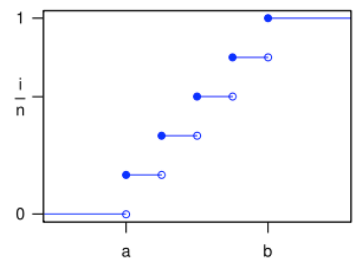
# REFERENCE PAGES

## PROBABILITY DISTRIBUTIONS

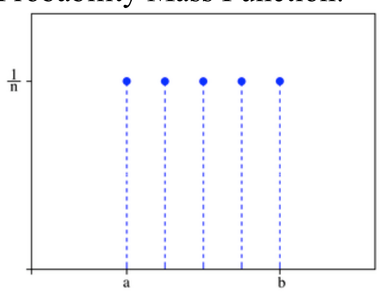
### DISCRETE UNIFORM DISTRIBUTION: $X \sim DUnif(a,b),\ b > a$

**1.** Cumulative Distribution Function:




$$F(X) = \begin{cases} 0,\ x < a \\ \dfrac{\lfloor x \rfloor - a + 1}{n},\ a \le x \le b \\ 1,\ \mathrm{x} > b \end{cases}$$

**2.** Probability Mass Function:




$$p(x) = \begin{cases} 0,\ \text{otherwise} \\ \dfrac{1}{n},\ a \le x \le b \end{cases}$$

**3.** Moment Generating Function: $m(t) = \dfrac{e^{t\theta_2} - e^{t\theta_1}}{t\left(\theta_2 - \theta_1\right)}$

**4.** Expected Value: $E(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx = \dfrac{b+a}{2}$

**5.** Variance: $V(X) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \dfrac{(b-a)^2}{12}$

**6.** Percentile: $F(\varphi_p) = \left(\dfrac{1}{b-a}\right)\left(\varphi_p - a\right) = p$

$\varphi_p = p(b-a) + a$

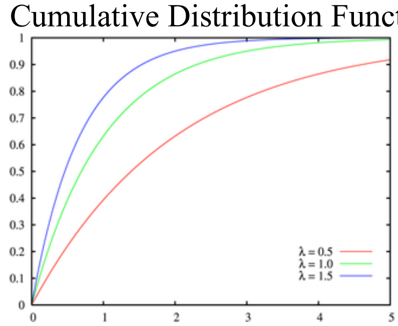
Median: $\varphi_{.5} = \dfrac{a+b}{2}$

Higher Quartile: $\varphi_{.75} = .75b + .25a$

Lower Quartile: $\varphi_{.25} = .25b + .75a$

### EXPONENTIAL DISTRIBUTION: $X \sim Exp(\lambda)$

**7.** Cumulative Distribution Function:




$$F(X) = \begin{cases} 0,\ x < 0 \\ -e^{-\lambda x},\ x > 0 \end{cases}$$

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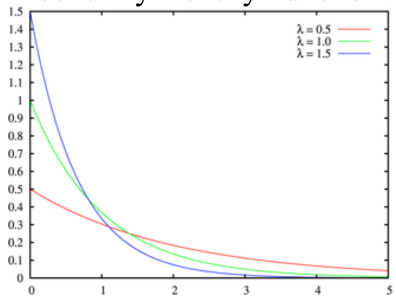
# PROBABILITY DISTRIBUTIONS

### EXPONENTIAL DISTRIBUTION: $X \sim Exp(\lambda)$

**8.** Probability Density Function:




$$f(X) = \begin{cases} \lambda e^{-\lambda x}, \ x > 0 \\ 0, \ x \le 0 \end{cases}$$

**9.** Probability Mass Function: $p(x) = \lambda e^{-x/\beta}, 0 < x < \infty$

**10.** Moment Generating Function: $m(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}$

**11.** Expected Value: $E(X) = \mu = \int_{-\infty}^{\infty} x\lambda e^{-\lambda x} dx = \frac{1}{\lambda}$

**12.** Variance: $V(X) = \sigma^2 = \int_{-\infty}^{\infty} x^2 \lambda e^{-\lambda x} dx = \frac{1}{\lambda^2}$

**13.** Percentile:

$$F(\varphi_p) = 1 - e^{-\lambda \varphi_p} = p$$

$$\varphi_p = \frac{\ln(1-p)}{-\lambda}$$

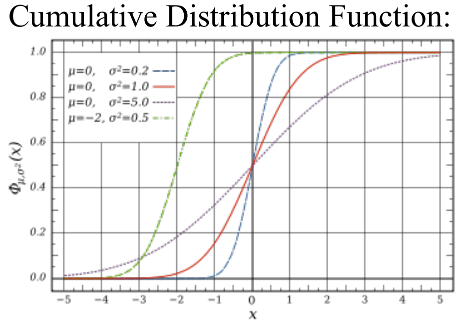
Median: $\varphi_{.5} = \frac{\ln 0.5}{-\lambda}$

Higher Quartile: $\varphi_{.75} = \frac{\ln 0.75}{-\lambda}$

Lower Quartile: $\varphi_{.25} = \frac{\ln 0.25}{-\lambda}$

### NORMAL DISTRIBUTION: $X \sim N(\mu, \sigma^2),\ Z \sim N(0,1)$

**14.** Cumulative Distribution Function:

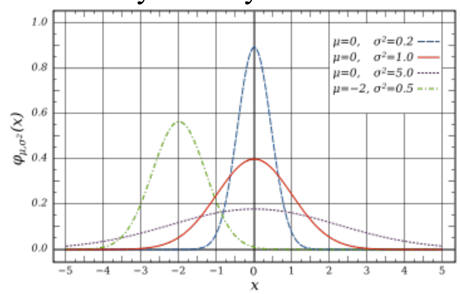



$$F(X) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$$

# PROBABILITY DISTRIBUTIONS

## NORMAL DISTRIBUTION: $X \sim N(\mu, \sigma^2)$

**15.** Probability Density Function:




$$f(X) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \text{ where } X = \sigma Z + \mu$$

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t}{2}}, \text{ where } t = \frac{z-\mu}{\sigma}$$

**16.** Probability Mass Function: $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(x-\mu)^2\right], -\infty < x < \infty$

**17.** Moment Generating Function: $m(t) = \exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$

**18.** Expected Value: $E(X) = \mu = E(\sigma Z + \mu) = \sigma E(Z) + \mu = \mu$

**19.** Variance: $V(X) = \sigma^2 = V(\sigma Z + \mu) = \sigma^2 V(Z) = \sigma^2$

**20.** Percentile:

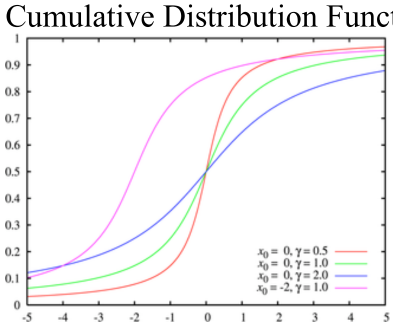
$$F(\varphi_p) = \Phi\left(\frac{\varphi_p - \mu}{\sigma}\right) = p$$

$$\varphi_p = \sigma\zeta_{1-p} + \mu, \quad \frac{\varphi_p - \mu}{\sigma} = \zeta_{1-p}$$

Median: $\varphi_{.5} = \mu$

## CAUCHY (LORENTZ) DISTRIBUTION: $X \sim f(X) = \frac{1}{\pi}\frac{1}{1+x^2}, \forall X$

**21.** Cumulative Distribution Function:




$$F(X) = \begin{cases} \frac{1}{\pi}\frac{1}{1+x^2}, & 0 \le x < \infty \\ 0, & x < 0 \end{cases}$$

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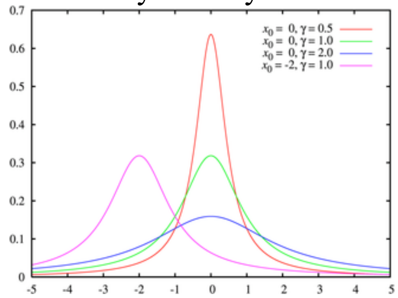
## PROBABILITY DISTRIBUTIONS

### CAUCHY (LORENTZ) DISTRIBUTION: $X \sim f(X) = \frac{1}{\pi}\frac{1}{1+x^2},\ \forall X$

**22.** Probability Density Function:




$$f(X) = \frac{1}{\pi}\arctan(x) + \frac{1}{2}$$

**23.** Expected Value: $E(X) = \mu = \int_{-\infty}^{\infty} x\frac{1}{\pi}\frac{1}{1+x^2}dx = \infty,\ \therefore DNE$

**24.** Variance: $V(X) = \sigma^2 = E(X^2) - \infty^2 = DNE$

**25.** Percentile:

$$F(\varphi_p) = \frac{1}{\pi}\arctan(\varphi_p) + \frac{1}{2} = p$$

$$\varphi_p = \tan\left(p\pi - \frac{\pi}{2}\right)$$

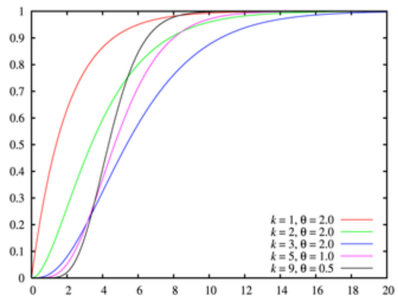
Median: $\varphi_{.5} = \tan(0) = 0$

Higher Quartile: $\varphi_{.75} = \tan\left(\frac{\pi}{4}\right) = 1$

Lower Quartile: $\varphi_{.25} = \tan\left(-\frac{\pi}{4}\right) = -1$

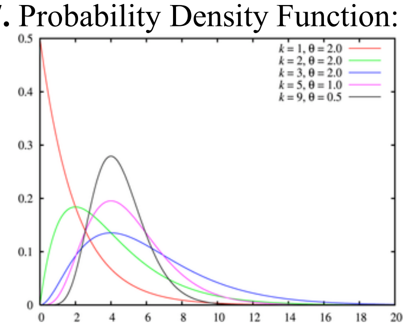
### GAMMA DISTRIBUTION: $X \sim Gamma\left(\alpha, \lambda = \frac{1}{\beta}\right),\ \alpha > 0,\ \lambda > 0$

**26.** Cumulative Distribution Function:




$$F(X) = \int_0^x \frac{\lambda^\alpha x^{\alpha-1}e^{-\lambda t}}{\Gamma(\alpha)}dt \text{ or } \int_0^x \frac{x^{\alpha-1}e^{-\frac{t}{\beta}}}{\beta^\alpha\Gamma(\alpha)}dt$$

**27.** Probability Density Function:




$$f(X) = \frac{\lambda^\alpha x^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)} \text{ or } \frac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^\alpha\Gamma(\alpha)},$$

$$\forall\Gamma(\alpha) = (\alpha-1)! : \alpha = \mathbb{Z}$$

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## PROBABILITY DISTRIBUTIONS

### GAMMA DISTRIBUTION: $X \sim Gamma\left(\alpha, \lambda = \frac{1}{\beta}\right),\ \alpha > 0,\ \lambda > 0$

**28.** Probability Mass Function: $p(x) = \left[\dfrac{1}{\Gamma(\alpha)\left(\frac{1}{\lambda}\right)^{\alpha}}\right] x^{\frac{1}{\lambda}-1} e^{-x\lambda}, 0 < x < \infty$

**29.** Moment Generating Function: $m(t) = \left(1 - \dfrac{t}{\lambda}\right)^{-\alpha}$

**30.** Expected Value: $E(X) = \mu = \dfrac{\alpha}{\lambda}$

**31.** Variance: $V(X) = \sigma^2 = \dfrac{\alpha}{\lambda^2}$

**32.** Percentile: $F(\varphi_p) = \dfrac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda\varphi_p}}{\Gamma(\alpha)} = p$ Median: No simple closed form

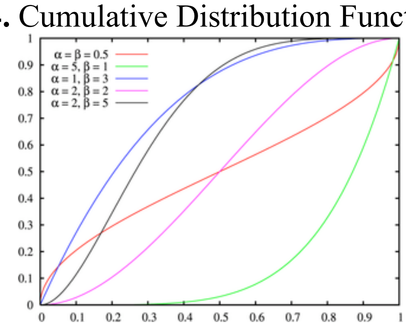
**33.** Special Cases:

$a)$ If $\alpha = 1, f(X) = \lambda e^{-\lambda x}$, for $x > 0$. $\therefore X \sim Exp(\lambda)$

$b)$ If $\alpha = \dfrac{\nu}{2}$, for $\nu = 1,2,3,\ldots = \mathbb{Z}$, then $\lambda = \dfrac{1}{2}$ or $\beta = 2$. $\therefore X \sim \chi^2_{\nu}$, where $\chi^2_p \triangleq \varphi_{1-p}$

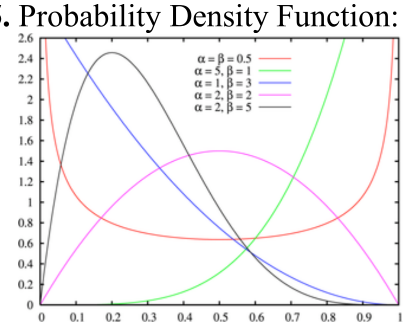
### BETA DISTRIBUTION: $X \sim Beta(\alpha, \beta)$

**34.** Cumulative Distribution Function:




$$F(X) = \int_0^x \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}\,dt = I_x(\alpha,\beta)$$

**35.** Probability Density Function:




$$f(X) = \begin{cases} \dfrac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, 0 \le X \le 1 \\ 0,\ \text{otherwise} \end{cases}$$

where $B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

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## PROBABILITY DISTRIBUTIONS

### BETA DISTRIBUTION: $X \sim Beta(\alpha,\beta)$

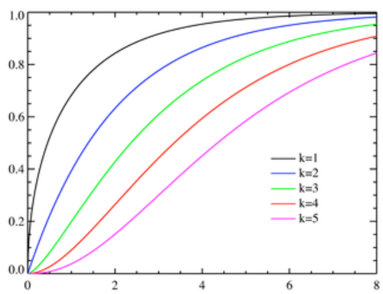
**36.** Probability Mass Function: $p(x) = B(\alpha,\beta)x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1$

**37.** Expected Value: $E(X) = \mu = \dfrac{\alpha}{\alpha+\beta}$

**38.** Variance: $V(X) = \sigma^2 = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

### CHI-SQUARE DISTRIBUTION: $X \sim \chi_k^2$

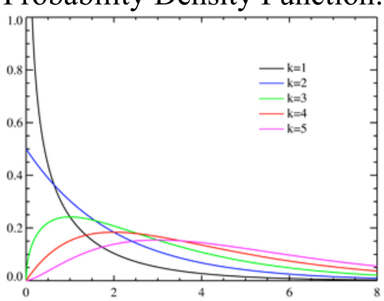
**39.** Cumulative Distribution Function:




$$F(X) = \frac{\gamma(k/2, x/2)}{\Gamma(k/2)} = P(k/2, x/2)$$

where $\begin{cases} \gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt \\ \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt \end{cases}$

**40.** Probability Density Function:




$$f(X) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{(k/2)-1} e^{-x/2}, x > 0$$

**41.** Probability Mass Function: $p(x) = \dfrac{x^{(k/2)-1}e^{-x/2}}{2^{k/2}\Gamma(k/2)}, x^2 > 0$

**42.** Moment Generating Function: $m(t) = (1-2t)^{-k/2}$

**43.** Expected Value: $E(X) = \mu = k$

**44.** Variance: $V(X) = \sigma^2 = 2k$

**45.** Median: $\varphi_{.5} \approx k - \dfrac{2}{3} + \dfrac{4}{27k} - \dfrac{8}{729k^2}$

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## PROBABILITY DISTRIBUTIONS

### ANY GIVEN CONTINUOUS PROBABILITY DISTRIBUTION FUNCTION

**46.** Properties of a Cumulative Distribution Function: $F(y)=\int_{-\infty}^{y} f(t)dt$

1. $F(-\infty)\equiv \lim_{y\to -\infty} F(y)=0$
2. $F(\infty)\equiv \lim_{y\to \infty} F(y)=1$
3. $F(y)$ is a nondecreasing function of $y$

**47.** Properties of a Probability Density Function: $f(y)=\frac{dF(y)}{dy}=F'(y)$

1. $f(y)\geq 0$ for all $y,\ -\infty<y<\infty$
2. $\int_{-\infty}^{\infty} f(y)dy=1$

**48.** Probability Mass Function: $P(a\leq Y\leq b)=\int_{a}^{b} f(y)dy$

**49.** Moment Generating Function: $E\left[e^{tg(Y)}\right]=\int_{-\infty}^{\infty} e^{tg(y)} f(y)dy$

**50.** Expected Value: $E(Y)=\mu=\int_{-\infty}^{\infty} yf(y)dy$

**51.** Variance: $V(Y)=\sigma^2=E(Y^2)-\left[E(Y)\right]^2=\int_{-\infty}^{\infty} y^2 f(y)dy-\mu^2$

### TCHEBYSHEFF'S INEQUALITY THEOREM

**52.** $P\left(|Y-\mu|<k\sigma\right)\geq 1-\frac{1}{k^2}$ $\qquad P\left(|Y-\mu|\geq k\sigma\right)\leq \frac{1}{k^2}$

### BIVARIATE/MULTIVARIATE PROBABILITY DISTRIBUTION

**53.** (Discrete) Joint Distribution Function: $F(y_1,y_2)=P\left(Y_1\leq y_1,Y_2\leq y_2\right),\ -\infty<y_1,y_2<\infty$

**54.** (Discrete) Joint Probability Function: $p(y_1,y_2)=P(Y_1=y_1,Y_2=y_2),\ -\infty<y_1,y_2<\infty$

**55.** (Discrete) Marginal Probability Functions: $p_1(y_1)=\sum_{\text{all } y_2} p(y_1,y_2)$ all $p_2(y_2)=\sum_{\text{all } y_1} p(y_1,y_2)$

**56.** (Continuous) Joint Distribution Function: $F(y_1,y_2)=\int_{-\infty}^{y_1}\int_{-\infty}^{y_2} f(t_1,t_2)dt_2dt_1$

**57.** (Continuous) Joint Probability Function: $P\left(a_1\leq Y_1\leq b_1,\ a_2\leq Y_2\leq b_2\right)=\int_{a_2}^{b_2}\int_{a_1}^{b_1} f(y_1,y_2)dy_1dy_2$

**58.** (Continuous) Marginal Density Functions: $f_1(y_1)=\int_{-\infty}^{\infty} f(y_1,y_2)dy_2$ and $f_2(y_2)=\int_{-\infty}^{\infty} f(y_1,y_2)dy_1$