Vidyalankar Institute of Technology

Prelim Question Paper : S.E. [INFT/CMPN] - Maths IV

Time : 3 Hrs.] **[Marks : 100**

N.B. : 1. Q No.**1** is **compulsory.**
2. Answer any **four** from the remaining.
3. Figures to the **right** indicate full marks.

1. (a) Define : **[3]**
(i) Absolute error
(ii) Relative error
(iii) Percentage error

(b) Prove that $\Delta\nabla = \delta^2 = \nabla\Delta$ **[2]**

(c) Apply Gauss elimination method to solve the equations **[4]**

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

(d) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ **[4]**

(e) Prove that characteristics roots of Hermitian matrix are real. **[3]**

(f) Evaluate $\int_{-2}^{2} \frac{2z-3}{z} dz$ when the path C is the upper half of the circle $|z| = 2$. **[4]**

2. (a) State and prove Cauchy's integral formula and hence evaluate $\oint_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{z^2 + 3z + 2} dz$

where C : (i) $|z| = 1/2$ (ii) $|z| = 3$ **[7]**

(b) Evaluate the following integrals using residue theorem

(i) $\oint_C \frac{dz}{4z^2 + 1}$ where C : $|z| = 1$ **[4]**

(ii) $\oint_C \frac{[z+4]^2}{z^4 + 5z^3 + 6z^2}$ where C : $|z| = 1$ **[3]**

(c) Expand $\frac{z^2 - 1}{z^2 + 5z + 6}$ about $z = 0$ **[6]**

3. (a) Choose the correct answer with proper justifications **[6]**

(i) The value of the complex integral $\int_C \tanh z dz$ where C is $|z| = 3$ is

(a) πi (b) $2\pi i$ (c) 0 (d) $4\pi i$

(ii) The residue of the f^n $f(z) = z^2 e^{1/e}$ at the point $z = 0$ is,

(a) $-1/6$ (b) $1/6$ (c) 0 (b) 00

(b) Expand $f(z) = \frac{1}{z^3 + 3z^2 + 2z}$ as Laurent's series about $z = 0$ for **[6]**

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(c) Using Contour Integration prove that – **[4]**

$$\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta} = \pi/2$$

4. (a) If $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ find the characteristic roots and the corresponding characteristic vectors of A. **[7]**

(b) If λ is a eigenvalue of a matrix A with corresponding eigenvector X, then prove that λ^n is eigenvalue of A^n with corresponding eigenvector X. Hence, find characteristic roots of A^4 where $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **[6]**

(c) Define the following : **[5]**

i) Characteristic matrix of matrix A.
ii) Characteristic Polynomial of matrix A.
iii) Characteristic equation of matrix A.
iv) Characteristic roots of matrix A.
v) Characteristic vector of matrix A.

5. (a) If λ is an eigen value of a non singular matrix A then prove that $1/\lambda$ is an eigen value of A^{-1} and find the eigen values and eigen vectors of A^{-1}, where $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ **[10]**

(b) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ **[6]**

and hence find A^{-2} and A^4 if it exists.

(c) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory and find its minimal polynomials. **[4]**

6. (a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using **[7]**

(i) Trapezoidal rule
(ii) Simpson's 1/3 rule
(iii) Simpson's 3/8 rule.

(b) A solid of revolution is formed rotating about the x–axis, the area between the x–axis, the lines x = 0 and x = a, and a curve, through the points (0, 1), (0.25, 0.9896), (0.50, 0.9589), (0.75, 0.9089) and (1, 0.8415). Estimate the volume of the solid formed. **[5]**

(c) Explain the geometrical significance of Trapezoidal rule and explain the occurrence of error and its minimization in Trapezoidal rule. **[4]**

7. (a) Using Newton's interpolation formula find the value of f(x) when x = 1.85 and 2.35 from the following table. **[5]**

x :	1.7	1.8	1.9	2.0	2.1	2.2	2.3
f(x) :	5.474	6.050	6.686	7.389	8.166	9.025	9.974

(b) Use Lagrange's formula to find the value of y when x = 10 from the following data. **[5]**

x :	5	6	9	11
y :	12	13	14	16

(c) Solve the following by Euler's modified method

$$\frac{dy}{dx} = \log(x+y), \qquad y(0) = 2 \quad \text{at } x = 0.8 \text{ taking } h = 0.2$$ **[7]**

(d) Evaluate $\Delta^2\left(\frac{5x+12}{x^2+5x+16}\right)$ **[3]**

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