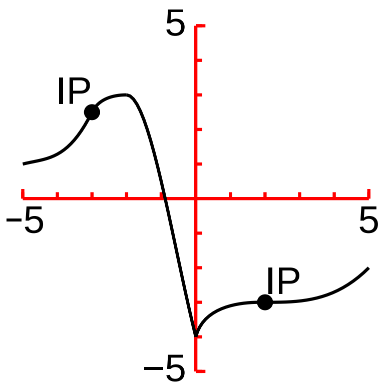
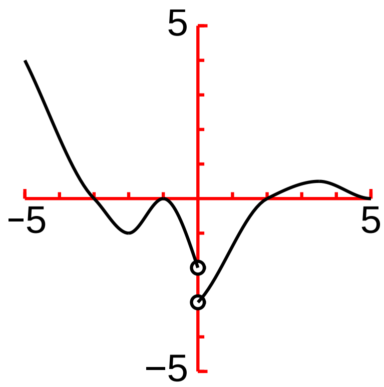
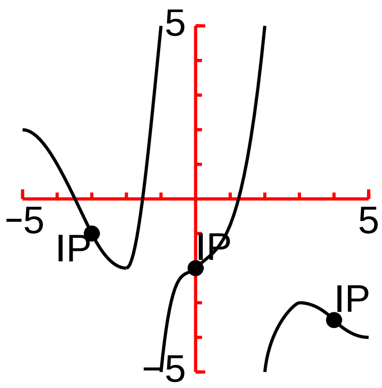
Math 1120 Fall 2000: Midterm 2 Review

1. We first see what the graph tells us:
$f'(x) = 0$ for $x = -2, 2$, $f'(x) > 0$ for $x \in (-\infty, -2) \cup (0, 2) \cup (2, \infty)$ and $f'(x) < 0$ for $x \in (-2, 0)$. $f''(x) > 0$ for $x \in (-\infty, -3) \cup (2, \infty)$ and $f''(x) < 0$ for $x \in (-3, -1) \cup (-1, 0) \cup (0, 2)$. There is also a jump in $f'(x)$ at $x = 0$, which means f will come to a point at $x = 0$. $\lim_{x\to\infty} f'(x) = 1$, so $f(x)$ will approach a line as x gets large. $f'(x)$ is concave up for $x \in (-2, -1) \cup (0, 3.5)$ and $f'(x)$ is concave down for $x \in (-5, -2) \cup (-1, 0) \cup (3.5, \infty)$. Putting all this together,





2.



3. $f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{(3+h)^2 + 3 - (9+3)}{h} =$
$\lim_{h\to 0} \frac{9 + 6h + h^2 + 3 - 12}{h} = \lim_{h\to 0} \frac{6h + h^2}{h} = \lim_{h\to 0}(6 + h) = 6.$
The tangent line is $y - f(3) = f'(3)(x - 3)$, $y - 12 = 6(x - 3)$ or $y = 6x - 6$.

4. Notice that $f'(x) > 1$ for all $x \neq 0$, and particularly for all $x \geq 3$. The line $l(x) = x - 2$ is such that $l(3) = 1 = f(3)$ and $l'(x) = 1 < f'(x)$ for $x \geq 3$, so by the Racetrack Principle, $f(5) > l(5) = 3$.