

Tutorial Sheet 25

(Binomial Distribution)

1. Let X be a Binomial random variable with distribution $\text{Bin}(5, 0.3)$. Find
 - a) $P(X = 4)$
 - b) $P(X > 1)$
 - c) $P(X \geq 4)$

2. Suppose that in a certain company, the probability of any member of staff taking a sick leave is 0.01. If there are 6 clerks in the company, find the probability that
 - a) none of them takes a sick leave
 - b) exactly two of them take a sick leave
 - c) more than one of them take a sick leave.

3. A salesman visits 8 customers to sell a product. The probability that any customer orders the product is $\frac{1}{6}$.
 - a) Tabulate the probability that x customers order the product for values of x from 0 to 8.
 - b) Find the probability that more than half of the customers will order the product.
 - c) Find the most likely number of customers that will order the product.

- *4. Consider a binomial random variable $X \sim \text{Bin}(n, \theta)$.
 - a) Write down $p(x)$ and $p(x + 1)$ in terms of n , x and θ .
 - b) Show that $p(x + 1) = p(x) \left(\frac{n - x}{x + 1} \cdot \frac{\theta}{1 - \theta} \right)$.
 - c) Show that $p(x + 1) > p(x)$ if and only if $x < (n + 1)\theta - 1$.
 - d) Using the result of (c), find the maximum value of $p(x)$ in each of the following cases:
 - (i) $n = 10, \theta = 0.1$
 - (ii) $n = 10, \theta = 0.5$

*Optional