

Valuation and Characteristics of

BONDS



Bonds and Their Features

Bond -- A long-term debt instrument with a final maturity generally being 10 years or more.

- **A long-term debt instrument issued by a corporation or government.**
- **It is a promise to pay interest and repay the borrowed money on terms specified in a contract a bond indenture**

Contrato de emisión de bonos

Bond markets

- Most bonds are owned by and traded among large financial institutions.
- A representative group of bonds is listed and traded on the bond division of the NYSE.

Bond Terms

➤ Face Value (=par value)

The amount of money the issuer must repay by the end of the bond's life (maturity).

So, the face value of bonds can be called maturity value.

Most corporate bonds issued in the United States have a face value of \$1,000 per bond.

Unless otherwise stated, we assume a \$1,000 face value.



Maturity

The maturity of a bond can represent both of a point of time or a period of time.

The maturity of a bond is the end of its life (a point of time).

When a bond is issued, the length of its life is its original maturity. (a period of time)

The amount of time remaining until maturity is called the remaining maturity (a period of time)

Bond Terms

– Coupon

- The amount of money the issuer promises to pay as nominal interest over the life of the bond. Coupon payments are the finance term for what are called interest payments in everyday language. (but they are still different) The large majority of corporate bonds make coupon payments semiannually – every six months.

– Coupon Rate

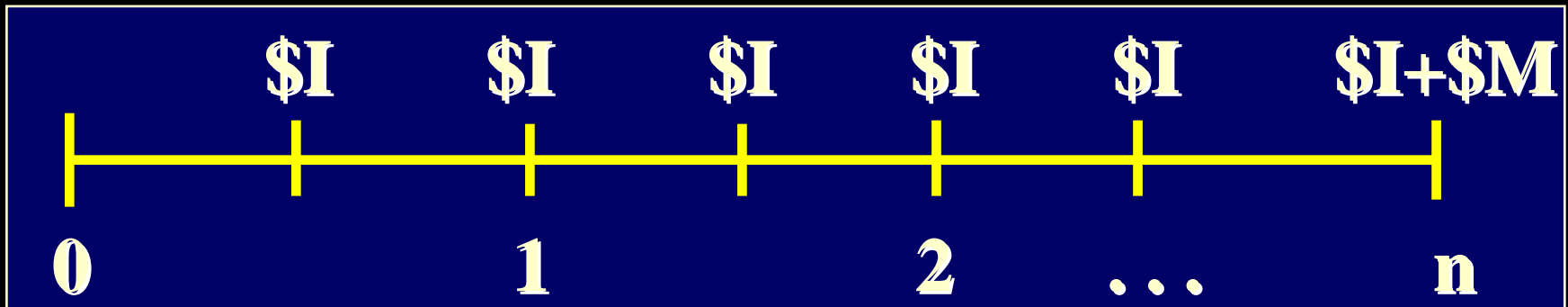
- The nominal annual interest rate of a bond stated on the bond's face value. Coupon payments are determined by the coupon rate and the number of payments per year, both of which are stated in the bond indenture.

Characteristics of Bonds

- Bonds pay fixed coupon (interest) payments at fixed intervals (usually every 6 months) and pay the par value at maturity.

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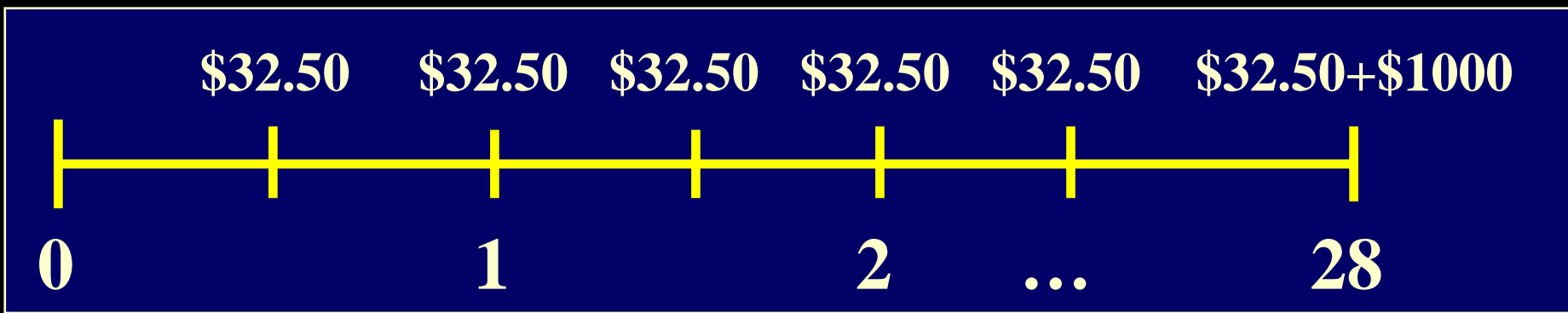


Example: ATT 6 1/2 28

- par value = **\$1000**
- coupon = 6.5% of par value per year.
= **\$65** per year (**\$32.50** every 6 months).
- maturity = 28 years (matures in **2033**).
- issued by AT&T.

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Rating Definitions and Terminology

STANDARD
&POOR'S

- A Standard & Poor's issue credit rating is a current opinion of the creditworthiness of an obligor with respect to a specific financial obligation, a specific class of financial obligations, or a specific financial program (including ratings on medium term note programs and commercial paper programs).

Long-Term Issue Credit Ratings

AAA

An obligation rated 'AAA' has the highest rating assigned by Standard & Poor's. The obligor's capacity to meet its financial commitment on the obligation is extremely strong.

AA

An obligation rated 'AA' differs from the highest rated obligations only in small degree. The obligor's capacity to meet its financial commitment on the obligation is very strong.

A

An obligation rated 'A' is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than obligations in higher rated categories. However, the obligor's capacity to meet its financial commitment on the obligation is still strong.



BBB

An obligation rated 'BBB' exhibits adequate protection parameters.

However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitment on the obligation.

Obligations rated 'BB', 'B', 'CCC', 'CC', and 'C' are regarded as having significant speculative characteristics. 'BB' indicates the least degree of speculation and 'C' the highest. While such obligations will likely have some quality and protective characteristics, these may be outweighed by large uncertainties or major exposures to adverse conditions.

BB

An obligation rated 'BB' is less vulnerable to nonpayment than other speculative issues. However, it faces major ongoing uncertainties or exposure to adverse business, financial, or economic conditions which could lead to the obligor's inadequate capacity to meet its financial commitment on the obligation.

B

An obligation rated 'B' is more vulnerable to nonpayment than obligations rated 'BB', but the obligor currently has the capacity to meet its financial commitment on the obligation. Adverse business, financial, or economic conditions will likely impair the obligor's capacity or willingness to meet its financial commitment on the obligation.

CCC

An obligation rated 'CCC' is currently vulnerable to nonpayment, and is dependent upon favorable business, financial, and economic conditions for the obligor to meet its financial commitment on the obligation. In the event of adverse business, financial, or economic conditions, the obligor is not likely to have the capacity to meet its financial commitment on the obligation.

CC

An obligation rated 'CC' is currently highly vulnerable to nonpayment.

C

A subordinated debt or preferred stock obligation rated 'C' is **CURRENTLY HIGHLY VULNERABLE** to nonpayment. The 'C' rating may be used to cover a situation where a bankruptcy petition has been filed or similar action taken, but payments on this obligation are being continued. A 'C' also will be assigned to a preferred stock issue in arrears on dividends or sinking fund payments, but that is currently paying.

D

An obligation rated 'D' is in payment default. The 'D' rating category is used when payments on an obligation are not made on the date due even if the applicable grace period has not expired, unless Standard & Poor's believes that such payments will be made during such grace period. The 'D' rating also will be used upon the filing of a bankruptcy petition or the taking of a similar action if payments on an obligation are jeopardized.

Plus (+) or minus(-)

The ratings from 'AA' to 'CCC' may be modified by the addition of a plus or minus sign to show relative standing within the major rating categories.

r

This symbol is attached to the ratings of instruments with significant noncredit risks. It highlights risks to principal or volatility of expected returns which are not addressed in the credit rating.

N.R.

This indicates that no rating has been requested, that there is insufficient information on which to base a rating, or that Standard & Poor's does not rate a particular obligation as a matter of policy.

Types of Bonds

- **Debentures** - unsecured bonds.
Obligación sin garantía hipotecaria
- **Subordinated debentures** - unsecured “junior” debt.
- **Mortgage bonds** - secured bonds.
hipotecarios
- **Zeros** - bonds that pay only par value at maturity; no coupons.
- **Junk bonds** - speculative or below-investment grade bonds; rated BB and below. High-yield bonds.

Types of Long-Term Debt Instruments

Debenture -- A long-term, unsecured debt instrument.

Obligación sin garantía hipotecaria

- Investors look to the earning power of the firm as their primary security.
- Investors receive some protection by the restrictions imposed in the bond indenture, particularly any *negative-pledge clause*.
contrato fianza
- A *negative-pledge clause* precludes the corporation from pledging any of its assets (not already pledged) to other creditors.
evita

Types of Long-Term Debt Instruments

Subordinated Debenture -- A long-term, unsecured debt instrument with a lower claim on assets and income than other classes of debt; known as junior debt.

demanda

- In this case, subordinated debenture holders rank behind debenture holders but ahead of preferred and common stockholders in the event of liquidation.
- Frequently, the security is convertible into common stock to lower the yield required by subordinated debenture holders (often less than regular debentures).

Types of Long-Term Debt Instruments

Income Bond -- A bond where the payment of interest is contingent upon sufficient earnings of the firm.

- Frequently, there is a *cumulative feature*, which provides that any unpaid interest in a particular year accumulates. The cumulative obligation is usually limited to no more than three years.
- The bonds are unpopular with investors (usually limited to reorganizations), but are still senior to preferred and common shareholders in the event of liquidation.

Types of Long-Term Debt Instruments

Junk Bond -- A high-risk, high-yield (often unsecured) bond rated below investment grade.



- These are bonds with a rating of Ba (Moody's) or lower.
- Principal investors are pension funds, high-yield bond mutual funds, and some individual investors.
- Liquidity varies depending on investor sentiments.
- Junk bonds were used frequently in the 1980s as a means of financing leveraged buyouts.

compra



Moody's Investors Service

Types of Long-Term Debt Instruments

Mortgage Bond -- A bond issue secured by a mortgage on the issuer's property.

- The issue is secured by a *lien* on *specific assets* of the corporation. ^{embargo}
- The market value of the collateral should exceed the amount of the bond issue by a reasonable margin of safety to help protect bondholders.

Types of Long-Term Debt Instruments

Mortgage Bond (Continued)

- If the corporation defaults, the trustee can foreclose on behalf of the bondholders. The bondholders become general creditors for any residual amount after the sale of the collateral.
- The corporation may have a first mortgage and a second mortgage on the same assets. The first mortgage has a senior claim on the assets.

demanda

Types of Long-Term Debt Instruments

Equipment Trust Certificate -- An intermediate- to long-term security, usually issued by a transportation company such as a railroad or airline, that is used to finance new equipment.

Let us look at an example using a railroad.

- A railroad arranges with a trustee to purchase equipment from a manufacturer.
- The railroad signs a contract with the manufacturer for the construction of specific equipment.
- When the equipment is delivered, equipment trust certificates are sold to investors.

Types of Long-Term Debt Instruments

Equipment Trust Certificates (Continued)

créditos

- Proceeds plus the railroad downpayment are used to pay the manufacturer. señala
- Title of the equipment is ■ held by the trustee and the trustee leases the equipment to the railroad. retenido
- Lease payments are used to pay a fixed dividend to the certificate holders and to retire a specified portion of the certificates at regular intervals.
- After the final lease payment (all certificates are retired), title to the equipment passes to the railroad.

Retirement of Bonds



Sinking Fund -- Fund established to hundidos periodically retire a portion of a security issue before maturity. The corporation is required to make periodic sinking-fund payments to a trustee.

Sinking Fund and the Retirement of Bonds

- Volatility in interest rates or a decline in the credit quality of the firm could lower the market price of the bond and enhance the value to the firm of having this option.
- Bondholders may benefit from the orderly retirement of debt (amortization effect) which reduces the default risk of the firm and adds liquidity to bonds outstanding.

Sin pagar

Sinking Fund and the Retirement of Bonds

Balloon Payment -- A payment on debt that is much larger than other payments.

- Many bond issues are designed to have a larger final payment to pay off the debt.
- For example, a corporation may undertake a \$10 million dollar, 15-year bond issue. The firm is obligated to make \$500,000 sinking fund payments in the 5th through 14th years. The final balloon payment in the 15th year would be for the remaining \$5 million dollars of bonds.

Serial Bonds

Serial Bonds -- An issue of bonds with different maturities, as distinguished from an issue where all bonds have identical maturities (term bonds).

- For example, a \$10 million dollar issue of serial bonds might have \$500,000 of predetermined bonds maturing each year for 20 years.
- Investors are able to choose the maturity that best fits their needs (wider investor appeal).

Types of Bonds

- **Eurobonds** - bonds denominated in one currency and sold in another country. (Borrowing overseas).
- *example* - suppose Disney decides to sell \$1,000 bonds in France. These are U.S. denominated bonds trading in a foreign country. Why do this?

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- ***example*** - suppose Disney decides to sell \$1,000 bonds in France. These are U.S. denominated bonds trading in a foreign country. Why do this?
 - If borrowing rates are lower in France,
 - To avoid SEC regulations.

evitar

Other types (features) of bonds

- Convertible bond – may be exchanged for common stock of the firm, at the holder's option, usually a lower coupon rate.
- Warrant – long-term option to buy a stated number of shares of common stock at a specified price.
garantía
- Puttable bond – allows holder to sell the bond back to the company prior to maturity.
putativos
- Income bond – pays interest only when interest is earned by the firm.
- Indexed bond – interest rate paid is based upon the rate of inflation.

The Bond Indenture

- The **bond contract** between the firm and the trustee representing the bondholders.
- Lists all of the bond's features:
coupon, par value, maturity, etc.
- Lists **restrictive provisions** which are designed to protect bondholders.
- Describes repayment provisions.

Call Provision

Call Provision -- A feature in an indenture that permits the issuer to repurchase securities at a fixed price (or series of fixed prices) before maturity; also called *call feature*.

- Not all bonds are callable. In periods of low interest (hence, low coupon) rates firms are more likely to issue noncallable bonds.
- When a bond is callable, the *call price* is usually above the par value of the bond and often decreases over time.

Call Price

Call Price -- The price at which a security with a call provision can be purchased by the issuer prior to the security's maturity.



Value of the Call Privilege

$$\text{Callable-bond value} = \text{Noncallable-bond value} - \text{Call-option value}$$

- The call privilege is valuable to the firm to the detriment of bondholders. As such, bondholders require a premium for this additional risk in the form of a higher yield.
- The greater the volatility of interest rates, the greater the probability that the firm will call the bonds. Thus, the **call-option** is more valuable all else equal.

Value

- **Book Value:** value of an asset as shown on a firm's balance sheet; historical cost.
- **Liquidation value:** amount that could be received if an asset were sold individually.
- **Market value:** observed value of an asset in the marketplace; determined by supply and demand.
- **Intrinsic value:** economic or fair value of an asset; the present value of the asset's ^{justo} expected future cash flows.

Intrinsic Value



Example

Harry, a college student, and Bill, a senior portfolio manager, need a new computer. Suppose there are two computers, X and Y, available to both Harry and Bill. Computer X and Y have exactly the same features, but consumer report says that there is a 1% chance that computer X will break down within one year and there is a 0.01% chance that Y will break down. If the computer breaks down, Harry and Bill need to spend a couple of days to have it fixed.

Harry

Suppose that Harry wants to spend a maximum of \$1,000 to purchase X. If he has to pay more than \$1,000 for X, he would like to buy another computer that has less favorite features. Also, Harry wants to pay only \$50 more for computer Y. Even though X is less reliable than Y, Harry is willing to spend a couple of days to fix the computer in case that the computer he bought breaks down rather than pay more than \$50 to purchase a more reliable one, because he has plenty of time.

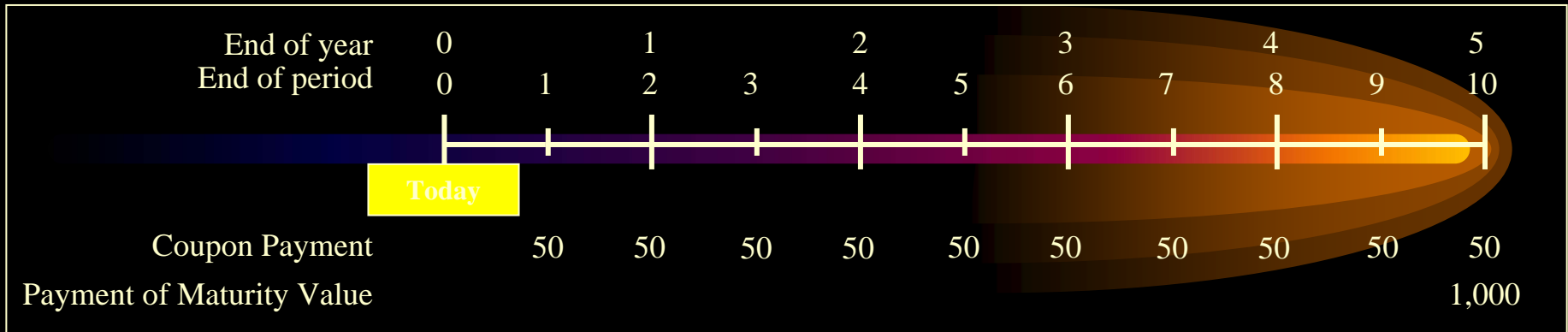
Therefore, we may say that the **intrinsic value** of X and Y to Harry would be \$1,000 and \$1,050, respectively.

Bill

Bill is a senior portfolio management, therefore, his hourly wage may be hundreds of dollars. Reliability of computers is more important than price to him. Suppose Bill would pay a maximum of \$1,300 for computer Y, however, he would pay only \$900 for X because the increased risk of break-down of X may be a serious problem to Bill. His opportunity cost to spend a couple of days to fix his computer is much higher than Harry's.

Therefore, the **intrinsic value** of X and Y to Bill would be \$900 and \$1,300, respectively.

Bond Valuation



Value (intrinsic) of the Bond

Present value of the expected cash flows discounted at the investor's required rate of return on the investment in the bond. Since the required rates of return are different among investors, the intrinsic values of the bond are different among investors.

Bond Valuation

Cash flows from the bond consist of ordinary annuity of coupon and maturity value at maturity of the bond. So present value of such cash flows is

$$V_B = PV_{CF} = PVA_{2n} \text{ of coupons} + PV_{2n} \text{ of maturity value}$$

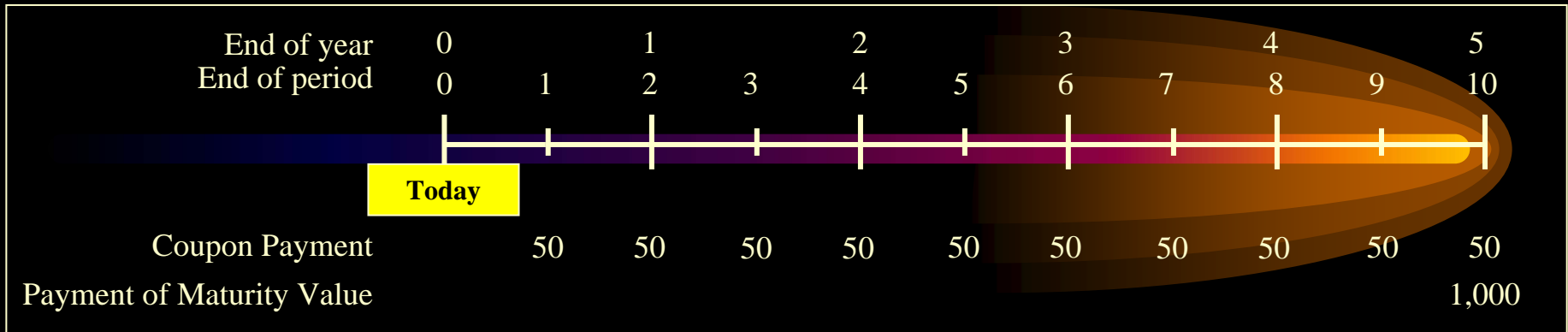
V_B : Value of a bond

PV_{CF} : Present value of cash flows

PVA_{2n} : Present value of a ordinary annuity for $2n$ periods discounted at the required rate of return per period (each payment is coupon per period)

PV_{2n} : Present value of the maturity value discounted at the required rate of return per period (\$1,000 unless otherwise stated)

Bond Valuation



Using Formula

$V_B = PV_{CF} = PVA_{2n}$ of coupons + PV_{2n} of maturity value

V_B : Value of a bond

C_A : Coupon payment per year

k_d : Required rate of return per year

MV : Maturity value

$$V_B = \frac{C_A/2}{(1+k_d/2)} + \frac{C_A/2}{(1+k_d/2)^2} + \dots + \frac{C_A/2}{(1+k_d/2)^{2n-1}} + \frac{C_A/2}{(1+k_d/2)^{2n}} + \frac{MV}{(1+k_d/2)^{2n}}$$

$$= C_A/2 \left[\frac{1 - (1+k_d/2)^{-2n}}{k_d/2} \right] + \frac{1,000}{(1+0.12/2)^{10}} = 368.00435 + 558.39478 = 926.39913$$

Security Valuation

- In general, the intrinsic value of an asset = the present value of the stream of expected cash flows discounted at an appropriate required rate of return.
- Can the **intrinsic value** of an asset differ from its **market value**?

Valuation

$$V = \sum_{t=1}^n \frac{\$C_t}{(1+k)^t}$$

- C_t = cash flow to be received at time t .
- k = the investor's required rate of return.
- V = the intrinsic value of the asset.

Bond Valuation

- **Discount the bond's cash flows at the investor's required rate of return.**



Bond Valuation

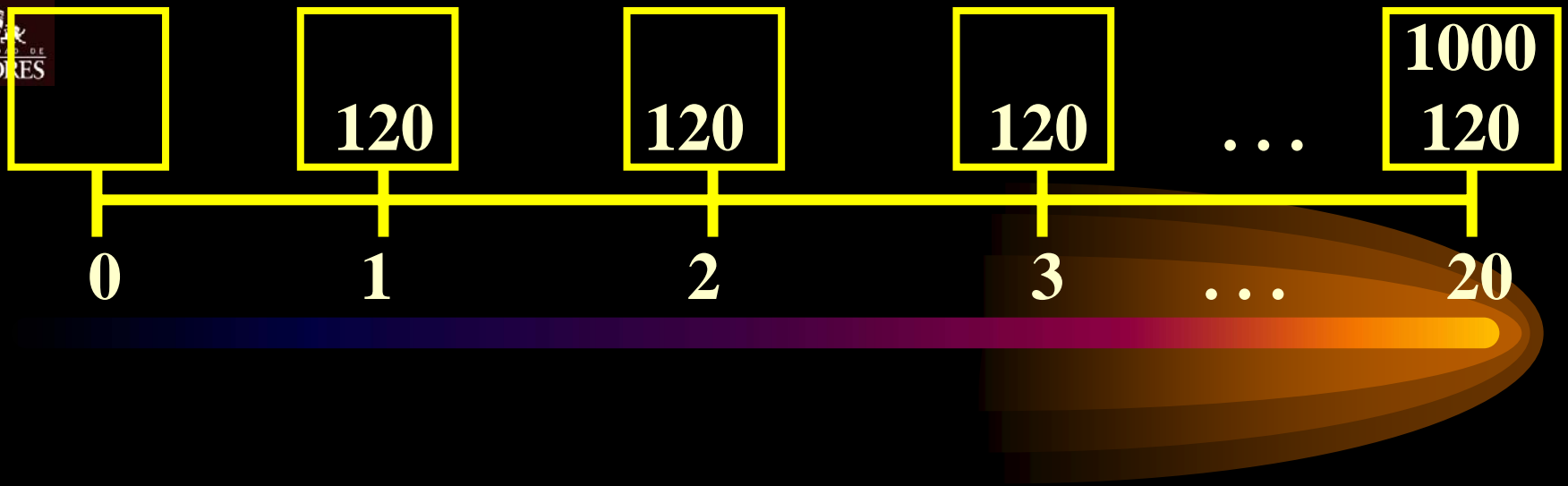
- **Discount the bond's cash flows at the investor's required rate of return.**
 - - **the coupon payment stream (an annuity).**

Bond Valuation

- **Discount the bond's cash flows at the investor's required rate of return.**
 -
 - **the coupon payment stream** (an annuity).
 - **the par value payment** (a single sum).

Bond Example

- Suppose our firm decides to issue **20-year** bonds with a par value of **\$1,000** and annual coupon payments. The return on other corporate bonds of similar risk is currently **12%**, so we decide to offer a **12% coupon** interest rate.
- What would be a fair price for these bonds?



Note: If the coupon rate = discount rate, the bond will sell for par value.⁴⁹

Bond Example

Mathematical Solution:

$$\mathbf{PV = PMT \left[\frac{1 - \frac{1}{(1 + i)^n}}{i} \right] + FV / (1 + i)^n}$$

Bond Example

Mathematical Solution:

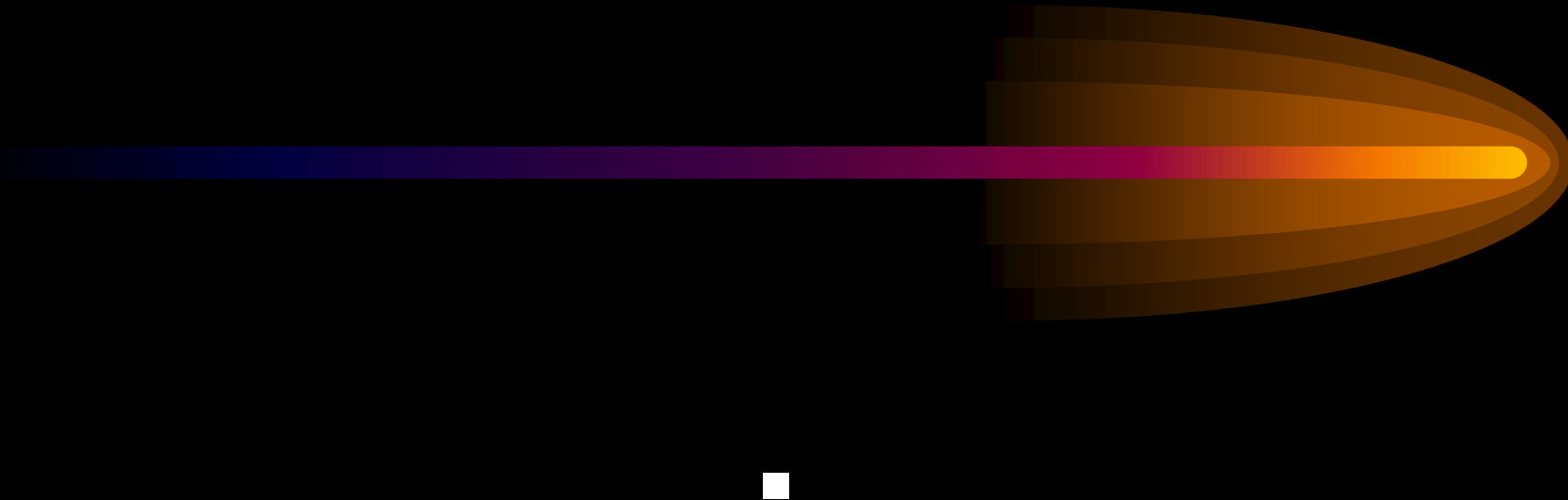
$$PV = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right] + FV / (1+i)^n$$

$$PV = 120 \left[\frac{1 - \frac{1}{(1.12)^{20}}}{.12} \right] + 1000 / (1.12)^{20} = \mathbf{\$1000}$$

- Suppose **interest rates fall** immediately after we issue the bonds. The required return on bonds of similar risk drops to **10%**.



- What would happen to the bond's intrinsic value?



**Note: If the coupon rate > discount rate,
the bond will sell for a premium.**

Bond Example

Mathematical Solution:

$V (PVIF_{k,n})$

■

Bond Example

Mathematical Solution:

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$$PV = 120 \left[\frac{1 - \frac{1}{(1.10)^{20}}}{.10} \right] + 1000 / (1.10)^{20} = \mathbf{\$1,170.27}$$

- Suppose **interest rates rise** immediately after we issue the bonds. The **required return on bonds of similar risk rises to 14%**.
- **What would happen to the bond's intrinsic value?**



**Note: If the coupon rate < discount rate,
the bond will sell for a discount.**

Bond Example

Mathematical Solution:

FV (PVIF_{k, n})

■

Bond Example

Mathematical Solution:

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Bond Example

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$$PV = 120 \left[\frac{1 - \frac{1}{(1.14)^{20}}}{.14} \right] + 1000 / (1.14)^{20} = \mathbf{\$867.54}$$

Bond Example

Mathematical Solution:

$$V = (PVIF_{k,p})$$



Bond Example

Mathematical Solution:

$$PV = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right] + FV / (1+i)^n$$

Bond Example

Mathematical Solution:

$$PV = PMT (PVIF_{k,p})$$

$$PV = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right] + FV / (1+i)^n$$

$$PV = 60 \left[\frac{1 - \frac{1}{(1.07)^{40}}}{.07} \right] + 1000 / (1.07)^{40} = \mathbf{\$866.68}$$

Semiannual Coupon-Payment Bonds

Example

ABC Corporation issues a 10 percent coupon bonds.

The bond have 5 years to maturity and will pay the coupons semiannually. If you buy one of the bonds, what are the expected cash flows from the bond?

Stated information from the example

Coupon Rate: 10%

Maturity: 5 years

Number of coupon Payments per Year: 2

Semiannual Coupon-Payment Bonds

Additional information you need to figure out from the stated information to know the expected cash flows

1. Maturity Value: \$1,000

If there is no stated maturity value (face value), you can assume \$1,000 as the maturity value.

2. Coupon Payments per Payment:



$$C_A = CR \times MV$$

$$C_A = 0.1 \times 1,000$$

$$C = \frac{C_A}{2} \text{ or } C = \frac{CR}{2} \times MV$$

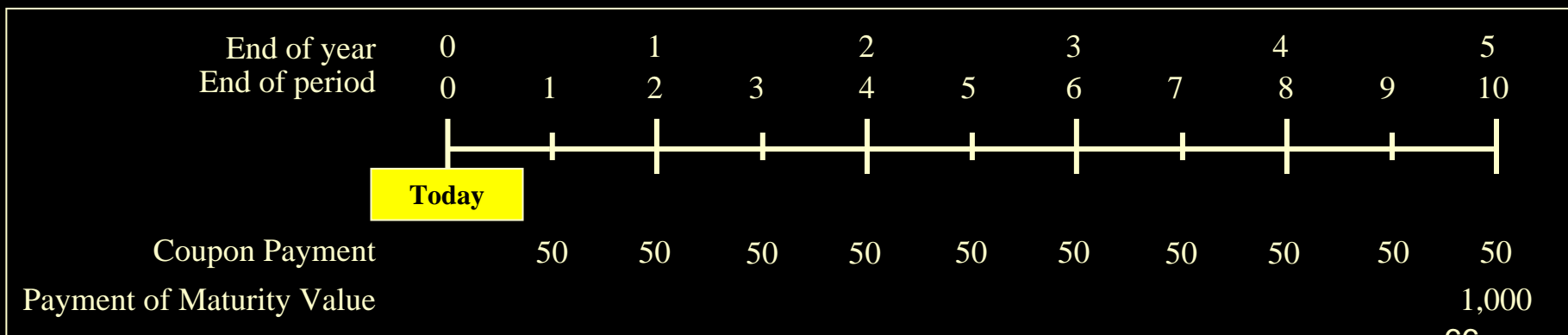
$$C = \frac{100}{2} = 50 \text{ or } C = \frac{0.1}{2} \times 1,000 = 50$$

C_A = Coupon per year

C = Coupon per payment

CR = Coupon Rate

MV = Maturity Value



Required Rate of Return

Example

Suppose both Mary and Harry ask you to lend \$1,000 to them for six months, you have an extra \$1,000 now, and you want to go on vacation with that money after six months.

Suppose that Mary is a very reliable person and you believe she will pay back the money with 99% probability.

However, you believe that Harry is not so reliable and he will pay back the money with only 90% probability.

Suppose that considering her reliability and interest rates from banks, your minimum interest rate for you to lend to Mary is 10%.

- If both of them promise the same rate of interest, you must lend to Mary. Therefore, in order to borrow from you, Harry should offer a higher interest rate than Mary. ■
- Suppose the minimum interest rate for you to lend to Harry is 15%. Also, suppose if Mary promises a 10% interest rate and Harry promises a 15% interest rate, you are indifferent as to whether you lend to Mary or Harry.

- Then, we may say that the **required rate of return** on your loan to Mary is 10% and that on your loan to Harry is 15%. (– *the riskier the investment is, the higher rate of return the investor requires* → *risk aversion*)
- → *The required rate of return depends on the riskiness of the investment.*

Example

Suppose both Mary and Harry also ask Jackie to lend the \$1,000 and Jackie has \$1,000 now but she needs to pay her tuition next semester with the money.

Because she needs to pay her tuition, she may not make a loan to a risky person unless the borrower promises significantly high interest rate. (more risk averse than you)

If Jackie requires at least a 25% interest rate for lending to Harry, her required rate of return on the loan to Harry is 25%.

Suppose Jackie would lend to Mary if Mary promises 12% interest rate because Mary is a reliable person.

Then, the required rate of return on her loan to Mary is 12%.
(the more risk averse investor is, the higher risk premium the investor requires for the same level of increased riskiness)

→ The required rate of return depends also on the investors.
(or on the degree of risk aversion of the investors)

Yield To Maturity

- The **expected rate of return** on a bond.
- The rate of return investors earn on a bond if they hold it to maturity.

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$$P_0 = \sum_{t=1}^n \frac{\$I_t}{(1 + \bar{k}_b)^t} + \frac{\$M}{(1 + \bar{k}_b)^n}$$

YTM Example

- Suppose we paid **\$898.90** for a **\$1,000** par **10%** coupon bond with 8 years to maturity and semi-annual coupon payments.
- What is our **yield to maturity?**

Bond Example

Mathematical Solution:

$$PV = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right] + FV / (1+i)^n$$

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$$898.90 = 50 \left[\frac{1 - \frac{1}{(1+i)^{16}}}{i} \right] + 1000 / (1+i)^{16}$$

Bond Example

Mathematical Solution:

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$$898.90 = 50 \left[\frac{1 - \frac{1}{(1+i)^{16}}}{i} \right] + 1000 / (1+i)^{16}$$

solve using trial and error

Rate of Return (Yield)

Market Required Rate of Return (Yield)

Consensus among market participants on required rate of return on the securities.

If we replace intrinsic value in our valuation equations with the market price of the securities, we can then solve for the market required rate of return.



Yield to Maturity (YTM) on Bonds

The market required rate of return on Bonds. Or,

The expected rate of return on a bond if bought at its current market price and held to maturity.

It is the discount rate that equates the present value of all expected coupon payments and the payment of maturity value at maturity with the bond's current market price.

Rate of Return (Yield)

$$P_B = \frac{C_A/2}{(1+YTM/2)} + \frac{C_A/2}{(1+YTM/2)^2} + \dots + \frac{C_A/2}{(1+YTM/2)^{2n-1}} + \frac{C_A/2}{(1+YTM/2)^{2n}} + \frac{MV}{(1+YTM/2)^{2n}}$$

$$P_B = C_A/2 \left[\frac{1}{YTM/2} \left(1 - \frac{1}{(1+YTM/2)^{2n}} \right) \right] + \frac{MV}{(1+YTM/2)^{2n}}$$

P_B : Current market price of a bond

C_A : Coupon payment per year

YTM : Yield to Maturity (Annual rate)

MV : Maturity value

Bond Prices vs. Yields

➤ **Discount Bond** – The current price of the bonds is less than the face value. (Price < MV)



(if and only if) Market Required Rate of Return (YTM) > CR

The difference between price and face value is **bond discount**. (MV – Price)

➤ **Premium Bond** – The current price of the bonds exceeds the face value. (Price > MV)



(if and only if) Market Required Rate of Return (YTM) < CR

The difference is **bond premium**. (Price – MV)

➤ **Par Bond** – The current price of the bonds is equal to the face value. (Price = MV)



(if and only if) Market Required Rate of Return (YTM) = CR

- If interest rates rise so that the market required rate of return increases, the bond's price will _____.
- If interest rates fall, the bond's price will _____.
- For a given change in market required rate of return, the longer its maturity is, the price of a bond will change by a greater amount.
- For a given change in market required rate of return, the lower the coupon rate, the price of a bond will change by proportionally more. In other words, bond price volatility is inversely related to coupon rate.

Bond Prices vs. Yields



1. When the market required rate of return is the same as the coupon rate, the bond price is the same as face value. (A)
2. When the market required rate of return increases, the prices of both bonds fall.
3. When the market required rate of return decreases, the prices increase.
4. When the market required rate of return decreases from 10% to 5%, the price of 15-year bond will increase by B-D, and the price of 5-year bond will increase by C-D. (B-D > C-D)

Rate of Return (Yield)

Market Required Rate of Return (Yield)

Consensus among market participants on required rate of return on the securities.

If we replace intrinsic value in our valuation equations with the market price of the securities, we can then solve for the market required rate of return.



Yield on Preferred Stocks or Common Stocks

The market required rate of return on preferred stocks or common stocks. (Since there is no maturity to preferred or common stock, yield to maturity on preferred stocks or common stocks makes no sense.)

The expected rate of return on a preferred or common stock if bought at its current market price.

It is the discount rate that equates the present value of all expected dividends with the stock's current market price.

Rate of Return (Yield)

Yield on Preferred Stock

$$Y_P = \frac{D_P}{P_P}$$

Y_P : Yield on a preferred stock

D_P : Stated annual dividend per share of preferred stock

P_P : Current market price of the preferred stock

Yield on Common Stock

$$P_E = \frac{D_1}{(1+Y_e)^1} + \frac{D_2}{(1+Y_e)^2} + \dots + \frac{D_\infty}{(1+Y_e)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1+Y_e)^t}$$

$$P_E = \frac{D_0(1+g)}{Y_e - g}$$

$$Y_e = \frac{D_0(1+g)}{P_E} + g$$

Y_e : Yield on a common stock

D_t : Expected dividend per share of common stock at t^{th} year

D_0 : Present dividend per share of common stock

P_E : Current market price of the common stock

Zero Coupon Bonds

- **No coupon interest payments.**
- **The bond holder's return is determined entirely by the price discount.**

Zero Example

- Suppose you pay **\$508** for a zero coupon bond that has **10 years** left to maturity.
- What is your **yield to maturity**?

Zero Example

- Suppose you pay **\$508** for a zero coupon bond that has **10 years** left to maturity.
- What is your **yield to maturity**?



Zero Example

$$PV = -508$$

$$FV = 1000$$

0

10

Mathematical Solution:

$$PV = FV (PVIF_{i, n})$$

$$508 = 1000 (PVIF_{i, 10})$$

$$.508 = (PVIF_{i, 10}) \quad [use PVIF table]$$

$$PV = FV / (1 + i)^{10}$$

$$508 = 1000 / (1 + i)^{10}$$

$$1.9685 = (1 + i)^{10}$$

$$i = 7\%$$

Other Types of Bond

➤ **Zero-Coupon Bond** : A bond that pays no coupon but sells at a deep discount from its face value.

- $V_B = PV_n$ of maturity value (← no coupon payments)

$$V_B = \frac{MV}{(1+k_d)^n}$$

➤ **Perpetual Bonds** : A bond that never matures; a perpetuity in the form of a bond. If a perpetual bond pays a coupon every year (annual payment) forever, value of the bond is

- $V_B = PVA$ of coupons (← no maturity value)

$$\begin{aligned} PVA &= \lim_{n \rightarrow \infty} PVA_n = \lim_{n \rightarrow \infty} C_A \left[\frac{1}{k_d} \left(1 - \frac{1}{(1+k_d)^n} \right) \right] \\ &= C_A \lim_{n \rightarrow \infty} \left[\frac{1}{k_d} \left(1 - \frac{1}{(1+k_d)^n} \right) \right] = C_A \frac{1}{k_d} \\ &\quad \left(\lim_{n \rightarrow \infty} \frac{1}{(1+k_d)^n} = 0 \right) \end{aligned}$$

The Financial Pages: Corporate Bonds

	Cur			Net
	Yld	Vol	Close	Chg
Polaroid 11 1/2 06	19.3	395	59 3/4	...

- **What is the yield to maturity for this bond?**

P/YR = 2, N = 10, FV = 1000,

PV = \$-597.50,

PMT = 57.50

- **Solve: I/YR = 26.48%**

The Financial Pages: Corporate Bonds

	Cur			Net
	Yld	Vol	Close	Chg
HewlPkd zr 17	...	20	51 1/2	+1

- **What is the yield to maturity for this bond?**

$$P/YR = 1, \quad N = 16, \quad FV = 1000,$$

$$PV = \$-515,$$

$$PMT = 0$$

- **Solve: I/YR = 4.24%**

The Financial Pages: Treasury Bonds

	Maturity				Ask
Rate	Mo/Yr	Bid	Asked	Chg	Yld
9	Nov 18	139:14	139:20	-34	5.46

- **What is the yield to maturity for this Treasury bond? (assume 35 half years)**

$$P/YR = 2, N = 35, FV = 1000,$$

$$PMT = 45,$$

$$PV = - 1,396.25 \quad (139.625\% \text{ of par})$$

- **Solve: $I/YR = 5.457\%$**

What is interest rate (or price) risk?

- Interest rate risk is the concern that rising k_d will cause the value of a bond to fall.

<u>% change</u>	<u>1 yr</u>	<u>k_d ■</u>	<u>10yr</u>	<u>% change</u>
+4.8%	\$1,048	5%	\$1,386	+38.6%
	\$1,000	10%	\$1,000	
-4.4%	\$956	15%	\$749	-25.1%

The 10-year bond is more sensitive to interest rate changes, and hence has more interest rate risk.

What is reinvestment rate risk?

- Reinvestment rate risk is the concern that k_d will fall, and future CFs will have to be reinvested at lower rates, hence reducing income.

EXAMPLE: Suppose you just won \$500,000 playing the lottery. You intend to invest the money and live off the interest.

Reinvestment rate risk example

- You may invest in either a 10-year bond or a series of ten 1-year bonds. Both 10-year and 1-year bonds currently yield 10%.
- If you choose the 1-year bond strategy:
 - After Year 1, you receive \$50,000 in income and have \$500,000 to reinvest. But, if 1-year rates fall to 3%, your annual income would fall to \$15,000.
- If you choose the 10-year bond strategy:
 - You can lock in a 10% interest rate, and \$50,000 annual income.

Conclusions about interest rate and reinvestment rate risk

	Short-term AND/OR High coupon bonds	Long-term AND/OR Low coupon bonds
Interest rate risk	Low	High
Reinvestment rate risk	High	Low

- **CONCLUSION: Nothing is riskless!**

Default risk

- If an issuer defaults, investors receive less than the promised return. Therefore, the expected return on corporate and municipal bonds is less than the promised return.
- Influenced by the issuer's financial strength and the terms of the bond contract.

Evaluating default risk:

Bond ratings

	Investment Grade	Junk Bonds
Moody's	Aaa Aa A Baa	Ba B Caa C
S & P	AAA AA A BBB	BB B CCC D

- Bond ratings are designed to reflect the probability of a bond issue going into default.

Factors affecting default risk and bond ratings

- Financial performance
 - Debt ratio
 - TIE ratio
 - Current ratio
- Bond contract provisions
 - Secured vs. Unsecured debt
 - Senior vs. subordinated debt
 - Guarantee and sinking fund provisions
 - Debt maturity

Other factors affecting default risk

- Earnings stability
- Regulatory environment
- Potential antitrust or product liabilities
- Pension liabilities
- Potential labor problems
- Accounting policies

Different Types of Bonds

A perpetual bond is a bond that *never* matures. It has an infinite life.

$$V = \frac{I}{(1 + k_d)^1} + \frac{I}{(1 + k_d)^2} + \dots + \frac{I}{(1 + k_d)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{I}{(1 + k_d)^t} \quad \text{or} \quad I (\text{PVIFA}_{k_d, \infty})$$

$$= I / k_d \quad [\text{Reduced Form}]$$

Perpetual Bond Example

Bond P has a \$1,000 face value and provides an 8% coupon. The appropriate discount rate is 10%. What is the value of the perpetual bond?

$$I = \$1,000 (8\%) = \$80.$$

$$k_d = 10\%.$$

$$V = I / k_d \quad [Reduced Form]$$

$$= \$80 / 10\% = \$800.$$

Different Types of Bonds

A non-zero coupon bond is a coupon paying bond with a finite life.

$$V = \frac{I}{(1 + k_d)^1} + \frac{I}{(1 + k_d)^2} + \dots + \frac{I + MV}{(1 + k_d)^n}$$

$$= \sum_{t=1}^n \frac{I}{(1 + k_d)^t} + \frac{MV}{(1 + k_d)^n}$$

$$= I (\text{PVIFA}_{k_d, n}) + MV (\text{PVIF}_{k_d, n})$$

Coupon Bond Example

Bond C has a \$1,000 face value and provides an 8% annual coupon for 30 years. The appropriate discount rate is 10%.

What is the value of the *coupon bond*?

$$\begin{aligned}
 V &= \$80 (\text{PVIFA}_{10\%, 30}) + \$1,000 (\text{PVIF}_{10\%, 30}) = \\
 & \$80 (9.427) + \$1,000 (.057) \\
 & = \$754.16 + \$57.00 \\
 & = \mathbf{\$811.16.}
 \end{aligned}$$

Different Types of Bonds

A zero coupon bond is a bond that pays no interest but sells at a deep discount from its face value; it provides compensation to investors in the form of price appreciation.

$$V = \frac{MV}{(1 + k_d)^n} = MV (PVIF_{k_d, n})$$

Zero-Coupon Bond

Example

Bond Z has a \$1,000 face value and a 30 year life. The appropriate discount rate is 10%. What is the value of the *zero-coupon bond*?

$$\begin{aligned}
 V &= \$1,000 (\text{PVIF}_{10\%, 30}) \\
 &= \$1,000 (.057) \\
 &= \$57.00
 \end{aligned}$$

Semiannual Compounding

Most bonds *in the U.S.* pay interest twice a year (1/2 of the annual coupon).

Adjustments needed:

- (1) Divide k_d by 2
- (2) Multiply n by 2
- (3) Divide I by 2

Semiannual Compounding

A non-zero coupon bond adjusted for semiannual compounding.

$$\begin{aligned}
 V &= \frac{I/2}{(1 + k_d/2)^1} + \frac{I/2}{(1 + k_d/2)^2} + \dots + \frac{I/2 + MV}{(1 + k_d/2)^{2*n}} \\
 &= \sum_{t=1}^{2*n} \frac{I/2}{(1 + k_d/2)^t} + \frac{MV}{(1 + k_d/2)^{2*n}} \\
 &= I/2 (PVIFA_{k_d/2, 2*n}) + MV (PVIF_{k_d/2, 2*n})
 \end{aligned}$$

Semiannual Coupon Bond

Example

Bond C has a \$1,000 face value and provides an 8% semiannual coupon for 15 years. The appropriate discount rate is 10% (annual rate). What is the value of the coupon bond?

$$\begin{aligned}
 V &= \$40 (\text{PVIFA}_{5\%, 30}) + \$1,000 (\text{PVIF}_{5\%, 30}) & = \\
 & \$40 (15.373) + \$1,000 (.231) \\
 & = \$614.92 + \$231.00 \\
 & = \mathbf{\$845.92}
 \end{aligned}$$

EJERCICIOS



Fast and Loose Company tiene un bono pendiente a 8%, a cuatro años, y un valor a la par de \$1,000 cuyos intereses se pagan anualmente.

- a. Si la tasa de rendimiento requerida de mercado es de 15%, ¿Cuál es el valor de mercado del bono?
- b. ¿Cuál sería su valor de mercado si la tasa de rendimiento requerida de mercado bajara al 2 y 8%?
- c. Si la tasa cupón fuera de 15%, en lugar de 8%, ¿Cuál sería el valor de mercado de acuerdo con el inciso (a)? Si la tasa de rendimiento requerida disminuyera a 8%, ¿Qué sucedería con el precio de mercado del bono?

SOLUCIÓN



1. a. b.

FINAL DE AÑO	PAGO	FACTOR DE DESCUENTO, 15%	VALOR PRESENTE, 15%	FACTOR DE DESCUENTO, 12%	VALOR PRESENTE, 12%
1-3	\$80	2.283	\$182.64	2.402	\$192.16
4	1,080	0.572	<u>617.76</u>	0.636	<u>686.88</u>
		Valor de mercado	\$800.40		\$879.04

Nota: A veces los errores de redondeo cometidos por usar tablas, provocan ligeras diferencias en las respuestas cuando se aplican métodos alternativos de resolución a los mismos flujos de efectivo.

El valor de mercado de un bono a 8% con un rendimiento de 8% en su valor nominal, de \$1,000.

c. El valor de mercado sería de \$1,000 si el rendimiento requerido fuera de 15%.

FINAL DE AÑO	PAGO	FACTOR DE DESCUENTO, 8%	VALOR PRESENTE, 8%
1—3	\$ 150	2.577	\$ 386.55
4	1,150	.735	<u>845.25</u>
	Valor de mercado		\$1,231.80

Un bono con un valor nominal de \$1,000 tiene un precio corriente de mercado de \$935, una tasa cupón de 8% y una vigencia de 10 años.

Los intereses se pagan de manera semestral.

Antes de hacer cualquier cálculo, determine si el rendimiento al vencimiento está por encima o por debajo de la tasa cupón. ¿Por qué?



- a. ¿Cuál es la tasa de descuento semestral determinada por el mercado (es decir, el rendimiento semestral al vencimiento de este bono)?
- b. Con base en su respuesta a la pregunta anterior, (i) ¿Cuál es el rendimiento (nominal anual) del bono al vencimiento?, (ii) ¿El rendimiento (efectivo anual) del bono al vencimiento?

SOLUCIÓN



3. El rendimiento al vencimiento es mayor que la tasa cupón de 8% porque los bonos se venden con un descuento de su valor nominal. El rendimiento (nominal anual) el vencimiento que se reporta en el mercado bonos es igual a $(2 \times RV \text{ semestral})$. El RV (anual en efectivo) es igual a $(1 + RV \text{ semestral})^2 - 1$. El problema se plantea de la siguiente manera:

$$\sum_{t=1}^{20} \frac{\$40}{(1 + k_d/2)^t} + \frac{\$1,000}{(1 + k_d/2)^{20}}$$

$$= (\$40)(FIVPA_{k_d/2, 20}) + VV(FIVP_{k_d/2, 20})$$

- a. Al determinar $k_d/2$ (el RV semestral) en la expresión con una calculadora, un proceso por computadora o tablas de valor presente se obtiene 4.5%.
- b. (i) El RV (nominal anual) es entonces $2 \times 4.5\% = 9\%$.
 (ii) El RV (efectivo anual) es entonces $(1 + 0.045)^2 - 1 = 9.2025\%$.

Problems on Bonds

Problem 1

The 9-percent-coupon-rate bonds of the ABC Company have exactly 15 years remaining maturity. The current market price of one of these \$1,000-par-value bonds is \$700. Coupon is paid semiannually. If your required rate of return on these bonds is 14 percent, should you buy some of these bonds or not?

Problem 2

What is the market required rate of return on the bonds in previous problem?