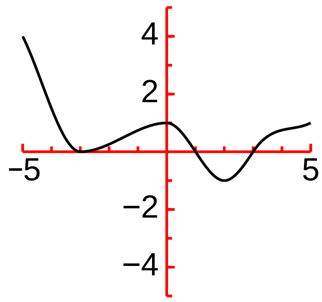
Math 1120 Fall 2000: Sample Midterm 2

1. $f'(x) > 0$ for $x \in (-\infty, -3) \cup (-3, -1) \cup (2, \infty)$ and $f''(x) > 0$ for $x \in (-3, -2) \cup (1, 3.5)$.

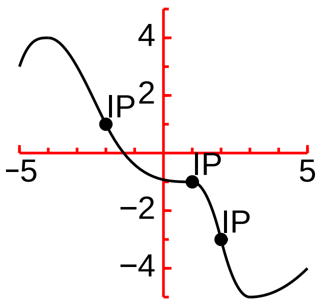
2. $f(x)$ is increasing on $(-\infty, 1) \cup (3, \infty)$, decreasing on $(1, 3)$, concave up on $(-3, 0) \cup (2, 4)$ and concave down on $(-\infty, -3) \cup (0, 2) \cup (4, \infty)$. Where f is increasing, f' is positive, where f is decreasing, f' is negative, where f is concave up f' is increasing and where f is concave down, f' is decreasing.

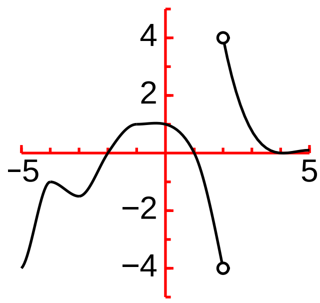


3. (a) Since $f'(x) < 0$ for $1 < x \leq 2$, f is decreasing on the interval $(1, 2)$ and so $f(2) < f(1)$.

   (b) $f'(x) > 0$, and hence f is increasing for $x \in (-\infty, -4) \cup (3, \infty)$, $f'(x) < 0$, and hence f is decreasing for $x \in (-4, 1) \cup (1, 3)$. f' is increasing, f is concave up, and $f''(x) > 0$ for $x \in (-2, 1) \cup (2, 4.2)$, f' is decreasing, f is concave down and $f''(x) < 0$ for $x \in (-\infty, -2) \cup (1, 2) \cup (4.2, \infty)$. $f'(x)$ is concave up and $f''(x)$ is increasing for $x \in (-\infty, -4) \cup (-3, -1) \cup (4, 2, \infty)$, $f'(x)$ is concave down, and hence $f''(x)$ decreasing for $x \in (-4, -3) \cup (-1, 2) \cup (2, 5)$. At $x = 2$ the graph of $f'(x)$ comes to a point, so there should be a discontinuity in the graph of $y = f''(x)$ at $x = 2$.

f f''





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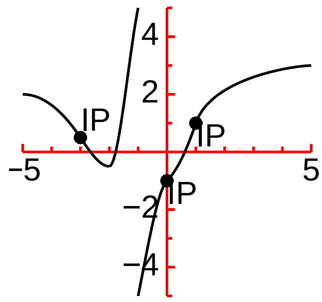
4.



5. (a) $\lim_{x\to 0} \dfrac{\frac{x-1}{x^2+1}+1}{x} = \lim_{x\to 0} \dfrac{\frac{x-1+x^2+1}{x^2+1}}{x} = \lim_{x\to 0} \dfrac{x+x^2}{x(x^2+1)} = \lim_{x\to 0} \dfrac{1+x}{x^2+1} = 1$

(b) $\lim_{x\to 0} \dfrac{x}{\sqrt{16+x}-\sqrt{16-x}} = \lim_{x\to 0} \dfrac{x}{\sqrt{16+x}-\sqrt{16-x}} \dfrac{\sqrt{16+x}+\sqrt{16-x}}{\sqrt{16+x}+\sqrt{16-x}} =$
$\lim_{x\to 0} \dfrac{x(\sqrt{16+x}+\sqrt{16-x})}{16+x-(16-x)} = \lim_{x\to 0} \dfrac{x(\sqrt{16+x}+\sqrt{16-x})}{2x} = \lim_{x\to 0} \dfrac{\sqrt{16+x}+\sqrt{16-x}}{2} = 4$

6. $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (x^2+1) = 2$, $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} (1-x) = 2$. Since the two 1-sided limits are both equal to 2, $\lim_{x\to -1} f(x) = 2$.

7. We need to take one-sided limits. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x^2+1)\dfrac{\sin x}{x} = \lim_{x\to 0^-} (x^2+1) \lim_{x\to 0^-} \dfrac{\sin x}{x} = 1\cdot 1 = 1$. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (1-x) = 1$. Since the two one-sided limits are both 1, $\lim_{x\to 0} f(x) = 1$. Since $f(0) = 1$, $\lim_{x\to 0} f(x) = f(0)$.

8. (a) $\lim_{x\to -\infty} \dfrac{4x^3+3x^2-x+7}{x^2+5x-2} \lim_{x\to -\infty} \dfrac{4x+3-1/x+7/x^2}{1+5/x-2/x^2} = -\infty$

(b) $\lim_{x\to -\infty} \dfrac{\sqrt[3]{x^4+x^2-3}}{x-7} = \lim_{x\to -\infty} \dfrac{\sqrt[3]{x^4}\sqrt[3]{1+1/x^2-3/x^4}}{x(1-7/x)} = \lim_{x\to -\infty} \dfrac{x^{4/3}\sqrt[3]{1+1/x^2-3/x^4}}{x(1-7/x)} =$
$\lim_{x\to -\infty} \dfrac{x^{1/3}\sqrt[3]{1+1/x^2-3/x^4}}{1-7/x} = -\infty$

(c) $f(x) = x\dfrac{x-5}{|x-5|}$. Treating this as a piecewise function, $f(x) = \begin{cases} x\frac{-(x-5)}{x-5} & \text{if } x<5 \\ x(\frac{x-5}{x-5}) & \text{if } x>5 \end{cases}$, or $f(x) = \begin{cases} -x & \text{if } x<5 \\ x & \text{if } x>5 \end{cases}$. $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} (-x) = -5$, $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^-} x = 5$.

9. $\lim_{h\to 0} \dfrac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \dfrac{(5+h)/((5+h)^2-16)-5/9}{h} = \lim_{h\to 0} \dfrac{9(5+h)-5((5+h)^2-16)}{9h((5+h)^2-16)} =$
$\lim_{h\to 0} \dfrac{45+9h-5(9+10h+h^2)}{9h((5+h)^2-16)} = \lim_{h\to 0} \dfrac{-41h-5h^2}{9h((5+h)^2-16)} = \lim_{h\to 0} \dfrac{-41-5h}{9((5+h)^2-16)} = -\dfrac{41}{81}$