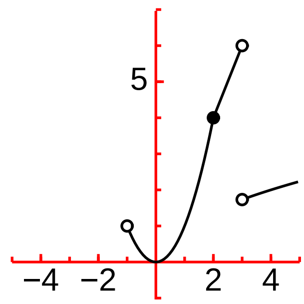
Math 1120, Fall 2000: Solutions to Sample Midterm 1

1. (a) y is not defined as a function of x since if we solve for y we find that $y = \pm\sqrt{x}$ and there are two possible choices of y for almost every x.

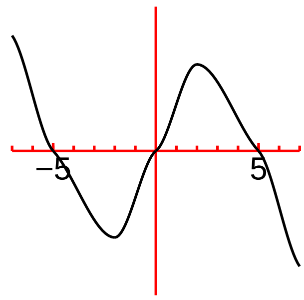
   (b) Only the first, second and fourth graphs represent functions. With each of these, if you choose any x in the domain of the function, you have only one associated y-value. For the third graph, this is violated.

2.



3. Again sketching this on computer is not easy... Recall that adding two to a function shifts it up by 2 and adding two to the argument shifts the graph to the left by two.

4.



5. The lengths of the sides of the box are x, $50-2x$ and $100-2x$. The volume is $V = x(50-2x)(100-2x)$.

6. The ranges of these functions were generally not easy to find, so they were ignored in grading.

   (a) Domain: $(0, \infty)$ ($\ln x$ is only defined if $x > 0$).

   (b) Domain: $(-\infty, 0) \cup (0, \infty)$.

   (c) Domain: all reals.

7. (a) $7 - 3 = 4^{x^2}$, $4 = 4^{x^2}$, $\log_4(4) = x^2 \log_4(4)$, $1 = x^2$ and $x = \pm 1$.

   (b) $4^{\log_4(x^2-x)} = 4^2$, $x^2 - x = 16$, $x^2 - x - 16 = 0$. $x = \dfrac{1 \pm \sqrt{1+4(16)}}{2} = \dfrac{1 \pm \sqrt{65}}{2}$. Either $x = -3.53$ or $x = 4.53$. Substituting into the original equation (to make sure we do not have nonsense),

$\log_4(-3.53^2 + 3.53) = \log_4(16) = 2$ and $\log_4(4.53^2 - 4.53) = \log_4(16) = 2$. Both roots work and so both are solutions.

8. $f \circ g$ will be even since $f(g(-x)) = f(-g(x)) = f(g(x))$ and $f(x)g(x)$ will be odd since $f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x)$.

9. Recall the Law of Cosines: if we label the friend by F the runner by R and the center of the circle by C, then $CF^2 = RF^2 + RC^2 - 2(RF)(RC)\cos\theta$, or $300^2 = RF^2 + 100^2 - 200RF\cos\frac{\pi}{6}$. Rearranging, $RF^2 - 100\sqrt{3}RF - 80000 = 0$. By the quadratic formula, $RF = \dfrac{100\sqrt{3} \pm \sqrt{30000 + 320000}}{2} = \dfrac{100\sqrt{3} \pm 100\sqrt{35}}{2} = 50\sqrt{3} \pm 50\sqrt{35}$. Since $50\sqrt{3} - 50\sqrt{35}$ is negative we only have one possible solution, namely $RF = 50(\sqrt{3} + \sqrt{35}) \sim 382.4$m.

   The distance between the runner and the friend is 382.4m

10. The function is decreasing for $x \in (-3, 0)$ and increasing for $x \in (0, 2) \cup (2, \infty)$. It is concave up for $x \in (-3, -2) \cup (1, 1) \cup (2, 3)$ and concave down for $x \in (-2, -1) \cup (1, 2) \cup (3, \infty)$.