Tutorial Sheet 12
(Answers)

1. a) The relation clearly is not reflexive and clearly is symmetric. It is not antisymmetric since both (2, 4) and (4, 2) are in the relation. It is not transitive, since although (2, 4) and (4, 2) are in the relation, (2, 2) is not.

   b) This relation is clearly not reflexive. It is not symmetric, since, for instance, (1, 2) is included but (2, 1) is not. It is antisymmetric, since there are no cases of (a, b) and (b, a) both being in the relation. It is not transitive, since although (1, 2) and (2, 3) are in the relation, (1, 3) is not.

   c) This relation is clearly reflexive and symmetric. It is trivially antisymmetric since there are no pairs (a, b) in the relation with $a \neq b$. It is trivially transitive.

2. a) The relation is not reflexive since it is not the case that $1 \neq 1$, for instance. It is symmetric: if $x \neq y$, then of course $y \neq x$. It is not antisymmetric, since, for instance, $1 \neq 2$ and also $2 \neq 1$. It is not transitive, since $1 \neq 2$ and $2 \neq 1$, for instance, but it is not the case that $1 \neq 1$.

   b) This relation is not reflexive, since (0, 0) is not included. It is symmetric, because the commutative property of multiplication tells us that $xy = yx$, so that one of these quantities is greater than or equal to 1 if the other is. It is not antisymmetric, since, for instance, (2, 3) and (3, 2) are both included. It is transitive.

   c) This relation is not reflexive, since (1, 1) is not included, for instance. It is symmetric; the equation $x = y - 1$ is equivalent to the equation $y = x + 1$, which is the same as the equation $x = y + 1$ with the roles of x and y reversed. It is not antisymmetric, since, for instance, both (1, 2) and (2, 1) are in the relation. It is not transitive, since, for instance, although both (1, 2) and (2, 1) are in the relation, (1, 1) is not.

3. a) This is reflexive (and not irreflexive), since the entries on the main diagonal are all 1's. It is symmetric, since the matrix is symmetric about the main diagonal. It is not antisymmetric, since there is a pair of 1's symmetrically placed on each side of the diagonal. It is transitive; this is most easily seen by drawing the directed graph.

   b) This relation is neither reflexive nor irreflexive, since the main diagonal is neither all 1's nor all 0's. It is not symmetric, but it is antisymmetric. It is trivially transitive.

   c) This relation is clearly symmetric, but it satisfies none of the other properties listed.