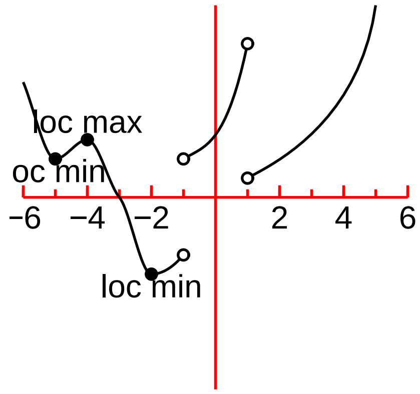
Math 1120 Fall 2000: Solutions to Midterm 2

1. (a) $f'(x) > 0$ for $x \in (-\infty, -3) \cup (-1, \infty)$, $f'(x) < 0$ for $x \in (-3, -1)$.

   (b) $\lim_{x \to -1^-} f(x) = -2$, $\lim_{x \to -1^+} f(x) = -4$

   (c) $f''(x) > 0$ for $x \in (-5, -4) \cup (-2, -1) \cup (-1, 1) \cup (1, \infty)$ and $f''(x) < 0$ for $x \in (-\infty, -5) \cup (-4, -2)$. $f''(x) > 0$ tells us that $f'(x)$ is increasing, and $f''(x) < 0$ tells us that $f'(x)$ is decreasing.

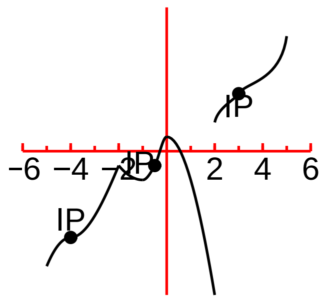
   (c)



2. (a) $f(x)$ is increasing for $x \in (-\infty, -4) \cup (-4, -2) \cup (-1, 0) \cup (2, \infty)$, decreasing for $x \in (-2, -1) \cup (0, 2)$, concave up for $x \in (-4, -2) \cup (-2, -0.5) \cup (3, \infty)$ and concave down for $x \in (-\infty, -5) \cup (-0.5, 2) \cup (2, 3)$.

   (b) From the graph, $f'(1) = -2$ and so the tangent line is $y - 3 = -2(x - 1)$, or $y = -2x + 5$.

   (c)



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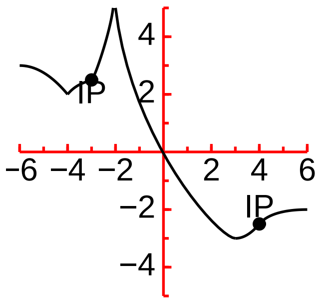
3.



4. (a) $\lim_{x\to 2} \dfrac{x^2+5x-14}{x^2-3x+2} = \lim_{x\to 2} \dfrac{(x+7)(x-2)}{(x-2)(x-1)} = \lim_{x\to 2} \dfrac{x+7}{x-1} = 9$

(b) $\lim_{x\to 0} \dfrac{\sqrt{9+x^2}-3}{x^2} = \lim_{x\to 0} \dfrac{\sqrt{9+x^2}-3}{x^2}\dfrac{\sqrt{9+x^2}+3}{\sqrt{9+x^2}+3} = \lim_{x\to 0} \dfrac{9+x^2-9}{x^2(\sqrt{9+x^2}+3)} = \lim_{x\to 0} \dfrac{x^2}{x^2(\sqrt{9+x^2}+3)} =$ $\lim_{x\to 0} \dfrac{1}{\sqrt{9+x^2}+3} = \dfrac{1}{6}$

5. First, since $f(x)$ is polynomial or rational except at $x=-2$, $x=2$ and $x=1$, it is continuous at least for all $x \neq \pm 2, 1$. At $x=-2$, $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} (3x^2-1) = 11$, $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \dfrac{1}{x-1} = -\dfrac{1}{3}$. Since the two one-sided limits do not agree, $\lim_{x\to -2} f(x)$ does not exist and f is not continuous at $x=-2$.

f is not continuous at 1 since $f(1)$ is not defined

At $x=2$, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \dfrac{1}{x-1} = 1$, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (2x-3) = 1$. Since the one-sided limits are equal at $x=2$, $\lim_{x\to 2} f(x) = 1$. Since $f(1) = \frac{1}{2-1} = 1$, $\lim_{x\to 2} f(x) = f(2)$ and f is continuous at $x=2$. f is continuous for all $x \neq -2, 1$.

6. If $l(x)$ is the tangent line to $y=f(x)$ at $x=3$, we know that $f(3)=l(3)$ and $f'(3)=l'(3)=l'(x)$ (remember that lines have constant slope!). Since $f'(x)$ is decreasing on the interval $[3,5]$, $f'(x) < f'(3) = l'(x)$ for all $x \in (3,5)$. The two conditions of the Racetrack Principle are satisfied (*i.e.* f(3)=l(3) and $f'(x) < l'(x)$ for $x \in (3,5)$), so $f'(x) < l'(x)$ for all $x \in (3,5)$ and in particular, $f'(4) < l'(4)$.

7. $f'(1) = \lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \dfrac{\frac{1}{1+h+1}-\frac{1}{2}}{h} = \lim_{h\to 0} \dfrac{2-(2+h)}{2h(2+h)} = \lim_{h\to 0} \dfrac{-h}{2h(2+h)} = \lim_{h\to 0} \dfrac{-1}{2(2+h)} =$ $-\dfrac{1}{4}$

8.