(x-4)}}$

$\Rightarrow \lim_{x \to 4} x - 3 = 4 - 3 = 1$

d) $\lim_{x \to 0+} \dfrac{x^3 - 5x^2 + x - 3}{5x^3 + 7x}$

All you have to do is plug in. You get $-3/+0 \Rightarrow -\infty$

e) $\lim_{x \to 2} \dfrac{\sqrt{6 - x} - 2}{\sqrt{3 - x} - 1}$

$$\frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} \times \frac{(\sqrt{3-x} + 1)\,(\sqrt{6-x} + 2)}{(\sqrt{3-x} + 1)\,(\sqrt{6-x} + 2)}$$

$$= \frac{[(6-x) - 2^2]\,(\sqrt{3-x} + 1)}{[(3-x) - 1^2]\,(\sqrt{6-x} + 2)}$$

$$= \frac{(6 - x - 4)(\text{ugly conjugate 1})}{(3 - x - 1)(\text{ugly conjugate 1})} = \frac{(\cancel{2-x})(\text{ugly 1})}{(\cancel{2-x})(\text{ugly 2})} \text{ now plug in}$$

$$= (\sqrt{3 - \mathbf{2}} + 1)/(\sqrt{6 - \mathbf{2}} + 2) = 2/4 = \mathbf{1/2}$$

f) $\lim_{x \to \infty} \dfrac{4x^4 + 2x^2 - 7x + 1}{9x^4 - 3x^3 - 5}$

Divide by the highest power

$$\frac{4x^4/x^4 + 2x^2/x^4 - 7x/x^4 + 1/x^4}{9x^4/x^4 - 3x^3/x^4 - 5/x^4} =$$

$$\frac{4 + 2/x^2 - 7/x^3 + 1/x^4}{9 - 3/x - 5/x^4} \Rightarrow \text{All the fractions go to zero}$$

$\Rightarrow (4 + 0 - 0 + 0)/(9 - 0 - 0) = 4/9 = \mathbf{.\overline{4}}$

g) $\lim_{x \to \infty} \dfrac{x + 4}{x}$

$= x/x + 4/x = 1 + 4/x$

As $x \Rightarrow \infty$, $4/x \Rightarrow 0$ so $1 + 0 = \mathbf{1}$

h) $\lim_{x \to 0+} \dfrac{4x^{-2} + x^{-3} - 5x^{-4}}{x^{-1} + 2x^{-4}}$

The lowest power is x^{-4} so divide by x^{-4}, which is the same as multiplying by x^4 $\Rightarrow$

$$\frac{4x^2 + x - 5}{x^3 + 2} \Rightarrow \frac{0 + 0 - 5}{0 + 2} = \mathbf{-5/2 = -2.5}$$

i) $\lim_{x \to -1} \dfrac{2x^4 + 8x^3 + 12x^2 + 8x + 2}{x + 1}$ $= \dfrac{2(x^4 + 4x^3 + 6x^2 + 4x + 1)}{x + 1}$ <- Pascal's triangle $\Rightarrow$

$= 2(x + 1)^4/(x + 1) = 2(x + 1)^3 = 0$

8. Let a, b, and c be real numbers.

a) What are all of the possible solutions to $\lim_{x \to \infty} \dfrac{x^a + x^b}{x^c}$?

The possible solutions are 0, ∞, and 2.

If we divide up the fraction, we get $x^a/x^c + x^b/x^c$

b) Explain what the limit would be given each scenario:

i. $b < a < c$
- Since $a < c$, with x^a/x^c there are x's left on the bottom (specifically $= 1/x^{c-a}$) so that part of the limit goes to 0.
- Since $b < c$, with x^b/x^c there are x's left on the bottom (specifically $= 1/x^{c-b}$) so that part of the limit goes to 0 also. **So 0 + 0 = 0**

ii. $c < b < a$
- Since $c < a$, with x^a/x^c there are x's left on the top (specifically $= x^{a-c}$) so that part of the limit goes to ∞.
- Since $c < b$, with x^b/x^c there are x's left on the top (specifically $= x^{b-c}$) so that part of the limit goes to ∞ also. **So ∞ + ∞ = ∞**

iii. $b < c < a$
- Since $c < a$, with x^a/x^c there are x's left on the top (specifically $= x^{a-c}$) so that part of the limit goes to ∞.
- Since $b < c$, with x^b/x^c there are x's left on the bottom (specifically $= 1/x^{c-b}$) so that part of the limit goes to 0. **So ∞ + 0 = ∞**

iv. $a = b = c$
- Since $a = c$, $x^a/x^c = x^a/x^a = 1$.
- Since $b = c$, $x^b/x^c = x^b/x^b = 1$. **So 1 + 1 = 2**

9. $\lim_{x \to n+} f(x) \neq \lim_{x \to n-} f(x)$ where **n** is a real number.

a) Find $\lim_{x \to n} f(x)$ This limit does not exist because the left side and the right side aren't equal.

b) Is the function continuous or discontinuous around $x = n$? Explain.

The function is discontinuous, because if the left side of the limit doesn't equal the right side, there must be a hole or asymptote in the graph.

10. Let f(x) and g(x) be two functions of x. $\lim_{x \to 3} g(x) = \lim_{x \to 3} f(x) = 8$

Find $\lim_{x \to 3} \dfrac{(g(x))^2 - (f(x))^2}{g(x) - f(x)}$ This problem relies on the fact that if we have a limit where two things are added together, we can take the limit of each part separately, and then add them. For example:

$\lim_{x \to \infty} 1/x + 3x = \lim_{x \to \infty} 1/x + \lim_{x \to \infty} 3x$ – this simply means that we would analyze the 1/x and the 3x separately and add the results- 0 for 1/x and ∞ for 3x and get 0 + ∞ = ∞.

Keeping this in mind: No matter what the functions are, $[g(x)]^2 - [f(x)]^2 = [g(x) + f(x)]\,[g(x) - f(x)]$ by difference of squares. Accordingly, the g(x) - f(x) cancels on top and bottom and we are left with just

$\lim_{x \to 3} g(x) + f(x) = \lim_{x \to 3} g(x) + \lim_{x \to 3} f(x) = 8 + 8 = \mathbf{16}$