

Math 0093 Fall 2000: Solutions to Sample Midterm 1

1. (a) Multiply both sides by 15:

$$10(x - 7) + 45 = 3x$$

$$10x - 70 + 45 = 3x$$

$$7x - 25 = 0$$

$$x = \frac{25}{7}$$

- (b)

$$x^2 + 7x + 10 = 0$$

$$(x + 5)(x + 2) = 0$$

Either $x = -5$ or $x = -2$.

- (c)

$$3x - 8 \geq 4$$

$$3x - 8 \leq -4$$

$$3x \geq 12$$

or

$$3x \leq 4$$

$$x \geq 4$$

$$x \leq \frac{4}{3}$$

$$\{x \in \mathbf{R} | x \geq 4 \text{ or } x \leq \frac{4}{3}\}$$

- (d) $w(w^2 - 16) = w(w - 4)(w + 4) = 0$. Either $w = 0$, $w = 4$ or $w = -4$.

- (e)

$$2|3x - 1| + 4 = 7$$

$$2|3x - 1| = 3$$

$$|3x - 1| = \frac{3}{2}$$

$$3x - 1 = \frac{3}{2}$$

$$3x - 1 = -\frac{3}{2}$$

$$3x > \frac{5}{2}$$

or

$$3x > -\frac{1}{2}$$

$$x > \frac{5}{6}$$

$$x < -\frac{1}{6}$$

Either $x = \frac{5}{6}$ or $x = -\frac{1}{6}$.

2. (a) $y = \frac{2}{5}x - \frac{1}{5}$ so the slope of the given line is $\frac{2}{5}$. The slope of the perpendicular line is $-\frac{5}{2}$ and $y + 3 = -\frac{5}{2}(x - 2)$, or $y = -\frac{5}{2}x + 5$.

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(b) The slope is $m = \frac{4-2}{-1-7} = -\frac{1}{4}$. the line is $y - 4 = -\frac{1}{4}(x + 1)$ or $y = -\frac{1}{4}x + \frac{15}{4}$.

3.

$$6x + 8y = 8$$

$$-6x + 9y = 6$$

Adding these two we see that $17y = 14$ and $y = \frac{14}{17}$. Substituting into the first equation, $3x + 4\frac{14}{17} = 4$ and $3x = 4 - \frac{56}{17} = \frac{12}{17}$. $x = \frac{4}{17}$. The solution is $(x, y) = (\frac{4}{17}, \frac{14}{17})$.

4. The midpoint is $(\frac{3-1}{2}, \frac{8-4}{2}) = (1, 2)$. The length is $\sqrt{(3 - (-1))^2 + (8 - (-4))^2} = \sqrt{16 + 144} = \sqrt{160} = 4\sqrt{10}$.

5. (a) This is a function since each person has one hair colour.

(b) This is not a function since everyone has two parents.

6. $\frac{x^{-9}y^6}{x^2y^3} = \frac{y^3}{x^{9+2}} = \frac{y^3}{x^{11}}$.

7.

	cost	amount	total cost
Aus	25	x	$25x$
BC	5	$150 - x$	$5(150 - x)$

$$25x + 5(150 - x) = 12 \cdot 150$$

$$20x + 750 = 1800$$

$$20x = 1050$$

$$x = 52.5$$

52.5l of BC wine and $150 - 52.5 = 97.5l$ of Australian wine should be used.

8. If x is the cost of the good wine and y is the cost of the bad, then

$$3x + 5y = 7 \cdot 8$$

$$6x + 7y = 8 \cdot 13$$

Multiplying the first equation by -2 ,

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$$-6x - 10y = -112$$

$$6x + 7y = 104$$

and adding them, $-3y = -8$ and $y = \frac{8}{3} = \$2.67$. Substituting into the first equation, $3x + 5\frac{8}{3} = 56$ and $3x = \frac{128}{3}$, or $x = \frac{128}{9} = \$14.22$. The bad wine costs \$2.67 per litre and the good \$14.22 per litre.

If 8 litres of each are used, the cost will be $8 \cdot 2.67 + 8 \cdot 14.22 = \135.12 .

9. (a) The saleswoman's earnings are linear with respect to her sales. If x = her sales and y = her earnings, $(1200000, 20000)$ and $(2100000, 25000)$ are two points on the line, the slope will be $m = \frac{25,000 - 20,000}{2,100,000 - 1,200,000} = \frac{5}{900} = \frac{1}{180}$. The equation will be $y - 20,000 = \frac{1}{180}(x - 1,200,000)$. The equation expressing her earnings as a function of her sales is $y = \frac{1}{180}x + \frac{40,000}{3}$.

- (b) If she earns 36,000 then $36,000 = \frac{1}{180}x + \frac{40,000}{3}$ and her sales must have been $x = \$4,080,000$.

10. The radius of the circle that passes through $(4, -6)$ contains the center of the circle, namely $(1, -2)$. From these two points its slope is $\frac{-2 + 6}{1 - 4} = -\frac{4}{3}$. The tangent line is perpendicular to the radius, and so has slope $\frac{3}{4}$. Its equation is $y + 6 = \frac{3}{4}(x - 4)$, or $y = \frac{3}{4}x - 9$.

- 11.