CHAPTER 3

ANSWERS TO QUESTIONS

Q3.1 No. Yes. The points could be widely separated. In this case, you can only determine the average velocity, which is $\overline{\mathbf{v}} = \Delta\mathbf{x} / \Delta t$.

Q3.2 (a) No. (b) Yes.
In the second case, the particle is continuously changing the direction of its velocity vector.

Q3.3 The *speed* is constant, not the velocity.

Q3.4 A parabola.

Q3.5 At the time the second ball is launched. Yes. 1 second. No.

Q3.6 No. The acceleration is a maximum in the opposite direction.

Q3.7 Yes. The top of the mast and the deck have the same horizontal velocity.

Q3.8 No, the projectile with the higher angle will be in the air longer. You can see this from $R = v_x t$, where v_x is greater for the projectile at the lower angle.

Q3.9 The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.

Q3.10 The quantities (b) acceleration and (c) horizontal component of velocity remain constant for a projectile in the absence of air resistance.

Q3.11 Less than 45°, so that more horizontal distance can be covered early in the motion, before air resistance has time to have much effect.

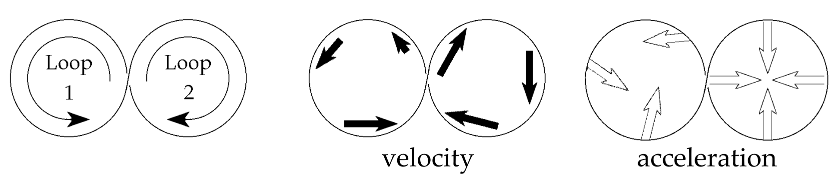
Q3.12 Velocity and acceleration both change in direction.

Q3.13 The projectile on the Moon has greater range and reaches greater altitude. The Apollo astronauts did the experiment with golf balls.

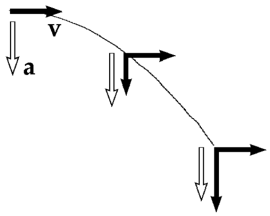
Q3.14 The horizontal velocity of the coin does not affect its vertical motion, which is identical to the vertical free fall of the ball.

Q3.15 (a) Drive straight ahead.
(b) Hold the steering wheel still to drive straight ahead or to follow any other path of constant curvature.

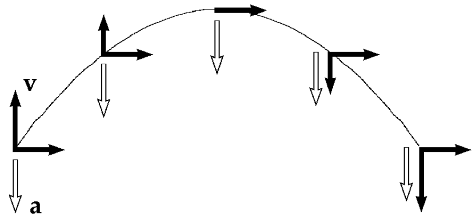
Q3.16 (Answer is the acceleration vectors, at far right.)



Q3.17



(a)



(b)

Q3.18 Let v_x and v_y represent its original velocity components.

(a) $x = v_x(v_y / g)$ and $y = v_y^2 / 2g$.

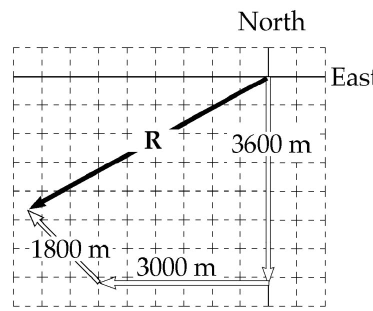
(b) Its velocity is horizontal and equal to v_x.

(c) Its acceleration is vertically downward, $-g$. With air resistance, the answers to (a) and (b) would be smaller. As for (c), the magnitude would be somewhat larger, because the acceleration would also have a component horizontally backwards.

PROBLEM SOLUTIONS

3.1

x(m)	y(m)
0	– 3600
–3000	0
–1270	1270
–4270 m	– 2330 m



(a) Net displacement = $\sqrt{x^2+y^2}$ = $\boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$

(b) Average speed = $\dfrac{20.0 \text{ m/s}(180 \text{ s}) + 25.0 \text{ m/s}(120 \text{ s}) + 30.0 \text{ m/s}(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$

(c) Average velocity $= \dfrac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \mathbf{R}}$

3.2 (a) For the average velocity, we have

$$\bar{\mathbf{v}} = \left(\frac{x(4.00) - x(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\mathbf{i} + \left(\frac{y(4.00) - y(2.00)}{4.00 \text{ s} - 2.00 \text{ s}}\right)\mathbf{j} = \left(\frac{5.00 \text{ m} - 3.00 \text{ m}}{2.00 \text{ s}}\right)\mathbf{i} + \left(\frac{3.00 \text{ m} - 1.50 \text{ m}}{2.00 \text{ s}}\right)\mathbf{j}$$

$$\bar{\mathbf{v}} = \boxed{(1.00\mathbf{i} + 0.750\mathbf{j}) \text{ m/s}}$$

(b) For the velocity components, we have $v_x = \dfrac{dx}{dt} = a = 1.00 \text{ m/s}$

and $v_y = \dfrac{dy}{dt} = 2ct = \left(0.250 \text{ m/s}^2\right)t$

Therefore, $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = (1.00 \text{ m/s})\mathbf{i} + \left(0.250 \text{ m/s}^2\right)t\mathbf{j}$

$$\boxed{\mathbf{v}(t = 2.00 \text{ s}) = (1.00 \text{ m/s})\mathbf{i} + (0.500 \text{ m/s})\mathbf{j}}$$

and the speed is $|\mathbf{v}(t = 2.00 \text{ s})| = \sqrt{(1.00 \text{ m/s})^2 + (0.500 \text{ m/s})^2} = \boxed{1.12 \text{ m/s}}$

3.3 (a) $\mathbf{r} = \boxed{18.0t\mathbf{i} + \left(4.00t - 4.90t^2\right)\mathbf{j}}$

(b) $\mathbf{v} = \boxed{(18.0 \text{ m/s})\mathbf{i} + [4.00 \text{ m/s} - \left(9.80 \text{ m/s}^2\right) t]\mathbf{j}}$

(c) $\mathbf{a} = \boxed{\left(-9.80 \text{ m/s}^2\right)\mathbf{j}}$

(d) $\mathbf{r}(3.00 \text{ s}) = \boxed{(54.0 \text{ m})\mathbf{i} - (32.1 \text{ m})\mathbf{j}}$

(e) $\mathbf{v}(3.00 \text{ s}) = \boxed{(18.0 \text{ m/s})\mathbf{i} - (25.4 \text{ m/s})\mathbf{j}}$

(f) $\mathbf{a}(3.00 \text{ s}) = \boxed{\left(-9.80 \text{ m/s}^2\right)\mathbf{j}}$

3.4 (a) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t} = \frac{(9.00\mathbf{i} + 7.00\mathbf{j}) - (3.00\mathbf{i} - 2.00\mathbf{j})}{3.00} = \boxed{(2.00\mathbf{i} + 3.00\mathbf{j}) \text{ m/s}^2}$$

(b) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = (3.00\mathbf{i} - 2.00\mathbf{j})t + \frac{1}{2}(2.00\mathbf{i} + 3.00\mathbf{j})t^2$

$\boxed{x = \left(3.00t + t^2\right) \text{ m}}$ and $\boxed{y = \left(1.50t^2 - 2.00t\right) \text{ m}}$

3.5 $\mathbf{v}_i = (4.00\mathbf{i} + 1.00\mathbf{j}) \text{ m/s}$ and $\mathbf{v}(20.0) = (20.0\mathbf{i} - 5.00\mathbf{j}) \text{ m/s}$

(a) $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{20.0 - 4.00}{20.0} \text{ m/s}^2 = \boxed{0.800 \text{ m/s}^2}$

$a_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-5.00 - 1.00}{20.0} \text{ m/s}^2 = \boxed{-0.300 \text{ m/s}^2}$

(b) $\theta = \tan^{-1}\left(\dfrac{-0.300}{0.800}\right) = -20.6° = \boxed{339° \text{ from } +x \text{ axis}}$

(c) At $t = 25.0$ s

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \tfrac{1}{2}(0.800)(25.0)^2 = \boxed{360 \text{ m}}$$

$$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \tfrac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

3.6 $\mathbf{a} = 3.00\mathbf{j}\ \text{m/s}^2;\ \mathbf{v}_i = 5.00\mathbf{i}\ \text{m/s};\ \mathbf{r}_i = 0\mathbf{i} + 0\mathbf{j}$

(a) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = \boxed{\left[5.00t\mathbf{i} + \frac{1}{2}3.00t^2\mathbf{j}\right]\ \text{m}}$

$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \boxed{(5.00\mathbf{i} + 3.00t\mathbf{j})\ \text{m/s}}$

(b) $t = 2.00\ \text{s},\ \mathbf{r}_f = 5.00(2.00)\mathbf{i} + \frac{1}{2}(3.00)(2.00)^2\mathbf{j} = (10.0\mathbf{i} + 6.00\mathbf{j})\ \text{m}$

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

$\mathbf{v}_f = 5.00\mathbf{i} + 3.00(2.00)\mathbf{j} = (5.00\mathbf{i} + 6.00\mathbf{j})\ \text{m/s}$

$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \text{m/s}}$

***3.7** (a) For the x-component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$0.01\ \text{m} = 0 + \left(1.80\times10^7\ \text{m/s}\right)t + \frac{1}{2}\left(8\times10^{14}\ \text{m/s}^2\right)t^2$$

$$\left(4\times10^{14}\ \text{m/s}^2\right)t^2 + \left(1.80\times10^7\ \text{m/s}\right)t - 10^{-2}\ \text{m} = 0$$

$$t = \frac{-1.80\times10^7\ \text{m/s} \pm \sqrt{\left(1.8\times10^7\ \text{m/s}\right)^2 - 4\left(4\times10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\ \text{m}\right)}}{2\left(4\times10^{14}\ \text{m/s}^2\right)} = \frac{-1.8\times10^7 \pm 1.84\times10^7\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation $t = \dfrac{4.39\times10^5\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2} = 5.49\times10^{-10}\ \text{s}$

Here $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}\left(1.6\times10^{15}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)^2 = 2.41\times10^{-4}\ \text{m}$

So, $\boxed{\mathbf{r}_f = (10.0\ \mathbf{i} + 0.241\ \mathbf{j})\ \text{mm}}$

(b) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 1.80\times10^7\ \text{m/s}\ \mathbf{i} + \left(8\times10^{14}\ \text{m/s}^2\ \mathbf{i} + 1.6\times10^{15}\ \text{m/s}^2\ \mathbf{j}\right)\left(5.49\times10^{-10}\ \text{s}\right)$

$= \left(1.80\times10^7\ \text{m/s}\right)\mathbf{i} + \left(4.39\times10^5\ \text{m/s}\right)\mathbf{i} + \left(8.78\times10^5\ \text{m/s}\right)\mathbf{j}$

$= \boxed{\left(1.84\times10^7\ \text{m/s}\right)\mathbf{i} + \left(8.78\times10^5\ \text{m/s}\right)\mathbf{j}}$

(c) $|\mathbf{v}_f| = \sqrt{\left(1.84\times10^7\ \text{m/s}\right)^2 + \left(8.78\times10^5\ \text{m/s}\right)^2} = \boxed{1.85\times10^7\ \text{m/s}}$

(d) $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) = \tan^{-1}\left(\dfrac{8.78\times10^5}{1.84\times10^7}\right) = \boxed{2.73°}$

3.8 (a) Coyote: $\Delta x = \frac{1}{2}at^2; \quad 70.0 = \frac{1}{2}(15.0)t^2$

Roadrunner: $\Delta x = v_i\, t; \quad 70.0 = v_i\, t$

Solving the above, we get $v_i = \boxed{22.9 \text{ m/s}}$ and $t = 3.06$ s

(b) At the edge of the cliff, $v_{xi} = at = (15.0)(3.06) = 45.8$ m/s

Substituting into $\Delta y = \frac{1}{2}a_y\, t^2$, we find $-100 = \frac{1}{2}(-9.80)t^2$

$t = 4.52$ s

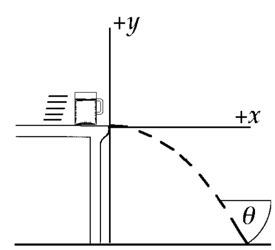
$$\Delta x = v_{xi}\, t + \frac{1}{2}a_x\, t^2 = (45.8)(4.52 \text{ s}) + \frac{1}{2}(15.0)(4.52 \text{ s})^2$$

Solving, $\Delta x = \boxed{360 \text{ m}}$

(c) For the Coyote's motion through the air $v_{xf} = v_{xi} + a_x\, t = 45.8 + 15(4.52) = \boxed{114 \text{ m/s}}$

$$v_{yf} = v_{yi} + a_y\, t = 0 - 9.80(4.52) = \boxed{-44.3 \text{ m/s}}$$

3.9 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$. i.e.,



$$t = \frac{x_f}{v_{xi}} = \frac{(1.40 \text{ m})}{v_{xi}}$$

In the same time it falls a distance of 0.860 m with acceleration downward of 9.80 m/s^2.

Then $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$: $\quad 0 = 0.860 \text{ m} + \frac{1}{2}(-9.80 \text{ m/s}^2)\left(\frac{1.40 \text{ m}}{v_{xi}}\right)^2$

Thus, $v_{xi} = \sqrt{\frac{(4.90 \text{ m/s}^2)(1.96 \text{ m}^2)}{0.860 \text{ m}}} = \boxed{3.34 \text{ m/s}}$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + (-9.80 \text{ m/s}^2)\left(\frac{1.40 \text{ m}}{3.34 \text{ m/s}}\right) = -4.11 \text{ m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}$$

***3.10** The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi}\,t + \tfrac{1}{2}a_x\,t^2 = v_{xi}\,t + 0 \quad \text{and} \quad y_f = v_{yi}\,t + \tfrac{1}{2}a_y\,t^2 = 0 - \tfrac{1}{2}g\,t^2$$

When the mug reaches the floor, $y_f = -h$, so $-h = -\tfrac{1}{2}g\,t^2$

which gives the time of impact as $t = \sqrt{\dfrac{2h}{g}}$

(a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi}\,t$ becomes $d = v_{xi}\sqrt{\dfrac{2h}{g}}$

giving the initial velocity as $\boxed{v_{xi} = d\sqrt{\dfrac{g}{2h}}}$

(b) Just before impact, the x-component of velocity is still $v_{xf} = v_{xi}$

while the y-component is $v_{yf} = v_{yi} + a_y\,t = 0 - g\sqrt{\dfrac{2h}{g}}$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right) = \tan^{-1}\left(\frac{g\sqrt{2h/g}}{d\sqrt{g/2h}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}$$

3.11 $y_f = v_i(\sin\ 3.00°)t - \tfrac{1}{2}g\,t^2$ and $v_{yf} = v_i \sin 3.00° - g\,t$

When $y_f = 0.330\text{ m}$, $v_{yf} = 0$ and $v_i \sin 3.00° = g\,t$ $\quad y_f = v_i(\sin 3.00°)\dfrac{v_i \sin 3.00°}{g} - \tfrac{1}{2}g\left(\dfrac{v_i \sin 3.00°}{g}\right)^2$

Solving, $y_f = \dfrac{v_i^2 \sin^2 3.00°}{2g} = 0.330\text{ m}$

Therefore, $v_i = \boxed{48.6\text{ m/s}}$

The 12.6 m is unnecessary information.

3.12 From Equation 3.16, $R = 15.0\text{ m},\ v_i = 3.00\text{ m/s},\ \theta_{\max} = 45.0°$

$$\therefore \quad g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600\text{ m/s}^2}$$

3.13 (a) $x_f = v_{xi}t = 8.00\cos 20.0°(3.00) = \boxed{22.6\text{ m}}$

(b) Taking y positive downwards, $y_f = v_{yi}t + \frac{1}{2}gt^2$

$$y_f = 8.00\sin 20.0°(3.00) + \tfrac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$ $\qquad 4.90t^2 + 2.74t - 10.0 = 0$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

3.14 Take the origin at the mouth of the cannon.

$x_f = v_{xi}t \qquad 2000\text{ m} = (1000\text{ m/s})\cos\theta_i t$

Therefore, $t = \dfrac{2.00\text{ s}}{\cos\theta_i}$

$y_f = v_{yi}t + \frac{1}{2}a_y t^2$: $\quad 800\text{ m} = (1000\text{ m/s})\sin\theta_i\, t + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)t^2$

$$800\text{ m} = (1000\text{ m/s})\sin\theta_i\left(\frac{2.00\text{ s}}{\cos\theta_i}\right) - \tfrac{1}{2}\left(9.80\text{ m/s}^2\right)\left(\frac{2.00\text{ s}}{\cos\theta_i}\right)^2$$

$$800\text{ m}\left(\cos^2\theta_i\right) = 2000\text{ m}\left(\sin\theta_i\cos\theta_i\right) - 19.6\text{ m}$$

$$19.6\text{ m} + 800\text{ m}\left(\cos^2\theta_i\right) = 2000\text{ m}\sqrt{1-\cos^2\theta_i}\left(\cos\theta_i\right)$$

$$384 + (31360)\cos^2\theta_i + (640000)\cos^4\theta_i = (4000000)\cos^2\theta_i - (4000000)\cos^4\theta_i$$

$$4640000\cos^4\theta_i - 3968640\cos^2\theta_i + 384 = 0$$

$$\cos^2\theta_i = \frac{3968640 \pm \sqrt{(3968640)^2 - 4(4640000)(384)}}{9280}$$

$\cos\theta_i = 0.925 \quad$ or $\quad 0.00984$

$\theta_i = \boxed{22.4° \text{ or } 89.4°}$ (Both solutions are valid.)

3.15 (a) We use Equation 3.14: $y_f = x_f \tan\theta_i - \dfrac{g x_f^2}{2 v_i^2 \cos\theta_i}$

With $x_f = 36.0 \text{ m}$, $v_i = 20.0 \text{ m/s}$, and $\theta = 53.0°$

we find $y_f = (36.0 \text{ m})\tan 53.0° - \dfrac{(9.80 \text{ m/s}^2)(36.0 \text{ m})^2}{2(20.0 \text{ m/s})^2 \cos^2(53.0°)} = 3.94 \text{ m}$

The ball clears the bar by $(3.94 - 3.05) \text{ m} = \boxed{0.889 \text{ m}}$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin\theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0°)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}$$

The time to travel 36.0 m horizontally is $t_2 = \dfrac{x_f}{v_{ix}}$

$t_2 = \dfrac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0°)} = 2.99 \text{ s}$ Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$

3.16 (a) The time of flight of the first snowball is the nonzero root of $y_f = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$

$0 = 0 + (25.0 \text{ m/s})(\sin 70.0°)t_1 - \frac{1}{2}(9.80 \text{ m/s}^2)t_1^2$ $t_1 = \dfrac{2(25.0 \text{ m/s})\sin 70.0°}{9.80 \text{ m/s}^2} = 4.79 \text{ s}$

The distance to your target is $x_f - x_i = v_{xi}t_1 = (25.0 \text{ m/s})\cos 70.0°(4.79 \text{ s}) = 41.0 \text{ m}$

Now the second snowball we describe by $y_f = y_i + v_{yi}t_2 + \frac{1}{2}a_y t_2^2$

$0 = (25.0 \text{ m/s})\sin\theta_2\, t_2 - (4.90 \text{ m/s}^2)t_2^2$ $t_2 = (5.10 \text{ s})\sin\theta_2$

$x_f - x_i = v_{xi}\, t_2$

$41.0 \text{ m} = (25.0 \text{ m/s})\cos\theta_2 (5.10 \text{ s})\sin\theta_2 = (128 \text{ m})\sin\theta_2\cos\theta_2$ $0.321 = \sin\theta_2\cos\theta_2$

Using $\sin 2\theta = 2\sin\theta\cos\theta$ we can solve $0.321 = \frac{1}{2}\sin 2\theta_2$

$2\theta_2 = \sin^{-1} 0.643$ and $\theta_2 = \boxed{20.0°}$

(b) The second snowball is in the air for time $t_2 = (5.10 \text{ s})\sin\theta_2 = (5.10 \text{ s})\sin 20° = 1.75 \text{ s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79 \text{ s} - 1.75 \text{ s} = \boxed{3.05 \text{ s}}$$

3.17 Consider the motion from original zero height to maximum height *h*:

$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$ gives $0 = v_{yi}^2 - 2g(h-0)$ or $v_{yi} = \sqrt{2gh}$

Now consider the motion from the original point to half the maximum height:

$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$ gives $v_{yh}^2 = 2gh + 2(-g)\left(\frac{1}{2}h - 0\right)$ so $v_{yh} = \sqrt{gh}$

At maximum height, $v_x = \frac{1}{2}\sqrt{v_x^2 + v_{yh}^2} = \frac{1}{2}\sqrt{v_x^2 + gh}$

Solving, $v_x = \sqrt{gh/3}$

Now the projection angle is $\theta_i = \tan^{-1}\dfrac{v_{yi}}{v_x} = \tan^{-1}\dfrac{\sqrt{2gh}}{\sqrt{gh/3}} = \tan^{-1}\sqrt{6} = \boxed{67.8°}$

***3.18** We interpret the problem to mean that the displacement from fish to bug is 2.00 m at $30° = (2.00\text{ m})\cos 30°\,\mathbf{i} + (2.00\text{ m})\sin 30°\,\mathbf{j} = (1.73\text{ m})\mathbf{i} + (1.00\text{ m})\mathbf{j}$. If the water should drop 0.03 m during its flight, then the fish must aim it at a point 0.03 m above the bug. The initial velocity of the water then is directed through the point with displacement $(1.73\text{ m})\mathbf{i} + (1.03\text{ m})\mathbf{j} = 2.015\text{ m at } 30.7°$.

For the time of flight of a water drop we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

$1.73\text{ m} = 0 + \left(v_i \cos 30.7°\right)t + 0$ so $t = \dfrac{1.73\text{ m}}{v_i \cos 30.7°}$

The vertical motion is described by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

The "drop on its path" is $-3.00\text{ cm} = \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(\dfrac{1.73\text{ m}}{v_i \cos 30.7°}\right)^2$

Thus, $v_i = \dfrac{1.73\text{ m}}{\cos 30.7°}\sqrt{\dfrac{9.80\text{ m/s}^2}{2\times 0.03\text{ m}}} = 2.015\text{ m}\left(12.8\text{ s}^{-1}\right) = \boxed{25.8\text{ m/s}}$

***3.19** From Equation 3.12, $x_f = v_{xi}t = \left(v_i \cos\theta_i\right)t$. Therefore, the time required to reach the building a distance *d* away is $t = d/v_i \cos\theta_i$.

At this time, the altitude of the water is $y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i\left(\dfrac{d}{v_i\cos\theta_i}\right) - \dfrac{g}{2}\left(\dfrac{d}{v_i\cos\theta_i}\right)^2$

Therefore the water strikes the building at a height *h* above ground level of

$$h = y_f = \boxed{d\tan\theta_i - \frac{gd^2}{2v_i^2\cos\theta_i}}$$

3.20 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad -40.0\text{ m} = 0 + 0 + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)t^2 \qquad t = 2.86\text{ s}$$

The extra time $3.00\text{ s} - 2.86\text{ s} = 0.143\text{ s}$ is the time required for the sound she hears to travel straight back to the player.

It covers distance $(343\text{ m/s})0.143\text{ s} = 49.0\text{ m} = \sqrt{x^2 + (40.0\text{ m})^2}$

where x represents the horizontal distance the rock travels.

$$x = 28.3\text{ m} = v_{xi}t + 0t^2 \qquad \therefore v_{xi} = \frac{28.3\text{ m}}{2.86\text{ s}} = \boxed{9.91\text{ m/s}}$$

*__3.21__ (a) For the horizontal motion, we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

$$24\text{ m} = 0 + v_i(\cos 53°)(2.2\text{ s}) + 0$$

$$v_i = \boxed{18.1\text{ m/s}}$$

(b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1\text{ m/s})(\sin 53°)(2.2\text{ s}) + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(2.2\text{ s})^2 = 8.13\text{ m}$$

So it clears the parapet by $8.13\text{ m} - 7\text{ m} = \boxed{1.13\text{ m}}$

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from Equation 3.14,

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2 \qquad \text{or} \qquad 6\text{ m} = (\tan 53°)x_f - \left(\frac{9.8\text{ m/s}^2}{2(18.1\text{ m/s})^2\cos^2 53°}\right)x_f^2$$

Solving, $\left(0.0412\text{ m}^{-1}\right)x_f^2 - 1.33x_f + 6\text{ m} = 0$

and
$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2\left(0.0412\text{ m}^{-1}\right)}$$

This yields two results: $x_f = 26.8\text{ m}$ or 5.44 m

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall $26.8\text{ m} - 24\text{ m} = \boxed{2.79\text{ m}}$

***3.22** From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\text{ m/s}^2)(1.85 - 1.02)\text{ m}$. From this, $v_{yi} = 4.03\text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\text{ m/s}^2)(0.900 - 1.85)\text{ m}$

Thus the vertical velocity just before he lands is $v_{yf} = -4.32\text{ m/s}$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$-4.32\text{ m/s} = 4.03\text{ m/s} + (-9.80\text{ m/s}^2)t$ or $t = \boxed{0.852\text{ s}}$

(b) Looking at the total horizontal displacement during the leap,

$x = v_{xi}t$ becomes $2.80\text{ m} = v_{xi}(0.852\text{ s})$

which yields $v_{xi} = \boxed{3.29\text{ m/s}}$

(c) $v_{yi} = \boxed{4.03\text{ m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\frac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\frac{4.03\text{ m/s}}{3.29\text{ m/s}}\right) = \boxed{50.8°}$

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$:

$0 = v_{yi}^2 + 2(-9.80\text{ m/s}^2)(2.50 - 1.20)\text{ m}$ so $v_{yi} = 5.04\text{ m/s}$

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$

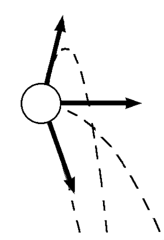
yields $v_{yf}^2 = 0 + 2(-9.80\text{ m/s}^2)(0.700 - 2.50)\text{ m}$

and $v_{yf} = -5.94\text{ m/s}$

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94\text{ m/s} = 5.04\text{ m/s} + (-9.80\text{ m/s}^2)t$

and $\boxed{t = 1.12\text{ s}}$

***3.23** For the smallest impact angle $\theta = \tan^{-1}(v_{yf}/v_{xf})$, we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y-component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is



$\theta = \tan^{-1}(v_{yf}/v_{xf}) = \boxed{\tan^{-1}(\sqrt{2gh}/v)}$

3.24 (a) $v = \frac{\Delta x}{\Delta y} = \frac{2\pi\left(3.84 \times 10^8 \text{ m}\right)}{(27.3 \text{ d})(24 \text{ h/d})(3600 \text{ s/h})} = \boxed{1.02 \times 10^3 \text{ m/s}}$

(b) Since v is constant and only direction changes,

$$a = \frac{v^2}{r} = \frac{(1.02 \times 10^3)^2}{3.84 \times 10^8} = \boxed{2.72 \times 10^{-3} \text{ m/s}^2}$$

3.25 $a_c = \frac{v^2}{r} = \frac{(20.0 \text{ m/s})^2}{1.06 \text{ m}} = \boxed{377 \text{ m/s}^2}$

The mass is unnecessary information.

***3.26** $a = \frac{v^2}{R}$ $T = 24 \text{ h}(3600 \text{ s/h}) = 86\,400 \text{ s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37 \times 10^6 \text{ m})}{86\,400 \text{ s}} = 463 \text{ m/s}$$

$$a = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = \boxed{0.0337 \text{ m/s}^2 \text{ directed toward the center of Earth.}}$$

3.27 $r = 0.500$ m;

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500 \text{ m})}{60.0 \text{ s}/200 \text{ rev}} = 10.47 \text{ m/s} \quad \boxed{10.5 \text{ m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219 \text{ m/s}^2 \text{ inward}}$$

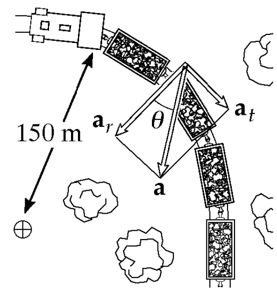
***3.28** $a_c = \frac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{3(9.8 \text{ m/s}^2)(9.45 \text{ m})} = 16.7 \text{ m/s}$$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45 \text{ m}) = 59.4 \text{ m}$. Then the rotation rate is

$$16.7 \text{ m/s}\left(\frac{1 \text{ rev}}{59.4 \text{ m}}\right) = \boxed{0.281 \text{ rev/s}}$$

3.29 We assume the train is still slowing down at the instant in question.



$$a_c = \frac{v^2}{r} = 1.29\ \text{m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0\ \text{km/h})(10^3\ \text{m/km})(1\ \text{h}/3600\ \text{s})}{15.0\ \text{s}} = -0.741\ \text{m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.29\ \text{m/s}^2)^2 + (-0.741\ \text{m/s}^2)^2}$$

at an angle of $\tan^{-1}(|a_t|/a_c) = \tan^{-1}(0.741/1.29)$

$\mathbf{a} =$ $\boxed{1.48\ \text{m/s}^2 \text{ inward and } 29.9^\circ \text{ backward}}$

3.30 (b) We do part (b) first. The tangential speed is described by $v_f = v_i + a_t t$

$0.7\ \text{m/s} = 0 + a_t(1.75\ \text{s})$ so $\boxed{a_t = 0.400\ \text{m/s}^2 \text{ forward}}$

(a) Now at $t = 1.25$ s, $v_f = v_i + a_t t = 0 + (0.4\ \text{m/s}^2)1.25\ \text{s}$

$v_f = 0.5\ \text{m/s}$

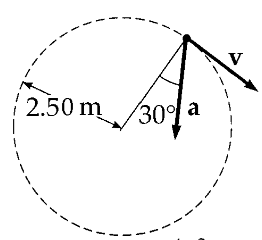
so $a_c = \frac{v^2}{r} = \frac{(0.5\ \text{m/s})^2}{0.2\ \text{m}} = \boxed{1.25\ \text{m/s}^2 \text{ toward the center}}$

(c) $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t = 0.4\ \text{m/s}^2 \text{ forward} + 1.25\ \text{m/s}^2 \text{inward}$

$\mathbf{a} = \sqrt{0.4^2 + 1.25^2}$ forward and inward at $\theta = \tan^{-1}(1.25/0.4)$

$\mathbf{a} =$ $\boxed{1.31\ \text{m/s}^2 \text{ forward and } 72.3^\circ \text{ inward}}$

3.31 $r = 2.50\ \text{m},\ a = 15.0\ \text{m/s}^2$



$a = 15.0\ \text{m/s}^2$

(a) $a_c = a\cos 30.0^\circ = (15.0\ \text{m/s}^2)(\cos 30^\circ) = \boxed{13.0\ \text{m/s}^2}$

(b) $a_c = \frac{v^2}{r}$

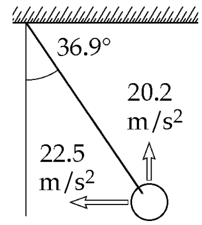
so $v^2 = ra_c = 2.50\ \text{m}(13.0\ \text{m/s}^2) = 32.5\ \text{m}^2/\text{s}^2$

$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\ \text{m/s}^2)^2 - (13.0\ \text{m/s}^2)} = \boxed{7.50\ \text{m/s}^2}$

3.32 (a) See figure to the right.



(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5 \text{ m/s}^2\right)\cos(90.0° - 36.9°) + \left(20.2 \text{ m/s}^2\right)\cos 36.9° = \boxed{29.7 \text{ m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67 \text{ m/s at } 36.9° \text{ above the horizontal}}$

***3.33** Total time in still water $t = \dfrac{d}{v} = \dfrac{2000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}$

Total time = time upstream plus time downstream: $t_{\text{up}} = \dfrac{1000}{(1.20 - 0.500)} = 1.43 \times 10^3 \text{ s}$

$$t_{\text{down}} = \frac{1000}{1.20 + 0.500} = 588 \text{ s}$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3 \text{ s}}$

***3.34** The bumpers are initially $100 \text{ m} = 0.100 \text{ km}$ apart. After time t the bumper of the leading car travels $40.0t$, while the bumper of the chasing car travels $60.0t$.

Since the cars are side by side at time t, we have

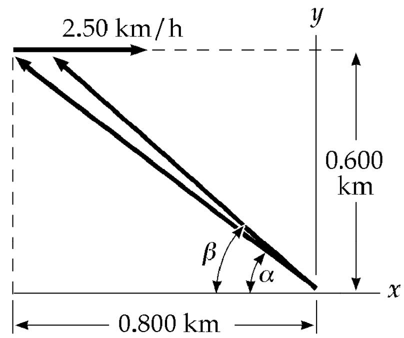
$0.100 + 40.0t = 60.0t$, yielding $t = 5.00 \times 10^{-3} \text{ h} = \boxed{18.0 \text{ s}}$

***3.35** $v = \sqrt{150^2 + 30.0^2} = \boxed{153 \text{ km/h}}$

$\theta = \tan^{-1}\left(\dfrac{30.0}{150.}\right) = \boxed{11.3° \text{ north of west}}$

***3.36** α = Heading with respect to the shore

β = Angle of boat with respect to the shore



(a) The boat should always steer for the child at heading

$$\alpha = \tan^{-1}\left(\frac{0.600}{0.800}\right) = \boxed{36.9°}$$

(b) $v_x = 20.0\cos\alpha - 2.50 = 13.5 \text{ km/h}$

$v_y = 20.0\sin\alpha = 12.0 \text{ km/h}$

$$\beta = \tan^{-1}\left(\frac{12.0 \text{ km/h}}{13.5 \text{ km/h}}\right) = \boxed{41.6°}$$

(c) $t = \dfrac{d_y}{v_y} = \dfrac{0.600 \text{ km}}{12.0 \text{ km/h}} = 5.00\times 10^{-2} \text{ h} = \boxed{3.00 \text{ min}}$

***3.37** Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

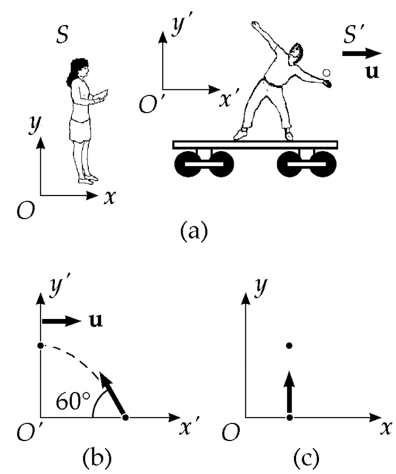
$$\frac{v_y'}{v_x'} = \tan 60.0° = \sqrt{3}$$

Let u represent the speed of S' relative to S. Then because there is no x–motion In S, we can write $v_x = v_x' + u = 0$ so that $v_x' = -u = -10.0 \text{ m/s}$. Hence the ball is thrown backwards in S'. Then,

$$v_y = v_y' = \sqrt{3}|v_x'| = 10.0\sqrt{3} \text{ m/s}$$

Using $v_y^2 = 2gh$ (from Eq. 3.15), we find

$$h = \frac{(10.0\sqrt{3} \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{15.3 \text{ m}}$$



The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).

***3.38** For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$

Therefore, the total time for Alan is $t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{2L/c}{1 - v^2/c^2}}$

For Beth, her cross-stream speed (both ways) is $\sqrt{c^2 - v^2}$

Thus, the total time for Beth is $t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{2L/c}{\sqrt{1 - v^2/c^2}}}$

Since $1 - v^2/c^2 < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

3.39 The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration.

$a_c = g$ so $\frac{v^2}{r} = g$

Solving for the velocity, $v = \sqrt{rg} = \sqrt{(6400 + 600)(10^3 \text{ m})(8.21 \text{ m/s}^2)} = \boxed{7.58 \times 10^3 \text{ m/s}}$

$v = \frac{2\pi r}{T}$ and $T = \frac{2\pi r}{v} = \frac{2\pi(7000 \times 10^3 \text{ m})}{7.58 \times 10^3 \text{ m/s}} = \boxed{5.80 \times 10^3 \text{ s}}$

$$T = 5.80 \times 10^3 \text{ s}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 96.7 \text{ min}$$

3.40 (a) The moon's gravitational acceleration is the probe's centripetal acceleration:

(For the moon's radius, see end papers of text.)

$a = \frac{v^2}{r}$ $\frac{1}{6}(9.80 \text{ m/s}^2) = \frac{v^2}{1.74 \times 10^6 \text{ m}}$

$v = \sqrt{2.84 \times 10^6 \text{ m}^2/s^2} = \boxed{1.69 \text{ km/s}}$

(b) $v = \frac{2\pi r}{T}$ $T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6 \text{ m})}{1.69 \times 10^3 \text{ m/s}} = 6.47 \times 10^3 \text{ s} = \boxed{1.80 \text{ h}}$

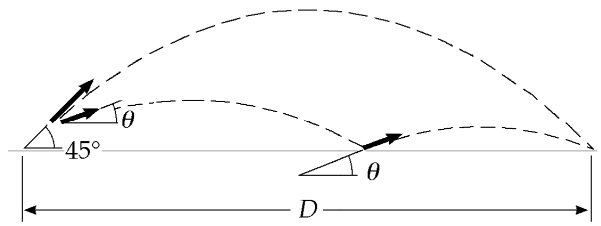
3.41 The special conditions allowing use of Equation 3.16 apply.

For the ball thrown at 45°, $D = R_{45} = \frac{v_i^2 \sin 90}{g}$

For the bouncing ball, $D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{(v_i/2)^2 \sin 2\theta}{g}$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:



$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$\sin 2\theta = \frac{4}{5}$ $\boxed{\theta = 26.6°}$

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0° $t_{45} = \frac{2v_i \sin 45.0°}{g}$

For the bouncing ball, $t = t_1 + t_2 = \frac{2v_i 26.6°}{g} + \frac{2(v_i/2)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$

The ratio of this time to that for no bounce is $\frac{3v_i \sin 26.6°/g}{2v_i \sin 45.0°/g} = \frac{1.34}{1.41} = \boxed{0.949}$

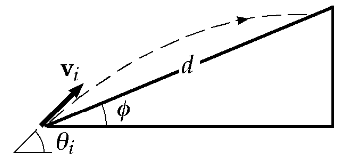
3.42 After the string breaks the ball is a projectile, and reaches the ground at time t: $y_f = v_{yi}t + \frac{1}{2}a_y t^2$

$-1.20 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$ so $t = 0.495 \text{ s}$

Its constant horizontal speed is $v_x = \frac{x}{t} = \frac{2.00 \text{ m}}{0.495 \text{ s}} = 4.04 \text{ m/s}$

so before the string breaks $a_c = \frac{v_x^2}{r} = \frac{(4.04 \text{ m/s})^2}{0.300 \text{ m}} = \boxed{54.4 \text{ m/s}^2}$

***3.43** (a) $y_f = (\tan\theta_i)(x_f) - \dfrac{g}{2v_i^2\cos^2\theta_i}x_f^2$



Setting $x_f = d\cos\phi$, and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2$$

Solving for d yields,

$$d = \frac{2v_i^2\cos\theta_i[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i]}{g\cos^2\phi}$$

or $d = \boxed{\dfrac{2v_i^2\cos\theta_i\sin(\theta_i - \phi)}{g\cos^2\phi}}$

(b) Setting $\dfrac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45° + \dfrac{\phi}{2}}$ and $\boxed{d_{\max} = \dfrac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$

***3.44** (a) The ice chest floats downstream 2 km in time t, so that $2\text{ km} = v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15\text{ min}$. The downstream motion is described by $d + 2\text{ km} = (v + v_w)(t - 15\text{ min})$. We eliminate $t = 2\text{ km}/v_w$ and d by substitution:

$$(v - v_w)15\text{ min} + 2\text{ km} = (v + v_w)(2\text{ km}/v_w - 15\text{ min})$$

$$v(15\text{ min}) - v_w(15\text{ min}) + 2\text{ km} = \frac{v}{v_w}2\text{ km} + 2\text{ km} - v(15\text{ min}) - v_w(15\text{ min})$$

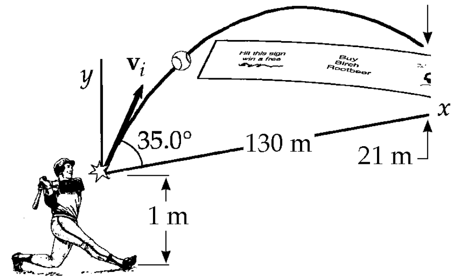
$$v(30\text{ min}) = \frac{v}{v_w}\ 2\text{ km}$$

$$v_w = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed v_w, traveling 2 km. Thus

$$v_w = \frac{\Delta x}{\Delta t} = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}.$$

3.45 Refer to the sketch:



(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i \cos 35.0°)t$

$\Delta y = v_{yi}t + \frac{1}{2}at^2$ substitution yields

$20.0 = (v_i \sin 35.0°)t + \frac{1}{2}(-9.80)t^2$

Solving the above gives $t = \boxed{3.81\text{ s}}$,

(a) $v_i = \boxed{41.7\text{ m/s}}$

(c) $v_{yf} = v_i \sin\theta_i - gt$ $\qquad v_x = v_i \cos\theta_i$

At $t = 3.81$ s, $\qquad v_{yf} = 41.7 \sin 35.0° - (9.80)(3.81) = \boxed{-13.4\text{ m/s}}$

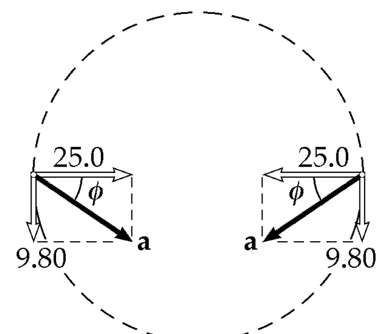
$$v_x = (41.7 \cos 35.0°) = \boxed{34.1\text{ m/s}}$$

$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.6\text{ m/s}}$$

***3.46** (a) $a_c = \frac{v^2}{r} = \frac{(5.00\text{ m/s})^2}{1.00\text{ m}} = \boxed{25.0\text{ m/s}^2}$

$a_t = g = \boxed{9.80\text{ m/s}^2}$

(b) See figure to the right.



(c) $a = \sqrt{a_c^2 + a_t^2} = \sqrt{(25.0\text{ m/s}^2)^2 + (9.80\text{ m/s}^2)^2} = \boxed{26.8\text{ m/s}^2}$

$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\frac{9.80\text{ m/s}^2}{25.0\text{ m/s}^2} = \boxed{21.4°}$

***3.47** $x_f = v_{ix}t = v_i t \cos 40.0°$

Thus, when $x_f = 10.0$ m, $\qquad t = \frac{10.0\text{ m}}{v_i \cos 40.0°}$

At this time, y_f should be $\qquad 3.05\text{ m} - 2.00\text{ m} = 1.05\text{ m}$

Thus, $\qquad 1.05\text{ m} = \frac{(v_i \sin 40.0°)10.0\text{ m}}{v_i \cos 40.0°} + \frac{1}{2}(-9.80\text{ m/s}^2)\left[\frac{10.0\text{ m}}{v_i \cos 40.0°}\right]^2$

From this, $\qquad v_i = \boxed{10.7\text{ m/s}}$

***3.48** Equation 3.15: $h = \dfrac{v_i^2 \sin^2 \theta_i}{2g}$

Equation 3.16: $R = \dfrac{v_i^2 \sin 2\theta_i}{g} = \dfrac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$

If $h = R/6$, Equation 3.15 yields $v_i \sin\theta_i = \sqrt{gR/3}$ (1)

Substituting Equation (1) above into Equation 3.16 gives $R = \dfrac{2\left(\sqrt{gR/3}\right)v_i \cos\theta_i}{g}$

which reduces to $v_i \cos\theta_i = \frac{1}{2}\sqrt{3gR}$ (2)

(a) From $v_{yf} = v_{yi} + a_y t$, the time to reach the peak of the path (where $v_{yf} = 0$) is found to be

$$t_{peak} = v_i \sin\theta_i / g$$

Using Equation (1), this gives $t_{peak} = \sqrt{R/3g}$

The total time of the ball's flight is then $\boxed{t_{flight} = 2t_{peak} = 2\sqrt{R/3g}}$

(b) At the path's peak, the ball moves horizontally with speed $v_{peak} = v_{xi} = v_i \cos\theta_i$

Using Equation (1), this becomes $v_{peak} = \boxed{\frac{1}{2}\sqrt{3gR}}$

(c) The initial vertical component of velocity is $v_{yi} = v_i \sin\theta_i$

From Equation (1), $v_{yi} = \boxed{\sqrt{gR/3}}$

(d) Squaring Eq. (1) and (2) and adding the results, $v_i^2\left(\sin^2\theta_i + \cos^2\theta_i\right) = \dfrac{gR}{3} + \dfrac{3gR}{4} = \dfrac{13gR}{12}$

Thus, the initial speed is $v_i = \boxed{\sqrt{\dfrac{13gR}{12}}}$

(e) Dividing Equation (1) by (2) yields $\tan\theta_i = \dfrac{v_i \sin\theta_i}{v_i \cos\theta_i} = \left[\dfrac{\left(\sqrt{gR/3}\right)}{\left(\frac{1}{2}\sqrt{3gR}\right)}\right] = \frac{2}{3}$

Therefore, $\theta_i = \tan^{-1}\left(\frac{2}{3}\right) = \boxed{33.7^\circ}$

(f) For a given initial speed, the projection angle yielding maximum peak height is $\theta_i = 90.0^\circ$. With the speed found in (d), Equation 3.15 then yields

$$h_{\max} = \frac{(13gR/12)\sin^2 90.0^\circ}{2g} = \boxed{\frac{13}{24}R}$$

(g) For a given initial speed, the projection angle yielding maximum range is $\theta_i = 45.0^\circ$. With the speed found in (d), Equation 3.16 then gives

$$R_{\max} = \frac{(13gR/12)\sin 90.0^\circ}{g} = \boxed{\frac{13}{12}R}$$

***3.49** Measure heights above the level ground.

The elevation y_b of the ball follows $$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $$y_b = R - gx^2 / 2v_i^2$$

(a) The elevation y_r of points on the rock is described by $$y_r^2 + x^2 = R^2$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$.

Then $$y_b^2 + x^2 > R^2$$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2 R}{v_i^2} + \frac{g^2 x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2 x^4}{4v_i^4} + x^2 > \frac{gx^2 R}{v_i^2}$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock:

$$1 > \frac{gR}{v_i^2}$$

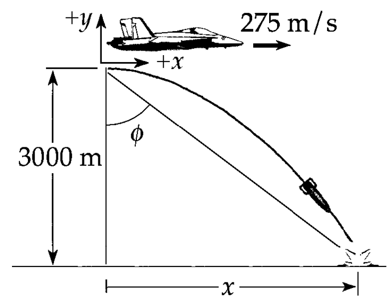
$$\boxed{v_i > \sqrt{gR}}$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $$0 = R - \frac{gx^2}{2gR}$$

or $$x = R\sqrt{2}$$

The distance from the rock's base is $$x - R = \boxed{(\sqrt{2} - 1)R}$$

3.50 (a) $\Delta y = -\frac{1}{2} g t^2$; $\Delta x = v_i t$



Combine the equations eliminating t: $\Delta y = -\frac{1}{2} g \left(\frac{\Delta x}{v_i}\right)^2$

. From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right) v_i^2$

thus $\Delta x = v_i \sqrt{\frac{-2\,\Delta y}{g}} = 275\sqrt{\frac{-2(-300)}{9.80}} = 6.80 \times 10^3 = \boxed{6.80 \text{ km}}$

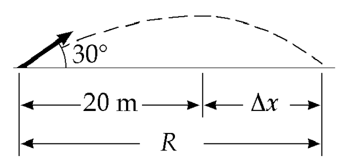
(b) The plane has the same velocity as the bomb in the x direction.

Therefore, the plane will be $\boxed{\text{3000 m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$

therefore, $\phi = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6800}{3000}\right) = \boxed{66.2°}$

3.51 The football travels a horizontal distance



$$R = \frac{v_i^2 \sin(2\theta_i)}{g} = \frac{(20.0)^2 \sin(60.0°)}{9.80} = 35.3 \text{ m}$$

Time of flight of ball is

$$t = \frac{2 v_i \sin\theta_i}{g} = \frac{2(20.0)\sin 30.0°}{9.80} = 2.04 \text{ s}$$

The receiver is Δx away from where the ball lands and $\Delta x = 35.3 - 20.0 = 15.3$ m

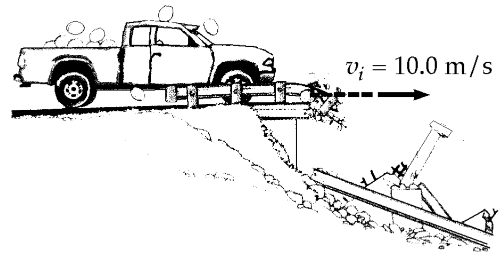
To cover this distance in 2.04 s, he travels with a velocity

$$v = \frac{15.3}{2.04} = \boxed{7.50 \text{ m/s in the direction the ball was thrown}}$$

***3.52** Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$y = -\frac{1}{2} g t^2$ (3)



Substitute for t from (2) into (3) $y = -\frac{1}{2} g \left(\frac{x^2}{v_i^2} \right)$

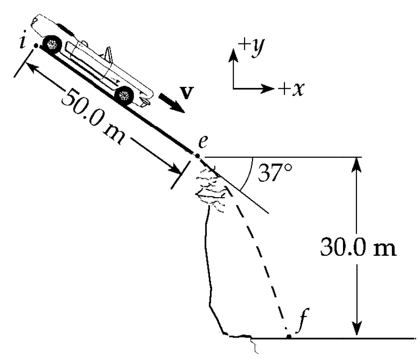
Equate y from the bank equation to y from the equations of motion:

$$16x = \left[-\frac{1}{2} g \left(\frac{x^2}{v_i^2} \right) \right]^2 \Rightarrow \frac{g^2 x^4}{4 v_i^4} - 16x = x \left(\frac{g^2 x^3}{4 v_i^4} - 16 \right) = 0$$

From this, $x = 0$ or $x^3 = \frac{64 v_i^4}{g^2}$ and $x = 4 \left(\frac{10^4}{9.80^2} \right)^{1/3} =$ **18.8 m**

Also, $y = -\frac{1}{2} g \left(\frac{x^2}{v_i^2} \right) = -\frac{1}{2} \frac{(9.80)(18.8)^2}{(10.0)^2} =$ **−17.3 m**

3.53 (a) While on the incline



$v_f^2 - v_i^2 = 2a\Delta x$ $\quad v_f - v_i = at$

$v_f^2 - 0 = 2(4.00)(50.0)$

$20.0 - 0 = 4.00t$

$v_f =$ **20.0 m/s** $\quad t =$ **5.00 s**

(b) Initial free–flight conditions give us $v_{xi} = 20.0 \cos 37.0° = 16.0$ m/s

and $v_{yi} = -20.0 \sin 37.0° = -12.0$ m/s

$v_{xf} = v_{xi}$ since $a_x = 0$

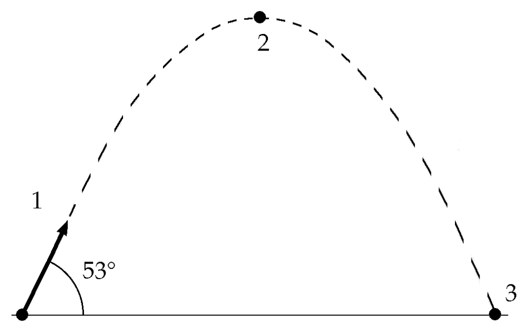
$$v_{yf} = -\sqrt{2 a_y \Delta y + v_{yi}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1 \text{ m/s}$$

$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(16.0)^2 + (-27.1)^2} =$ **31.5 m/s at 59.4° below the horizontal**

(c) $t_1 = 5$ s; $\quad t_2 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-27.1 + 12.0}{-9.80} = 1.54$ s $\quad t = t_1 + t_2 =$ **6.54 s**

(d) $\Delta x = v_{xi} t_1 = 16.0(1.54) =$ **24.6 m**

3.54 Consider the rocket's trajectory in 3 parts as shown in the sketch. Our initial conditions give:



$$a_y = 30.0 \sin 53.0° = 24.0 \text{ m/s}^2$$
$$a_x = 30.0 \cos 53.0° = 18.1 \text{ m/s}^2$$

$$v_{yi} = 100 \sin 53.0° = 79.9 \text{ m/s}$$
$$v_{xi} = 100 \cos 53.0° = 60.2 \text{ m/s}$$

The distances traveled during each phase of the motion are given in the table.

Path #1: $v_{yf} - 79.9 = 24.0(3.00)$ or $v_{yf} = 152 \text{ m/s}$

$v_{xf} - 60.2 = 18.1(3.00)$ or $v_{xf} = 114 \text{ m/s}$

$$\Delta y = 79.9(3.00) + \frac{1}{2}(24.0)(3.00)^2 = 347 \text{ m}$$

$$\Delta x = 60.2(3.00) + \frac{1}{2}(18.1)(3.00)^2 = 262 \text{ m}$$

Path #2: $a_x = 0,\ v_{xf} = v_{xi} = 114 \text{ m/s}$

$0 - 152 = -(9.80)t$ or $t = 15.5 \text{ s}$

$$\Delta x = 114(15.5) = 1.77 \times 10^3 \text{ m};$$

$$\Delta y = 152(15.5) - \frac{1}{2}(9.80)(15.5)^2 = 1.17 \times 10^3$$

Path #3: $(v_{yf})^2 - 0 = 2(-9.80)(-1.52 \times 10^3)$

$v_{yf} = -173 \text{ m/s}$ $v_{xf} = v_{xi} = 114 \text{ m/s, since } a_x = 0$

$-173 - 0 = -(9.80)t$ or $t = 17.6 \text{ s}$

$$\Delta x = 114(17.6) = 2.02 \times 10^3 \text{ m}$$

(a) $\Delta y(\text{max}) = \boxed{1.52 \times 10^3 \text{ m}}$

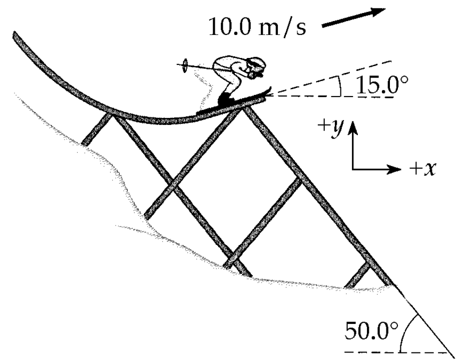
(b) $t(\text{net}) = 3.00 + 15.5 + 17.6 = \boxed{36.1 \text{ s}}$

(c) $\Delta x(\text{net}) = 262 + 1.77 \times 10^3 + 2.02 \times 10^3$

$\Delta x(\text{net}) = \boxed{4.05 \times 10^3 \text{ m}}$

	Path Part		
	#1	#2	#3
a_y	24.0	–9.80	–9.80
a_x	18.1	0.0	0.00
v_{yf}	152	0.0	–173
v_{xf}	114	114	114
v_{yi}	79.9	152	0.00
v_{xi}	60.2	114	114
Δy	347	1.17×10^3	-1.52×10^3
Δx	262	1.77×10^3	2.02×10^3
t	3.00	15.5	17.6

3.55 (a) $\Delta x = v_{xi}t, \qquad \Delta y = v_{yi}t + \frac{1}{2}gt^2$



$$d\cos 50.0° = (10.0\cos 15.0°)t$$

and $$-d\sin 50.0° = (10.0\sin 15.0°)t + \frac{1}{2}(-9.80)t^2$$

Solving, $d = \boxed{43.2\text{ m}}$ and $t = 2.88$ s

(b) Since $a_x = 0$,

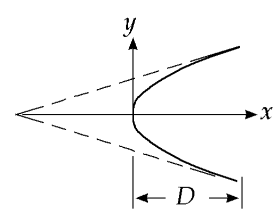
$$v_{xf} = v_{xi} = 10.0\cos 15.0° = \boxed{9.66\text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 10.0\sin 15.0° - 9.80(2.88) = \boxed{-25.6\text{ m/s}}$$

Air resistance would decrease the values of the range and maximum height.

As an airfoil, he can get some lift and increase his distance.

***3.56** For one electron, we have



$y = v_{iy}t$, $D = v_{ix}t + \frac{1}{2}a_x t^2 \cong \frac{1}{2}a_x t^2$, $v_{yf} = v_{yi}$, and $v_{xf} = v_{xi} + a_x t \cong a_x t$.

The angle its direction makes with the x-axis is given by

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\frac{v_{yi}}{a_x t} = \tan^{-1}\frac{v_{yi}t}{a_x t^2} = \tan^{-1}\frac{y}{2D}$$

Thus the horizontal distance from the aperture to the virtual source is $2D$. The source is at coordinate $\boxed{x = -D}$.

3.57 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s.

Its speed is $$\frac{\frac{1}{4}(2\pi)(0.6\text{ m})}{0.1\text{ s}} \cong 9\text{ m/s}$$

and its centripetal acceleration is $$\frac{v^2}{r} \cong \frac{(9\text{ m/s})^2}{0.6\text{ m}}\ \boxed{\sim 10^2\text{ m/s}^2}$$

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

3.58 Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are $> 45°$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$ $\qquad$ $x_f = v_{xi}t = v_i(\cos\theta)t$

Thus $\qquad t = \dfrac{x_f}{v_i \cos\theta}$

Substitute into the expression for y_f $\qquad y_f = v_i(\sin\theta)\dfrac{x_f}{v_i\cos\theta} - \dfrac{1}{2}g\left(\dfrac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2\cos^2\theta}$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ $\qquad$ so $\qquad y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$$

Substitute values, use the quadratic formula and find

$\tan\theta = 3.905$ or 1.197, which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$

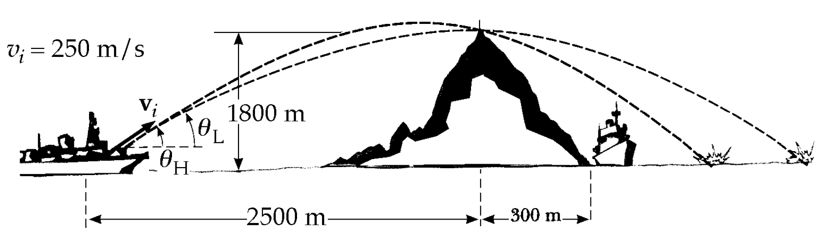
Range $(\text{at } \theta_H) = \dfrac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3$ m from enemy ship

$3.07\times10^3 - 2500 - 300 = 270$ m from shore

Range $(\text{at } \theta_L) = \dfrac{v_i^2\sin 2\theta_L}{g} = 6.28\times10^3$ m from enemy ship

$6.28\times10^3 - 2500 - 300 = 3.48\times10^3$ from shore

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48\times10^3 \text{ m}}$ from the shore.



ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) $(1.00\,\mathbf{i} + 0.750\,\mathbf{j})$ m/s (b) $(1.00\,\mathbf{i} + 0.500\,\mathbf{j})$ m/s, 1.12 m/s

4. (a) $(2.00\,\mathbf{i} + 3.00\,\mathbf{j})$ $\mathrm{m/s^2}$ (b) $\mathbf{r} = \left(3.00t + t^2\right)\mathbf{i}\text{ m} + \left(1.50t^2 - 2t\right)\mathbf{j}\text{ m}$

6. (a) $\mathbf{r} = \left[5.00t\,\mathbf{i} + \frac{1}{2}\left(3.00t^2\right)\mathbf{j}\right]$ m; $\mathbf{v} = \left[5.00\,\mathbf{i} + 3.00t\,\mathbf{j}\right]$ m/s
(b) (10.0 m, 6.00 m), 7.81 m/s

8. (a) 22.9 m/s (b) 360 m from the base of the cliff
(c) $\mathbf{v} = (114\,\mathbf{i} - 44.3\,\mathbf{j})$ m/s

10. (a) $v = d\sqrt{g/2h}$ horizontally (b) $\theta = \tan^{-1}(2h/d)$ below the horizontal

12. 0.600 $\mathrm{m/s^2}$ down

14. 22.4° or 89.4°

16. (a) 20.0° (b) 3.05 s

18. 25.8 m/s

20. 9.91 m/s

22. (a) 0.852 s (b) 3.29 m/s (c) 4.03 m/s
(d) 50.8° (e) 1.12 s

24. (a) 1.02×10^3 m/s (b) 2.72×10^{-3} $\mathrm{m/s^2}$ toward the Earth

26. 0.033 7 $\mathrm{m/s^2}$ toward the center of the Earth

28. 0.281 rev/s

30. (a) 1.25 $\mathrm{m/s^2}$ toward the center (b) 0.400 $\mathrm{m/s^2}$ forward
(c) 1.31 $\mathrm{m/s^2}$ forward and 72.3° inward

32. (a) See the solution. (b) 29.7 $\mathrm{m/s^2}$
(c) 6.67 m/s at 36.9° above the horizontal

34. 18.0 s

36. (a) 36.9° (b) 41.6° (c) 3.00 min

38. $t_{\text{Alan}} = \dfrac{2L/c}{1 - v^2/c^2}$, $t_{\text{Beth}} = \dfrac{2L/c}{\sqrt{1 - v^2/c^2}}$. Beth returns first.

40. (a) 1.69 km/s (b) 1.80 h

42. 54.4 $\mathrm{m/s^2}$

44. 4.00 km/h

46. (a) $25.0\ \text{m/s}^2;\ \ 9.80\ \text{m/s}^2$ (b) See the solution.
(c) $26.8\ \text{m/s}^2$ inward at 21.4° below the horizontal

48. (a) $2\sqrt{R/3g}$ (b) $\frac{1}{2}\sqrt{3gR}$ (c) $\sqrt{gR/3}$
(d) $\sqrt{\frac{13}{12}gR}$ (e) 33.7° (f) $\frac{13}{24}R$
(g) $\frac{13}{12}R$

50. (a) 6.80 km (b) 3.00 km vertically above the impact point
(c) 66.2°

52. (18.8, –17.3) m

54. (a) 1.52 km (b) 36.1 s (c) 4.05 km

56. $x = -D$

58. Safe distances are less than 270 m or greater than 3.48×10^3 m from the western shore.