

## Tutorial Sheet 25 (Binomial Distribution)

1. Let  $X$  be a Binomial random variable with distribution  $\text{Bin}(5, 0.3)$ . Find
  - a)  $P(X = 4)$
  - b)  $P(X > 1)$
  - c)  $P(X \geq 4)$
  
2. Suppose that in a certain company, the probability of any member of staff taking a sick leave is 0.01. If there are 6 clerks in the company, find the probability that
  - a) none of them takes a sick leave
  - b) exactly two of them take a sick leave
  - c) more than one of them take a sick leave.
  
3. A salesman visits 8 customers to sell a product. The probability that any customer orders the product is  $\frac{1}{6}$ .
  - a) Tabulate the probability that  $x$  customers order the product for values of  $x$  from 0 to 8.
  - b) Find the probability that more than half of the customers will order the product.
  - c) Find the most likely number of customers that will order the product.
  
- \*4. Consider a binomial random variable  $X \sim \text{Bin}(n, \theta)$ .
  - a) Write down  $p(x)$  and  $p(x + 1)$  in terms of  $n$ ,  $x$  and  $\theta$ .
  - b) Show that  $p(x + 1) = p(x) \left( \frac{n - x}{x + 1} \cdot \frac{\theta}{1 - \theta} \right)$ .
  - c) Show that  $p(x + 1) > p(x)$  if and only if  $x < (n + 1)\theta - 1$ .
  - d) Using the result of (c), find the maximum value of  $p(x)$  in each of the following cases:
    - (i)  $n = 10, \theta = 0.1$
    - (ii)  $n = 10, \theta = 0.5$

\*Optional