Solution of Test 2 (2001\2002)

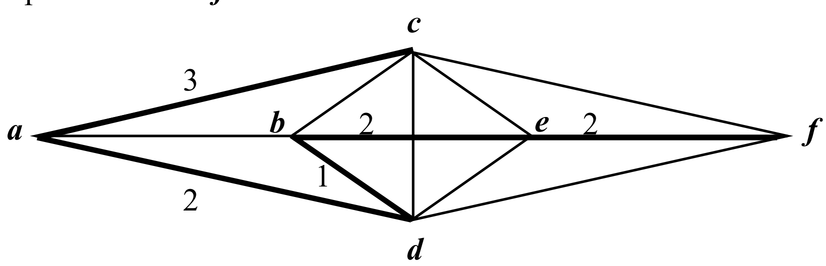
1. (a) (i) R is **not reflexive** since the statement $(2-1)(2-1)>1$ is false, i.e. $(2,\,2)\notin R$.
   (ii) R is **symmetric** since $(x-1)(y-1)>1$ then $(y-1)(x-1)>1$, i.e. $\forall(x,\,y)\in R,\,(y,\,x)\in R$.
   (iii) R is **not anti-symmetric** since $\exists(x,\,y)\in R$ and $(y,\,x)\in R$.
   (iv) R is **not transitive** since $(2-1)(3-1)>1$ and $(3-1)(2-1)>1$ but $(2-1)(2-1)\le 1$.

(b) (i) $\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$ (ii) Number of walks $=\begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}=2$ (iii) ***acb*** and ***adb***

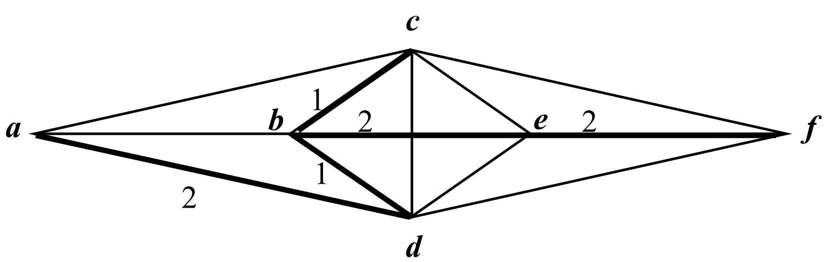
2. (a)

Solved Nodes Directly Connected to Unsolved Nodes	Closest Connected Unsolved Node	Total Distance Involved	nth Nearest Node	Minimum Distance	Last Connection
a	***d***	2	***d***	2	***ad***
a ***d***	***c*** ***b***	3 2 + 1 = 3	***c*** ***b***	3 3	***ac*** ***db***
c ***d*** ***b***	***e*** ***e*** ***e***	3 + 3 = 6 2 + 4 = 6 3 + 2 = 5	 ***e***	 5	 ***be***
e ***c*** ***d***	***f*** ***f*** ***f***	5 + 2 = 7 3 + 5 = 8 2 + 6 = 8	***f***	7	***ef***

The shortest path from ***a*** to ***f***:



(b) The minimum weight is **8** and the minimum spanning tree is as shown below:



3. (a) ${}_5P_3=60$ (b) $\dfrac{5!}{2!\times 3!}=10$ (c) ${}_4P_3+{}_3C_1\times\dfrac{3!}{2!}=33$