Tutorial Sheet 8
(Proof Techniques)

1.   Use direct proof to show that if the sum of any two integers is even, then so is their difference.

2.   Find the mistake in the following "proof".

   Theorem: The product of an even integer and an odd integer is even.
   Proof:
           Suppose m is an even integer and n is an odd integer. If $m \times n$ is even, then by definition of even there exists an integer r such that $m \times n = 2r$. Also since m is even, there exists an integer p such that $m = 2p$, and since n is odd there exists an integer q such that $n = 2q + 1$. Thus
$$m \times n = (2p)(2q+1) = 2r.$$

   where r is an integer. By definition of even, then, $m \times n$ is even.

3.   Prove by contraposition that if a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10.

4.   Prove by contraposition that if a sum of two real numbers is less than 50, then at least one of the numbers is less than 25.

5.   Prove that if a, b and c are integers and a divides b, then a divides bc.

6.   Prove that $n^3 + n$ is even if n is an integer.
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7.   Find the mistake in the following "proof".

   Theorem: Every integer is rational is incorrect.
   Proof
           Suppose not. Suppose every integer is irrational. Then the integer 1 is irrational. But $1 = 1/1$, which is rational. This is a contradiction.
   Hence the supposition is false and the theorem is true.