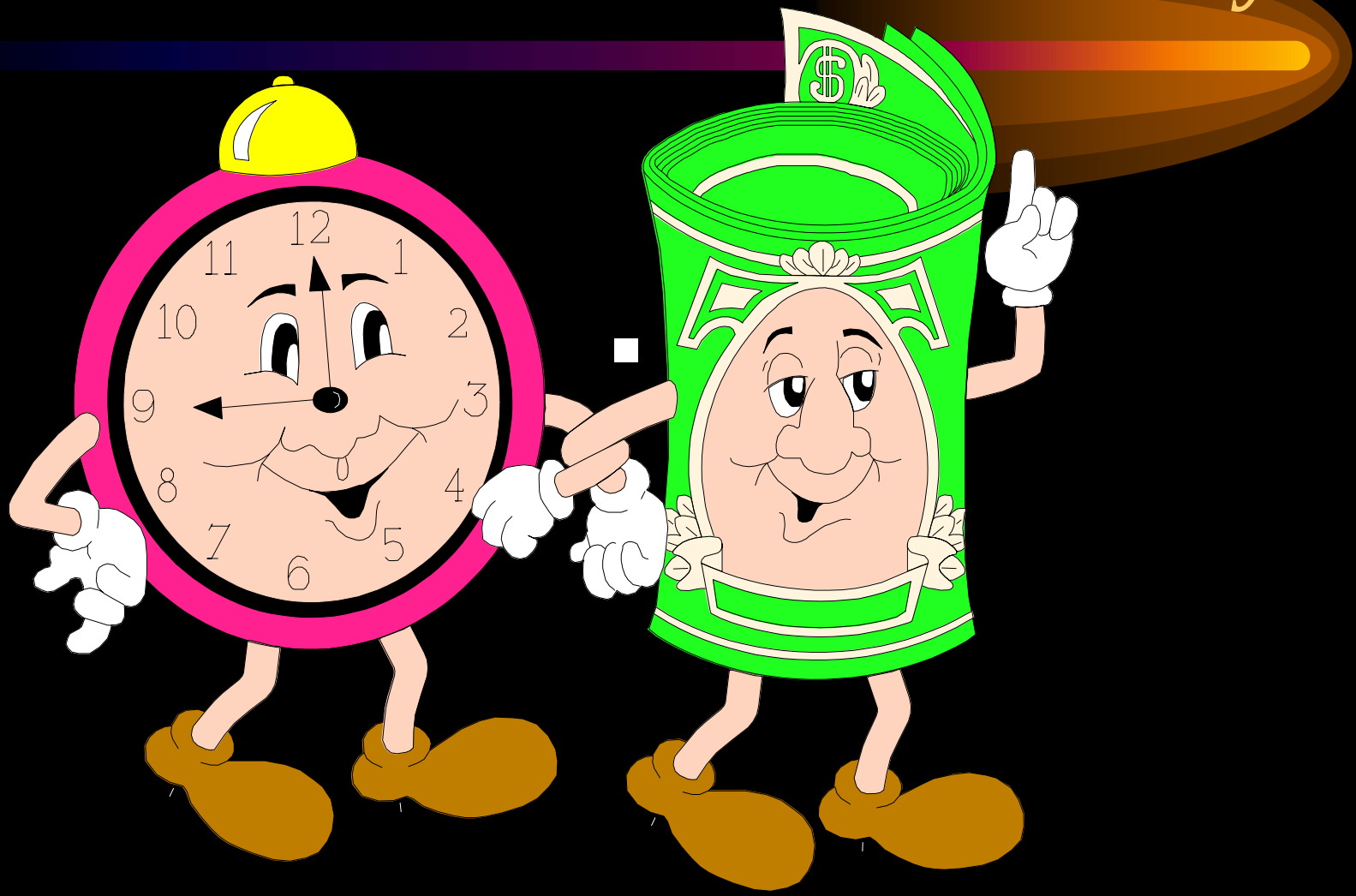
UNIVERSIDAD DE LONDRES

The Time Value of Money



M. En C. Eduardo Bustos Farías

Time Value of Money

- Business investments extend over long periods of time, so we must recognize the time value of money.
- Investments that promise returns earlier in time are preferable to those that promise returns later in time.

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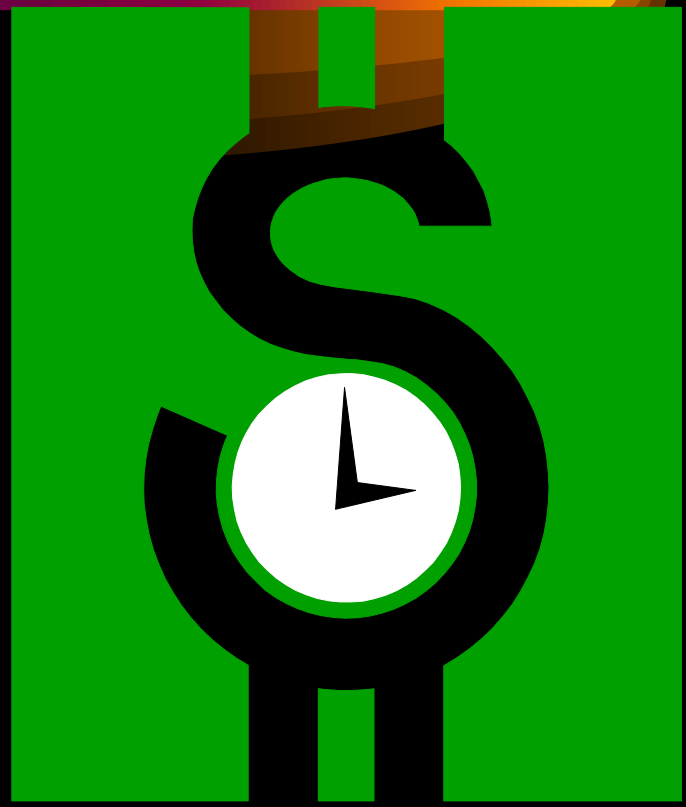
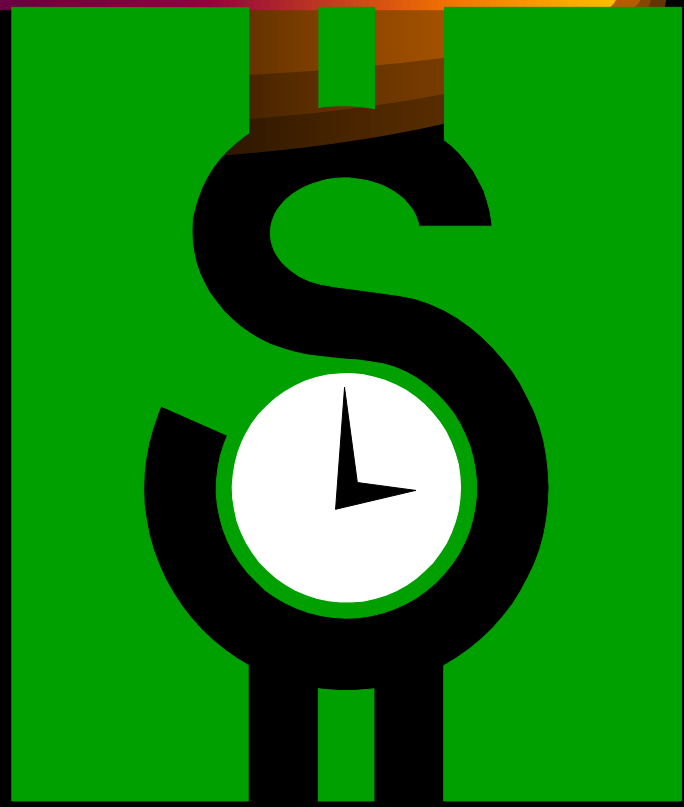


Time Value of Money

A dollar today is worth more than a dollar a year from now since a dollar received today can be invested, yielding more than a dollar a year from now.

vale



Interest and the Time Value of Money

If $100 is invested today at 8% interest, how much will you have in two years?

At the end of one year:

$$\$100 + 0.08 \times \$100 = (1.08) \times \$100 = \$108$$

At the end of two years:

$$(1.08) \times \$108 = \$116.64$$

or

$$(1.08)2 \times \$100 = \$116.64$$

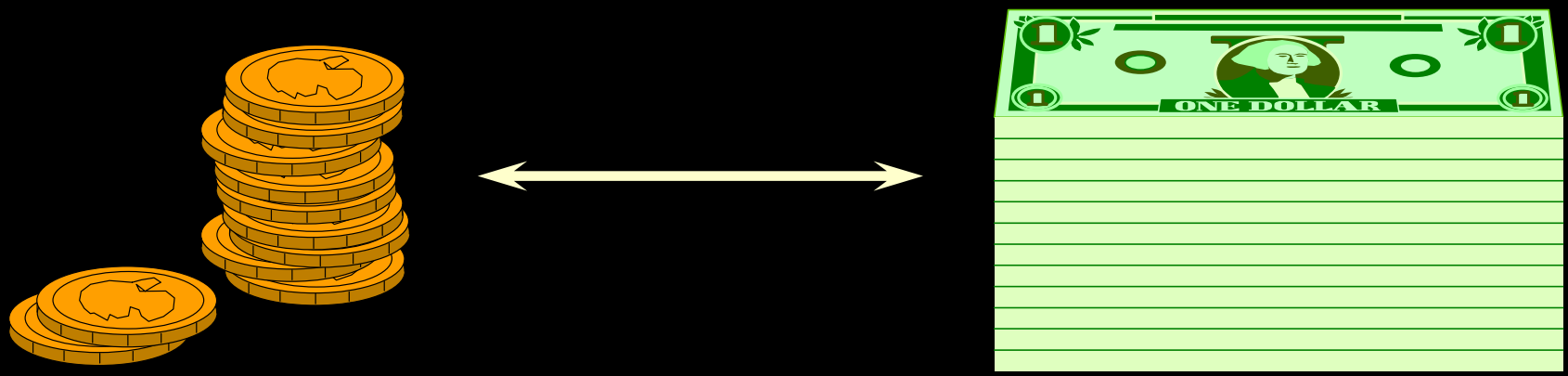
Interest and the Time Value of Money

If P dollars are invested today at the annual interest rate r, then in n years you would have F_n dollars computed as follows:

$$FV = PV\,(1 + r)^n$$

The Time Value of Money

Compounding and
capitalizar
Discounting Single Sums
descontar



We know that receiving $1 today is worth *more* than $1 in the future. This is due to **opportunity costs**.

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Debido

The opportunity cost of receiving $1 in the future is the **interest** we could have earned if we had received the $1 sooner.

Today

Future

If we can measure this opportunity cost, we can:

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- **Translate $1 today into its equivalent in the future *(compounding).***

  capitalizar

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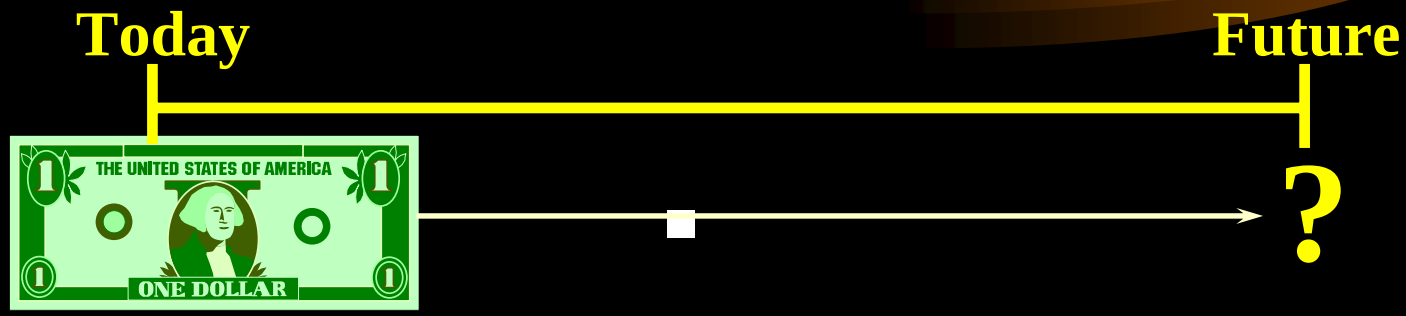
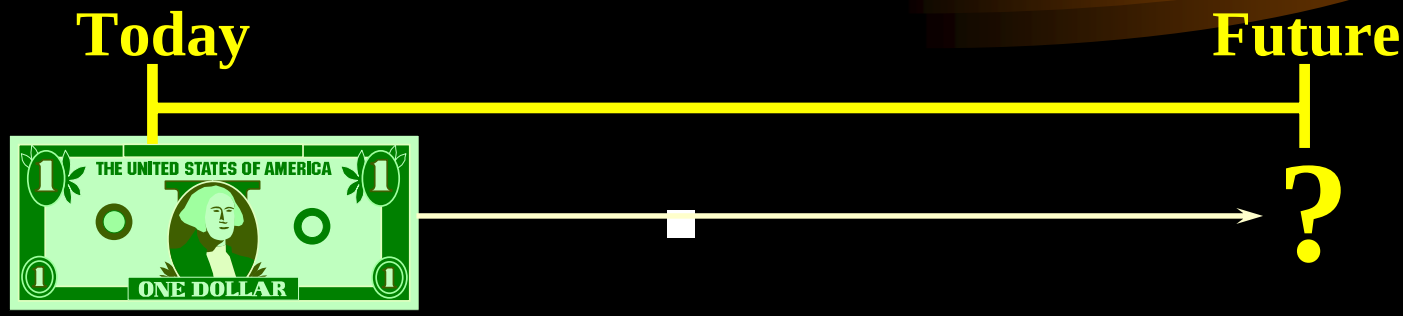


If we can measure this opportunity cost, we can:

- **Translate $1 today into its equivalent in the future *(compounding).***

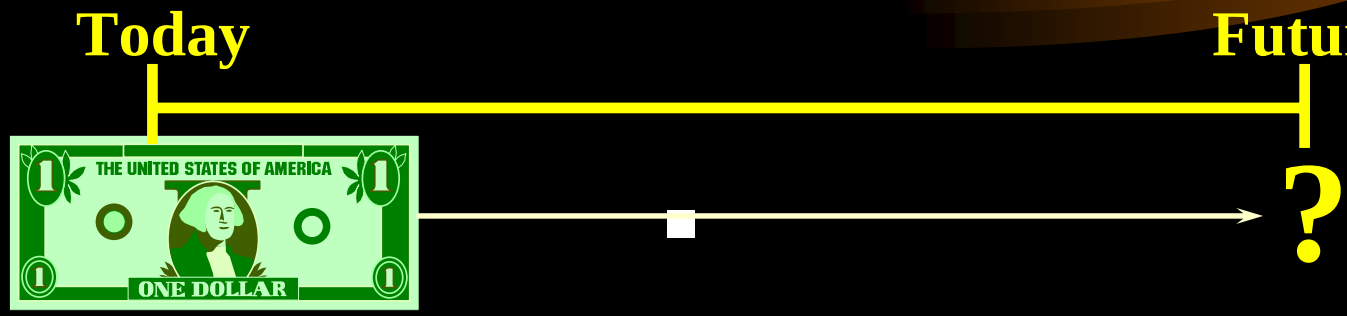


- **Translate $1 in the future into its equivalent today *(discounting).***

descontar

If we can measure this opportunity cost, we can:

- **Translate $1 today into its equivalent in the future *(compounding).***



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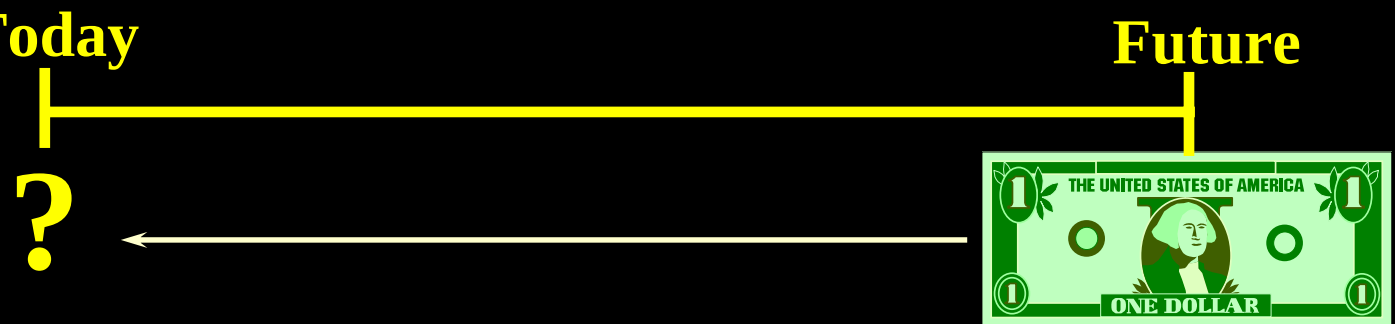

Future Value

Future Value: Time Value of Money

Assume the investment is $1,000. The interest rate is 8%. What is the future value if the money is invested for one year? Two? Three?

Future Value: Time Value of Money

$F = \$1{,}000(1.08) = \$1{,}080.00$ (after one year)

$F = \$1{,}000(1.08)^2 = \$1{,}166.40$ (after two years)

$F = \$1{,}000(1.08)^3 = \$1{,}259.71$ (after three years)

Future Value - single sums

If you deposit $100 in an account earning 6%, how much would you have in the account after 1 year?

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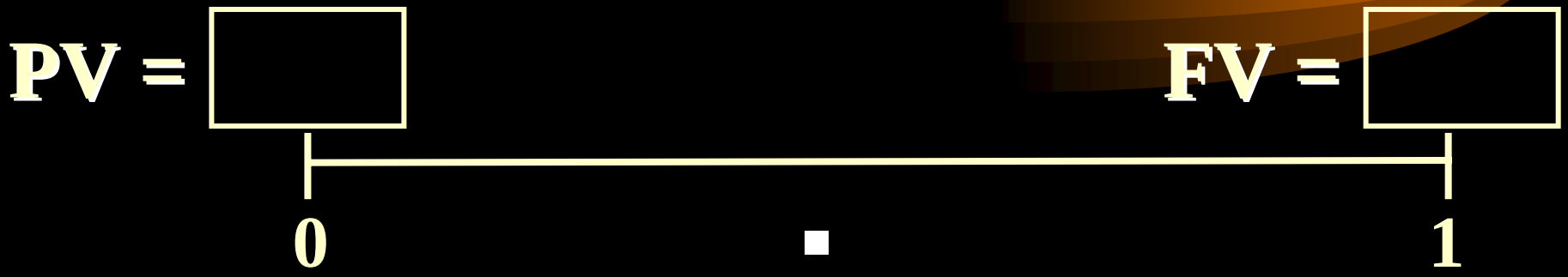

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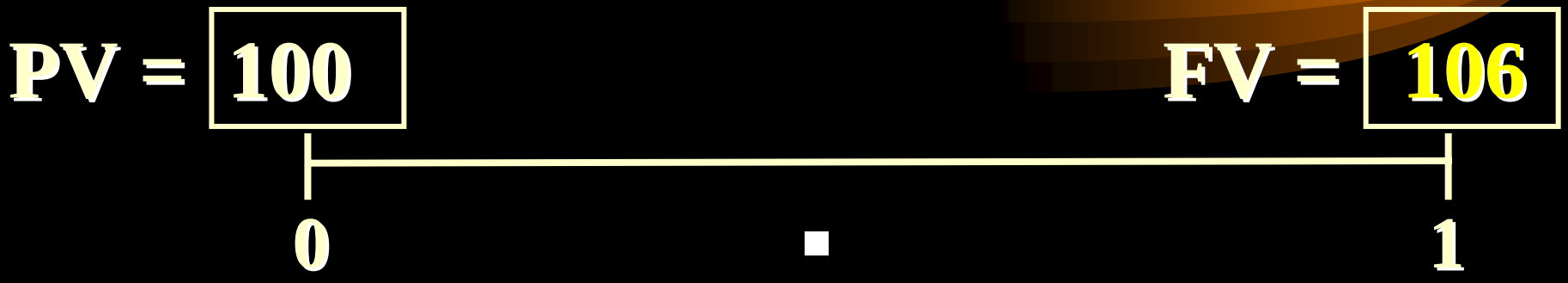

Mathematical Solution:

$$FV = PV (1 + i)^n$$

$$FV = 100 (1.06)^1 = \$106$$

Future Value - single sums

If you deposit $100 in an account earning 6%, how much would you have in the account after 5 years?

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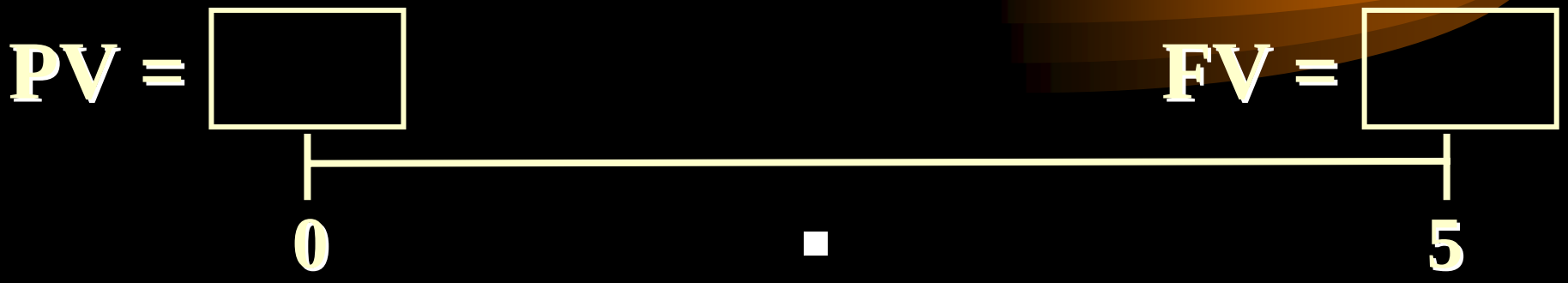

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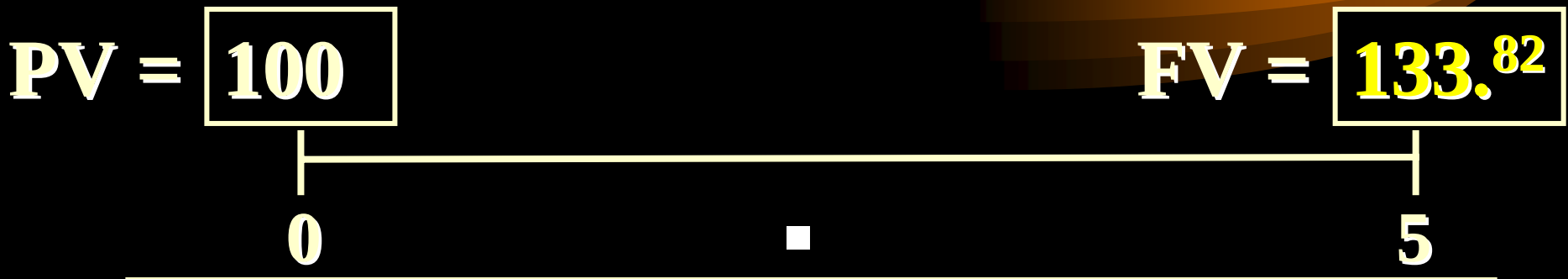


Mathematical Solution:

$FV = PV (1 + i)^n$

$FV = 100 (1.06)^5 = \$133.82$

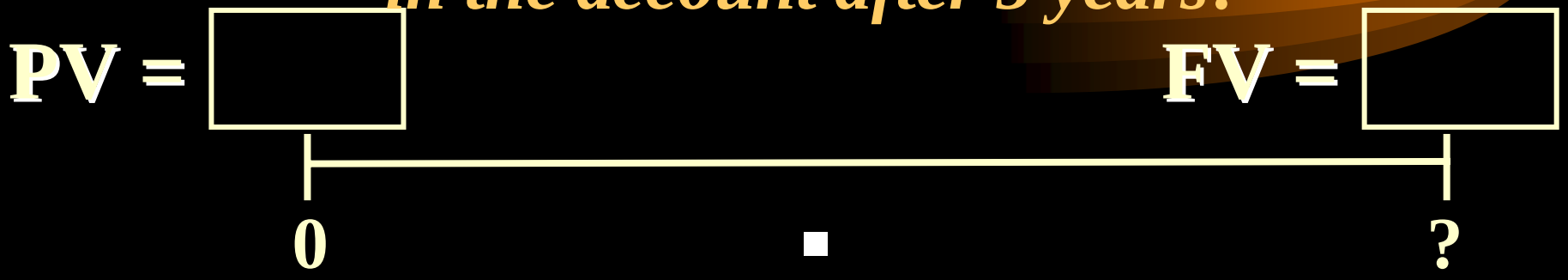
Future Value - single sums

If you deposit $100 in an account earning 6% with **quarterly compounding**, *how much would you have in the account after 5 years?*

capitalización

Future Value - single sums

If you deposit $100 in an account earning 6% with quarterly compounding, how much would you have in the account after 5 years?



Future Value - single sums

If you deposit $100 in an account earning 6% with quarterly compounding, how much would you have in the account after 5 years?

Mathematical Solution:

$$FV = PV\,(1 + i/m)^{m \times n}$$

$$FV = 100\,(1.015)^{20} = \$134.68$$

Future Value - single sums

If you deposit $100 in an account earning 6% with **monthly compounding**, *how much would you have in the account after 5 years?*

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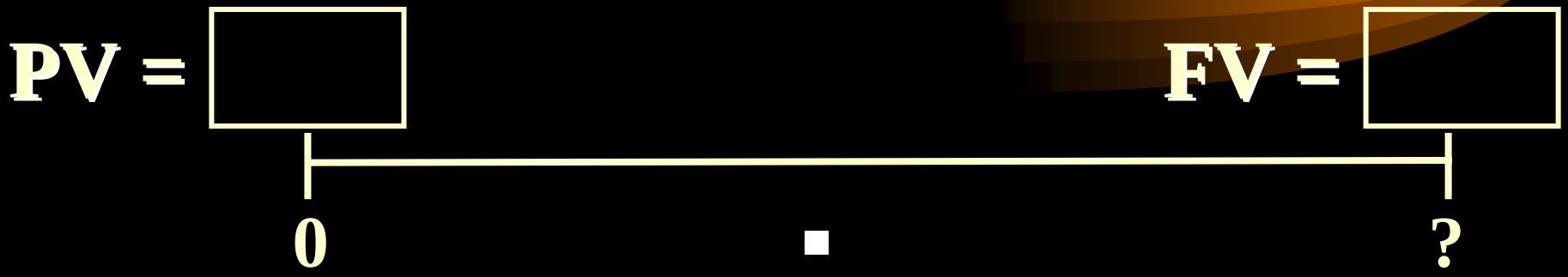
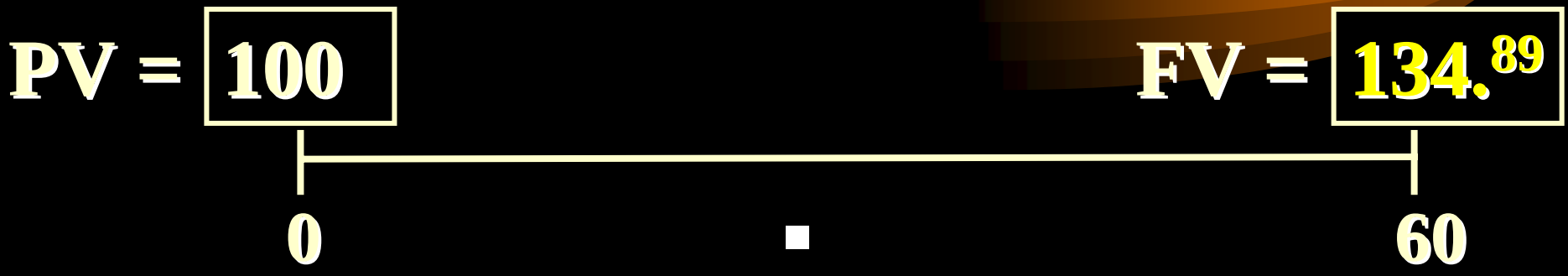


Future Value - single sums

If you deposit $100 in an account earning 6% with **monthly compounding**, *how much would you have in the account after 5 years?*



Mathematical Solution:

$$FV = PV\,(1 + i/m)^{m \times n}$$

$$FV = 100\,(1.005)^{60} = \$134.89$$

Future Value - continuous compounding

What is the FV of $1,000 earning 8% with **continuous compounding,** *after 100 years?*

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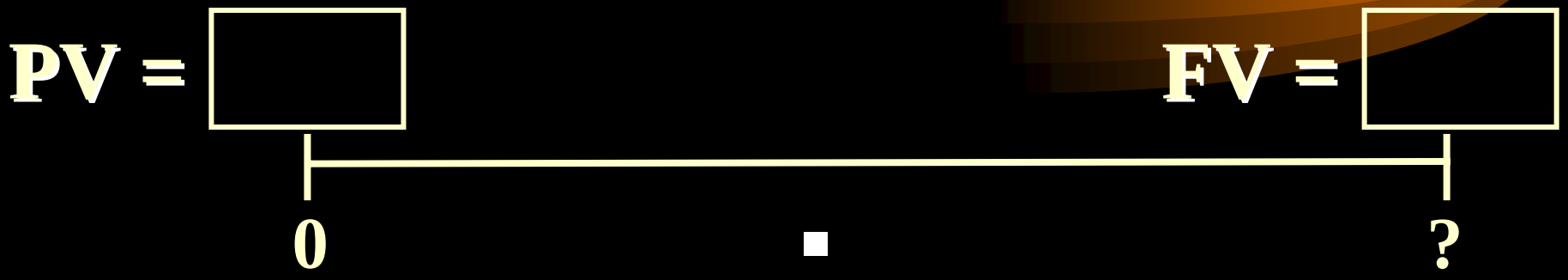

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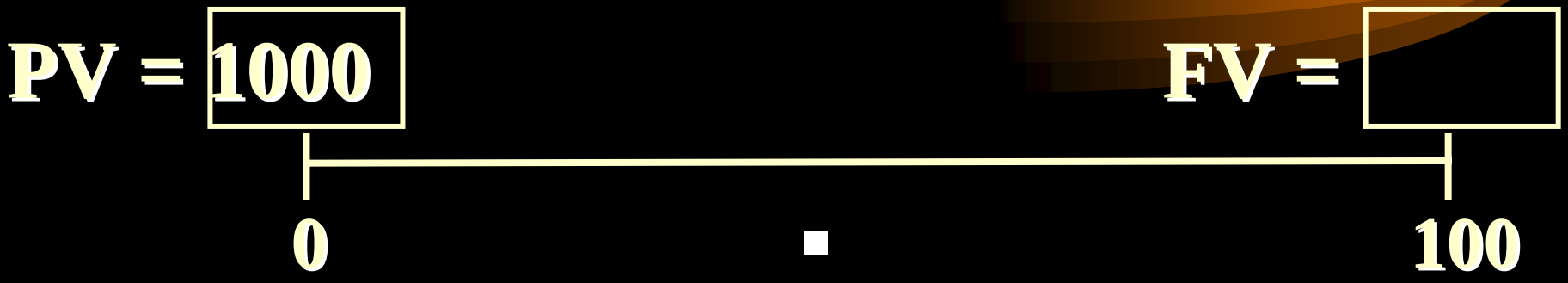

Mathematical Solution:

$$FV = PV\,(e^{in})$$

$$FV = 1000\,(e^{.08 \times 100}) = 1000\,(e^{8})$$

$$FV = \$2{,}980{,}957.^{99}$$

Future Value - continuous compounding

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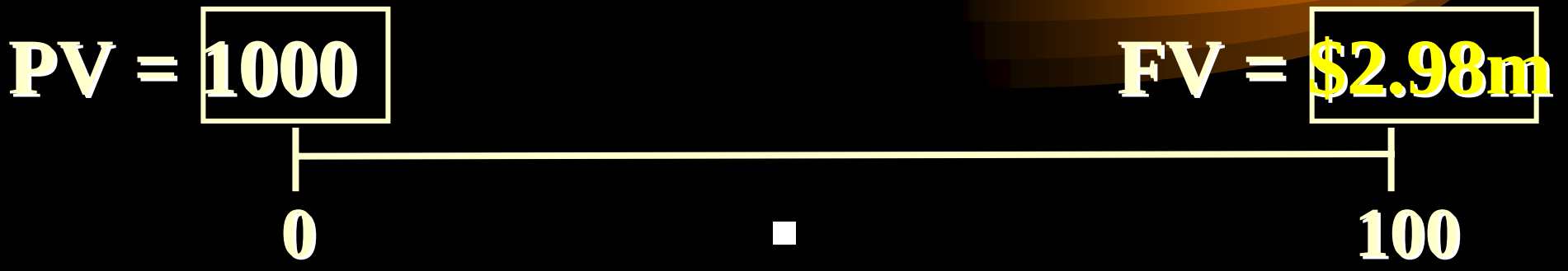

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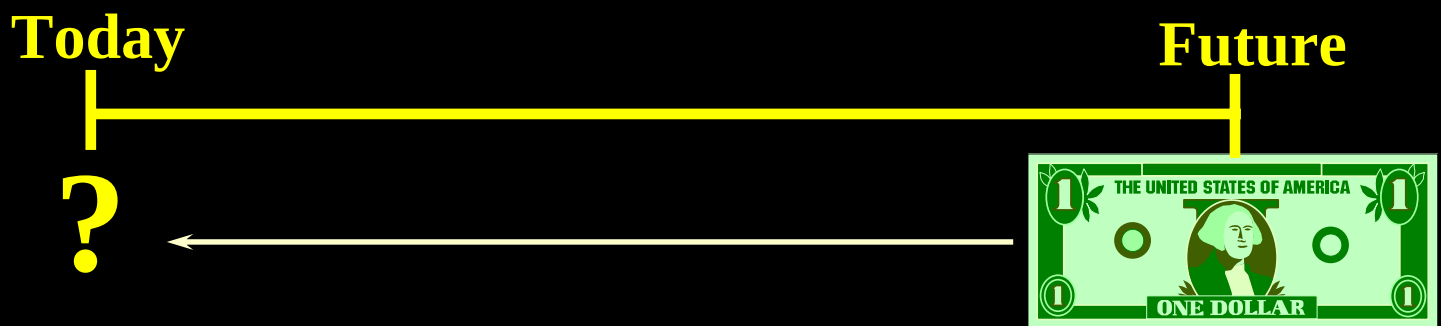
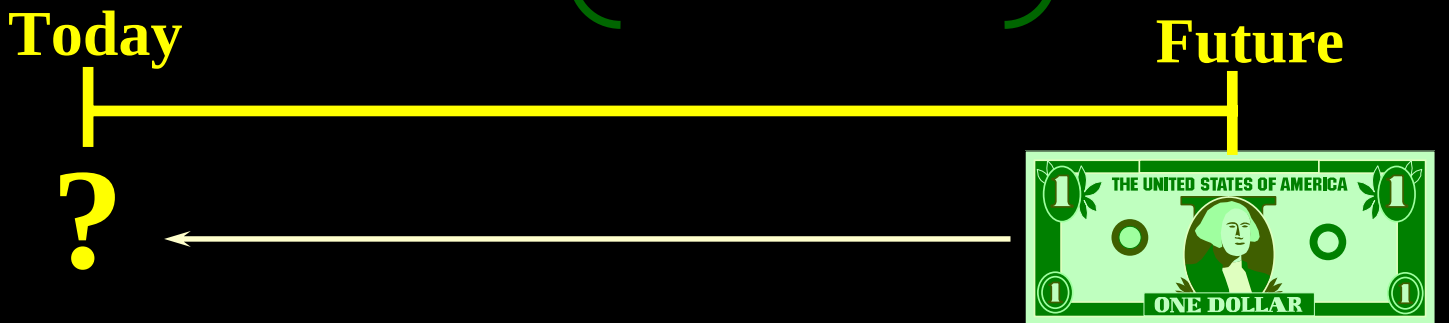
Present Value



Interest and the Time Value of Money

The present value of any sum to be received in the future can be computed by turning the interest formula around and solving for P:

$$P = FV\left[\frac{1}{(1+r)^n}\right]$$



Interest and the Time Value of Money

A bond will pay $100 in two years. What is the present value of the $100 if an investor can earn a return of 12% on investments?

$$P = 100 \left(\frac{1}{(1 + .12)^2} \right)$$

$$P = \$100\ (0.797)$$

$$P = \$79.70$$

Interest and the Time Value of Money

A bond will pay $100 in two years. What is the present value of the $100 if an investor can earn a return of 12% on investments?

Present Value = $79.70

What does this mean?
If $79.70 is put in the bank today,
it will be worth $100 in two years.
valdrá
In that sense, $79.70 today is
equivalent to $100 in two years.

Interest and the Time Value of Money

Let's verify that if we put $79.70 in the bank today at 12% interest that it would grow to $100 at the end of two years.

	Year 1	*Year 2*
Beginning balance	$ 79.70	$ 89.26
Interest @ 12%	$ 9.56	$ 10.71
Ending balance	$ 89.26	$ 99.97

We can also determine the present value using present value tables.

Time Value of Money

Present value tables

Periods	Rate		
	10%	12%	14%
1	0.909	0.893	0.877
2	0.826	0.797	0.769
3	0.751	0.712	0.675
4	0.683	0.636	0.592
5	0.621	0.567	0.519

Time Value of Money

$100 × 0.797 = $79.70 present value

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	10%	12%	14%
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Present value factor of $1 for 2 periods at 12%.

Quick Check ✓

How much would you have to put in the bank today to have $100 at the end of five years if the interest rate is 10%?

a. $62.10

b. $56.70

c. $90.90

d. $51.90

Quick Check ✓

How much would you have to put in the bank today to have $100 at the end of five years if the interest rate is 10%?

a. $62.10

b. $56.70

c. $90.90

d. $51.90

$$\$100 \times 0.621 = \$62.10$$

	Rate		
Periods	10%	12%	14%
1	0.909	0.893	0.877
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Present Value - single sums

If you receive $100 one year from now, what is the PV of that $100 if your opportunity cost is 6%?

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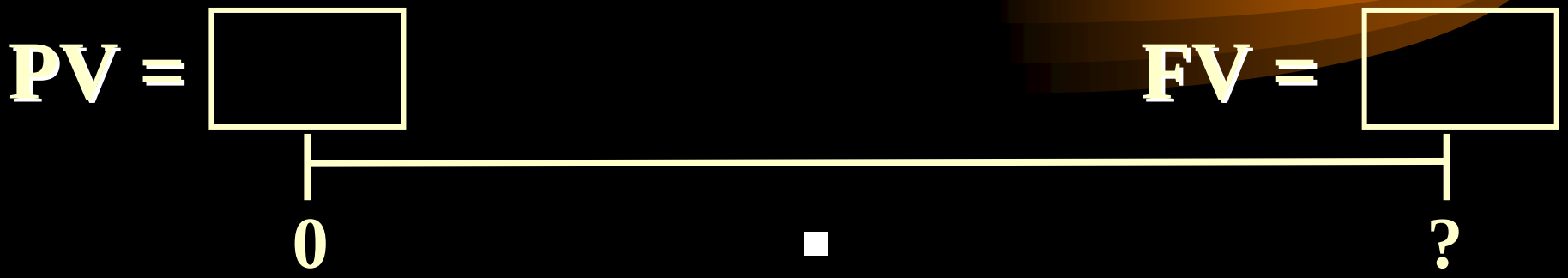

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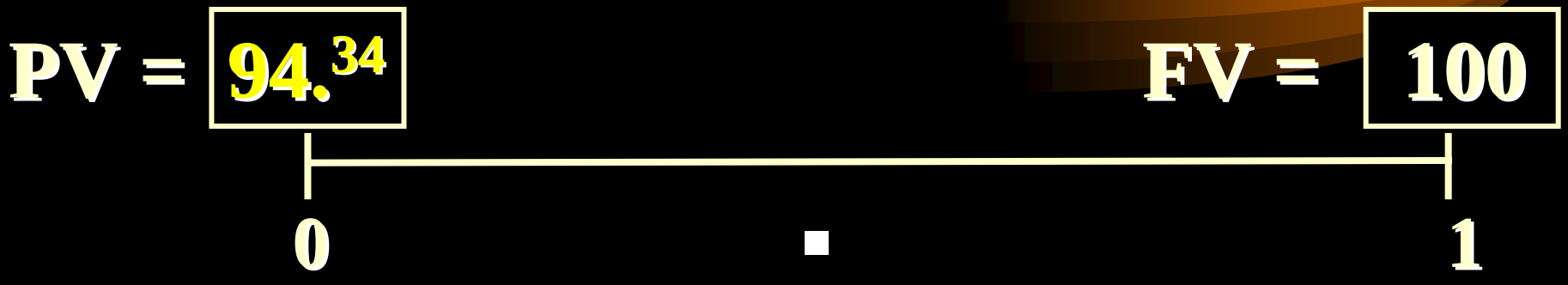

Mathematical Solution:

$PV = FV / (1 + i)^n$

$PV = 100 / (1.06)^1 = \$94.34$

Present Value - single sums

If you receive $100 five years from now, what is the PV of that $100 if your opportunity cost is 6%?

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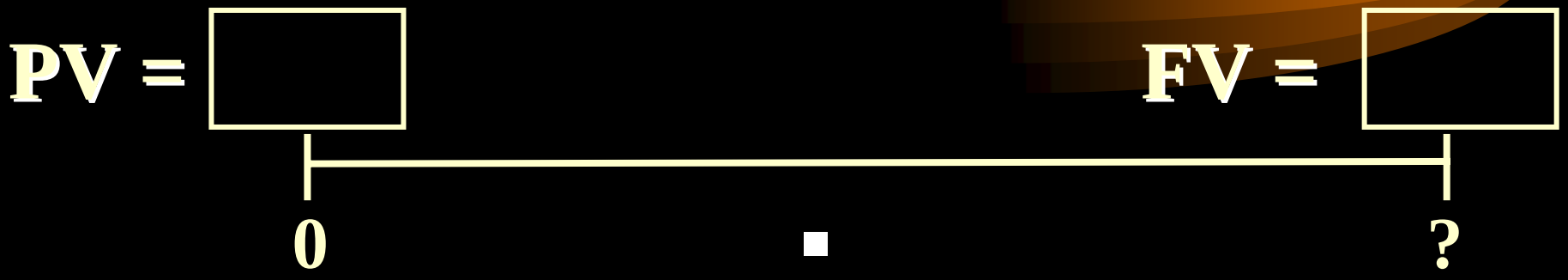

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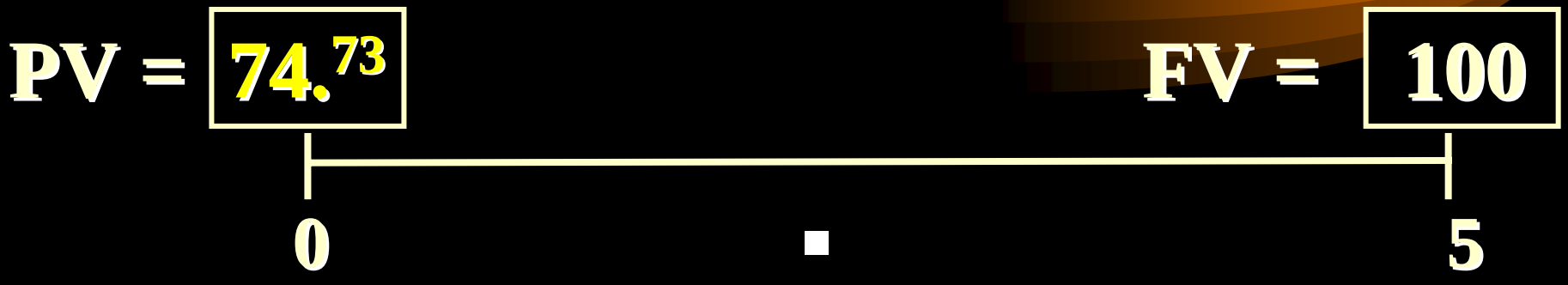

Mathematical Solution:

$$PV = FV / (1 + i)^n$$

$$PV = 100 / (1.06)^5 = \$74.73$$

Present Value - single sums

What is the PV of $1,000 to be received 15 years from now if your opportunity cost is 7%?

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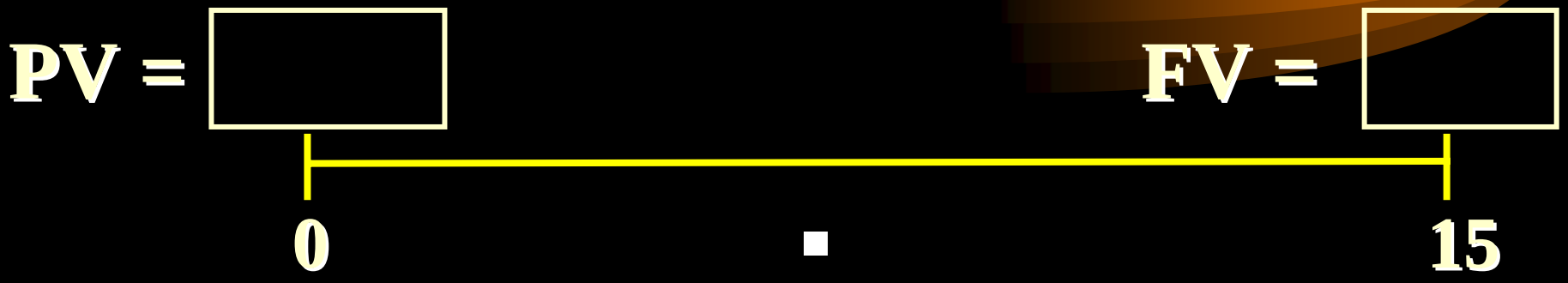

Present Value - single sums

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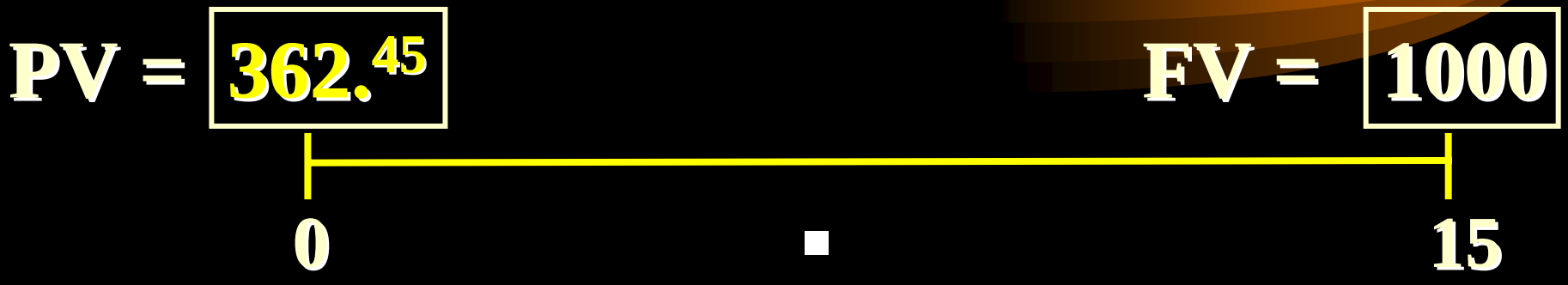

Mathematical Solution:

$$PV = FV / (1 + i)^n$$

$$PV = 100 / (1.07)^{15} = \$362.45$$

Present Value - single sums

If you sold land for $11,933 that you bought 5 years ago for $5,000, what is your annual rate of return?

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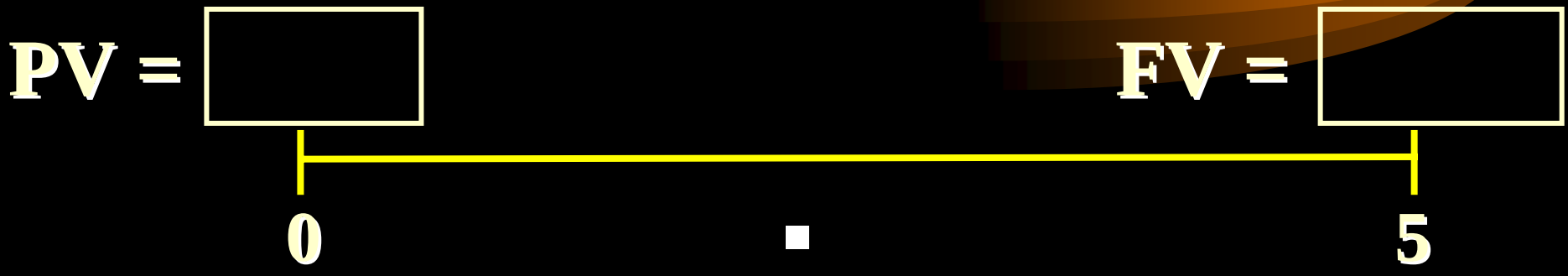

Present Value - single sums

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Mathematical Solution:

$$PV = FV / (1 + i)^n$$

$$5{,}000 = 11{,}933 / (1+ i)^5$$

$$.419 = ((1/ (1+i)^5)$$

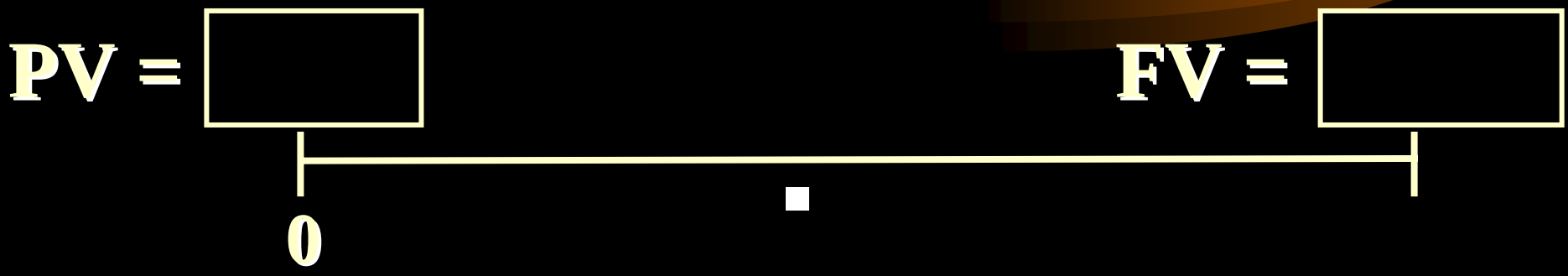
$$2.3866 = (1+i)^5$$

$$(2.3866)^{1/5} = (1+i) \qquad i = .19$$

Present Value - single sums

Suppose you placed $100 in an account that pays 9.6% interest, compounded monthly. How long will it take for your account to grow to $500?

capitalizable



Present Value - single sums

Suppose you placed $100 in an account that pays 9.6% interest, compounded monthly. How long will it take for your account to grow to $500?

Mathematical Solution:

$$PV = FV / (1 + i)^n$$

$$100 = 500 / (1+ .008)^N$$

$$5 = (1.008)^N$$

$$\ln 5 = \ln (1.008)^N$$

$$\ln 5 = N \ln (1.008)$$

$$1.60944 = .007968\ N \qquad N = 202 \text{ months}$$

Hint for single sum problems:

- **In every single sum future value and present value problem, there are 4 variables:**
- **FV, PV, i, and n**
- **When doing problems, you will be given 3 of these variables and asked to solve for the 4th variable.**
- **Keeping this in mind makes "time value" problems much easier!**

The Time Value of Money

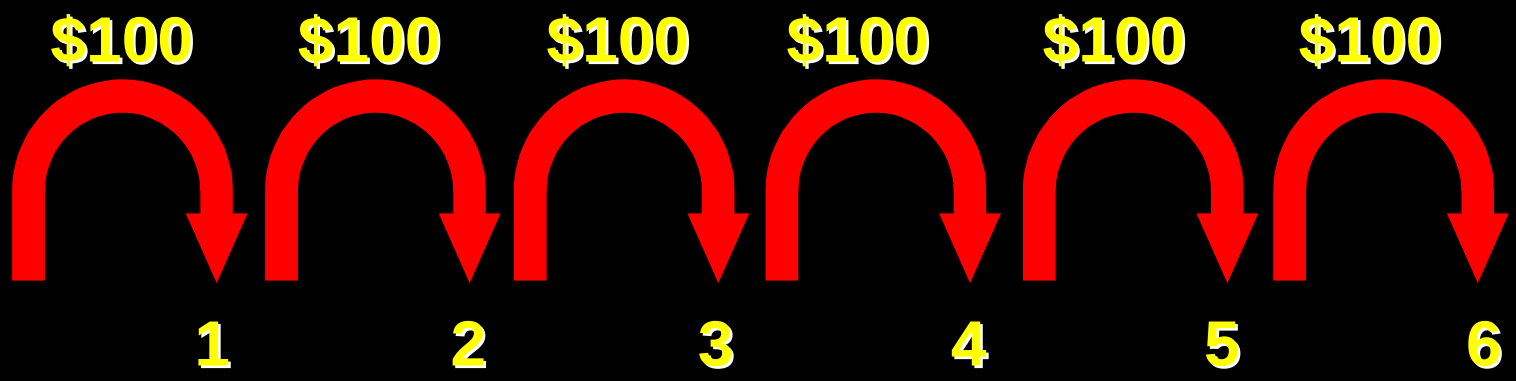
Compounding and Discounting

capitalizar descontar

Cash Flow Streams

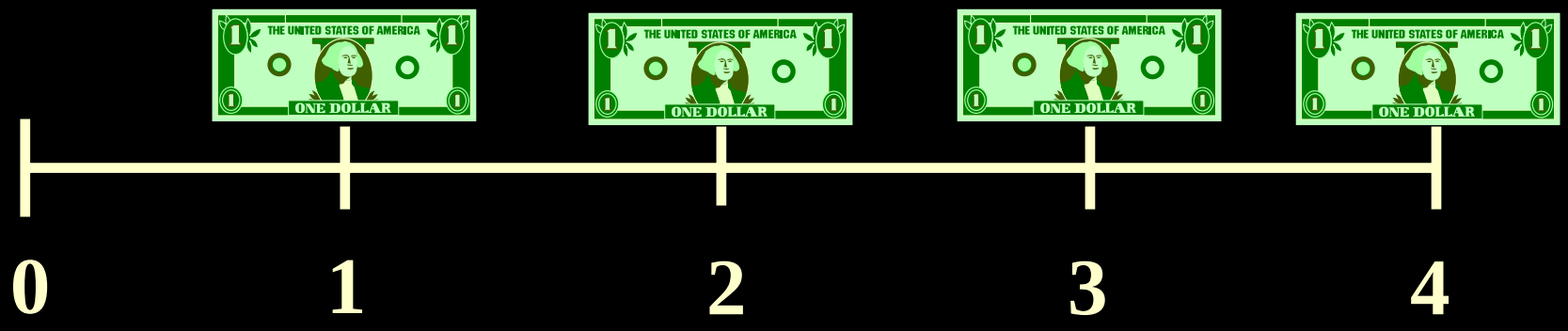
Time Value of Money

An investment that involves a series of identical cash flows at the end of each year is called an annuity.



Annuities

- **Annuity:** a sequence of **equal** cash flows, occurring at the end of each period.

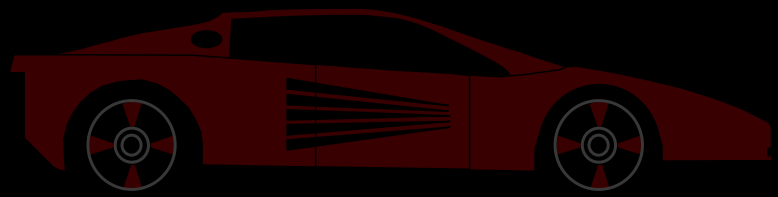


Examples of Annuities:

- **If you buy a bond, you will receive equal semi-annual coupon interest payments over the life of the bond.**
- **If you borrow money to buy a house or a car, you will pay a stream of equal payments.**

Pides prestado

flujo



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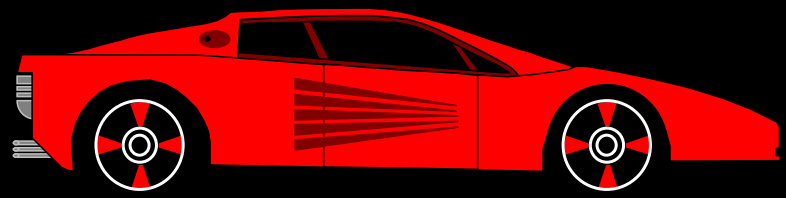

Future Value - annuity

If you invest $1,000 each year at 8%, how much would you have after 3 years?

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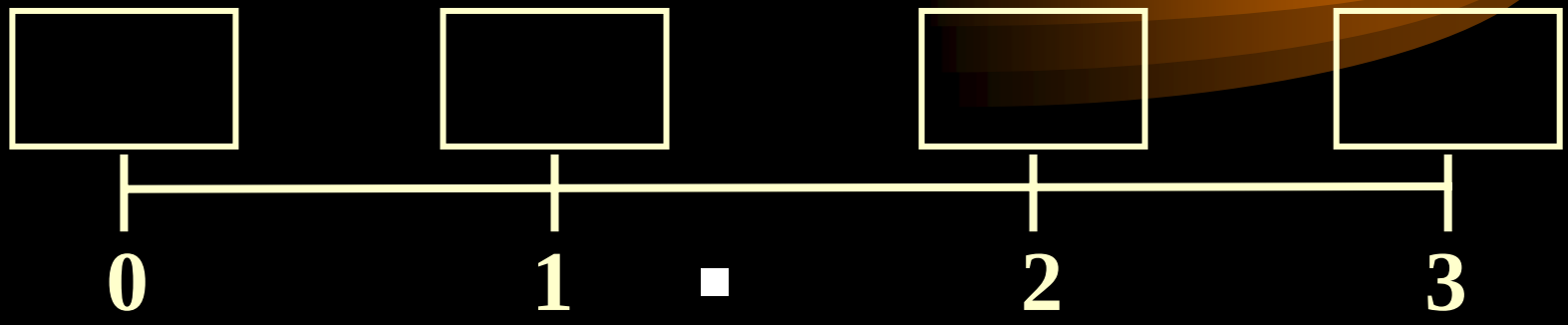

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Mathematical Solution:

Future Value - annuity

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Mathematical Solution:

$$FV = PMT\ (FVIFA_{i,\ n})$$

Future Value - annuity

If you invest $1,000 each year at 8%, how much would you have after 3 years?

Mathematical Solution:

$$FV = PMT\,(FVIFA_{i,\,n})$$

$$FV = 1{,}000\,(FVIFA_{.08,\,3}) \quad \text{(use table, or)}$$

Future Value - annuity

If you invest $1,000 each year at 8%, how much would you have after 3 years?

Mathematical Solution:

$$FV = PMT\,(FVIFA_{i,\,n})$$

$$FV = 1{,}000\,(FVIFA_{.08,\,3}) \quad \text{(use table, or)}$$

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

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$$FV = 1{,}000\left[\frac{(1.08)^3 - 1}{.08}\right] = \$3246.40$$

Present Value - annuity

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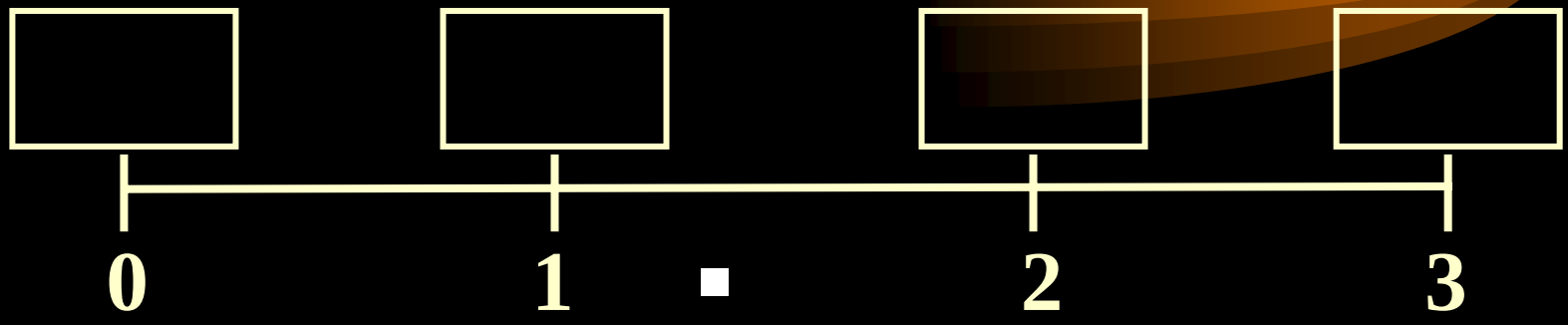

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$$PV = 1000 \left[\frac{1 - \frac{1}{(1.08)^3}}{.08} \right] = \$2,577.10$$

Time Value of Money

Lacey Inc. purchased a tract of land on which a $60,000 payment will be due each year for the next five years. What is the present value of this stream of cash payments when the discount rate is 12%?

flujo

Time Value of Money

We could solve the problem like this . . .

Table Present Value of series of $1 Cash Flows

Periods	10%	12%	14%
1	0.909	0.893	0.877
2	1.736	1.690	1.647
3	2.487	2.402	2.322
4	3.170	3.037	2.914
5	3.791	3.605	3.433

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$60,000 × 3.605 = $216,300

Quick Check ✓

If the interest rate is 14%, how much would you have to put in the bank today so as to be able to withdraw $100 at the end of each of the next five years?

retirar

a. $34.33

b. $500.00

c. $343.30

d. $360.50

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$100 × 3.433 = $343.30

Quick Check ✓

If the interest rate is 14%, what is the present value of $100 to be received at the end of the 3rd, 4th, and 5th years?

a. $866.90

b. $178.60

c. $ 86.90

d. $300.00

Quick Check ✓

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$100×(3.433-1.647)= $100×1.786 = $178.60

The Time Value of Money

Other Cash Flow Patterns

Perpetuities

- Suppose you will receive a fixed payment every period (month, year, etc.) forever. This is an example of a perpetuity.
- You can think of a perpetuity as an annuity that goes on forever.

Present Value of a Perpetuity

- **When we find the PV of an annuity, we think of the following relationship:**
  - 

Present Value of a Perpetuity

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$$PV = PMT\ (PVIFA_{i,n})$$

Mathematically,

$$(PVIFA\ \ i, n\ ) = \frac{1 - \frac{1}{(1+i)^n}}{i}$$

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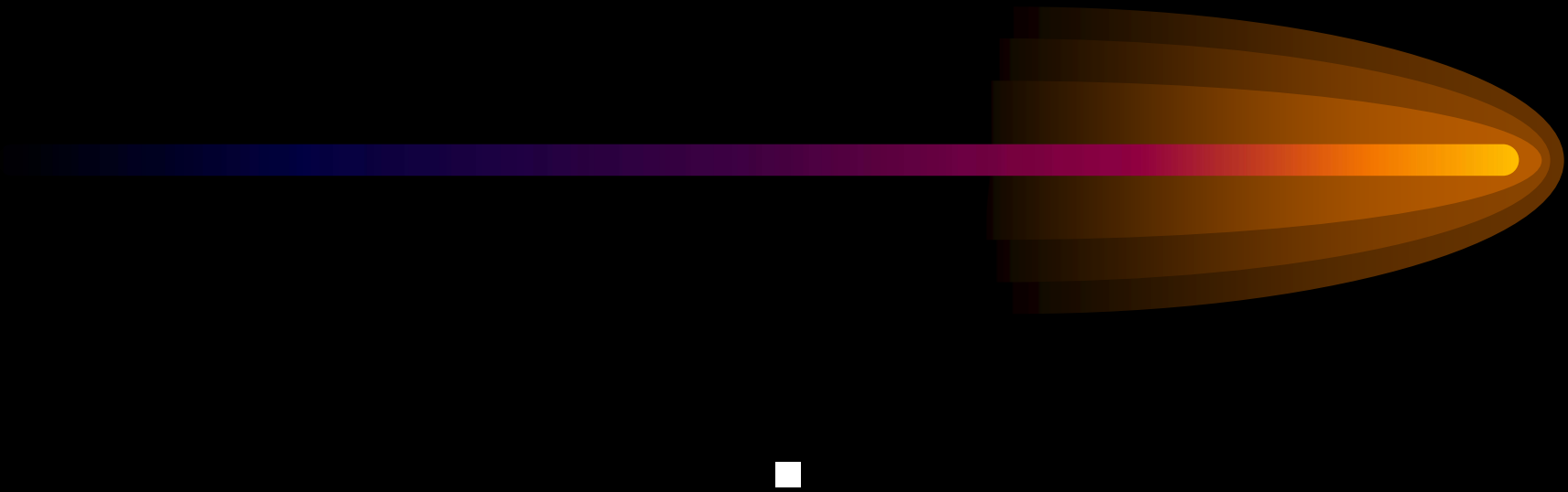
$$(PVIFA\ \ i, n\ ) = \frac{1 - \frac{1}{(1+i)^n}}{i}$$

We said that a perpetuity is an annuity where n = infinity. What happens to this formula when n gets very, very large?

When n gets very large,



When n gets very large,

$$\frac{1}{\frac{1 - (1 + i)^{n}}{i}}$$

.

When n gets very large,

$$\frac{1 - \frac{1}{(1+i)^n}}{i}$$

this becomes zero.

■

When n gets very large,

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this becomes zero.

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So we're left with PVIFA =

$$\frac{1}{i}$$

Present Value of a Perpetuity

- So, the PV of a perpetuity is very simple to find:

$$\blacksquare$$

Present Value of a Perpetuity

- So, the PV of a perpetuity is very simple to find:

$$PV = \frac{PMT}{i}$$

What should you be willing to pay in order to receive $10,000 annually forever, if you require 8% per year on the investment?

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$$PV = \frac{PMT}{i} = \frac{\$10,000}{.08}$$

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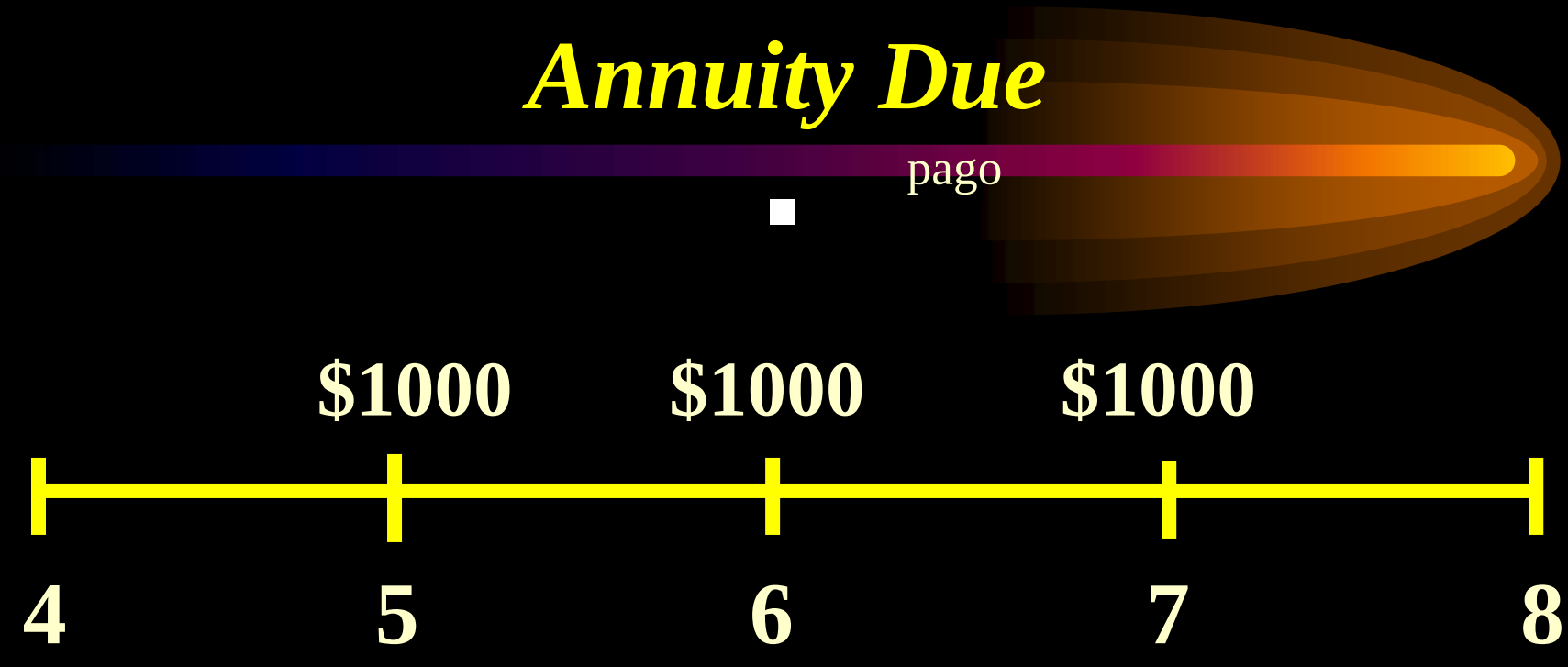
$$PV = \frac{PMT}{i} = \frac{\$10,000}{.08}$$

$$= \$125,000$$

Ordinary Annuity

vs.

Annuity Due



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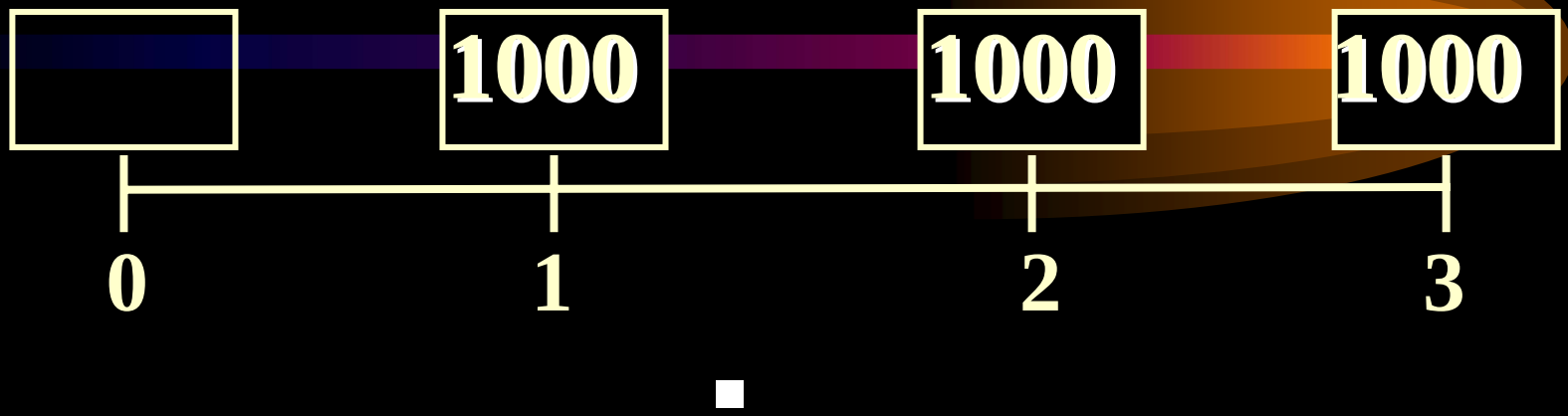

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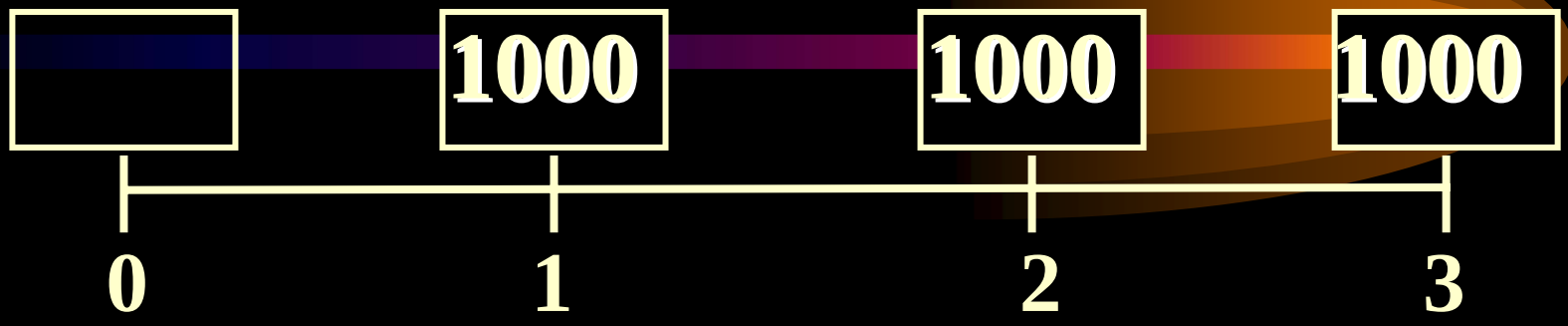

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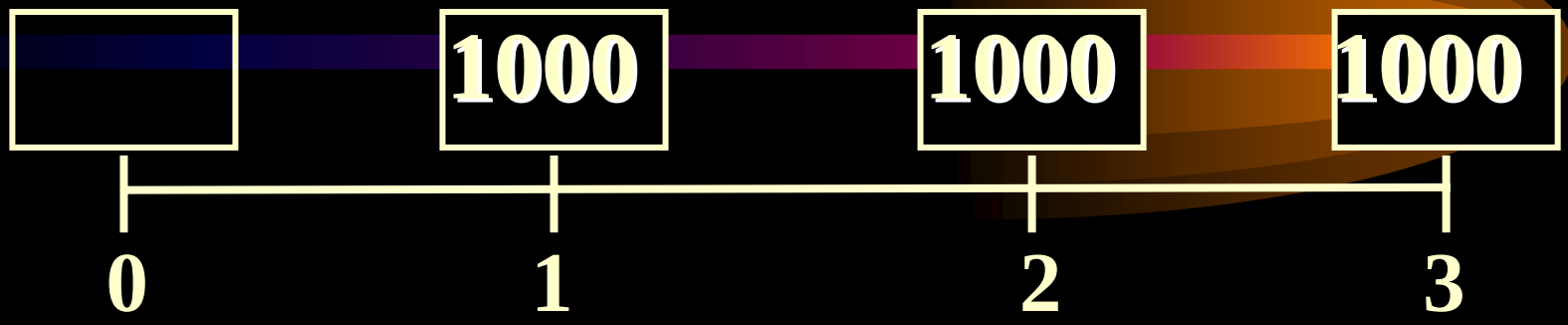

Using an interest rate of 8%, we find that:

- **The Future Value (at 3) is $3,246.40.**

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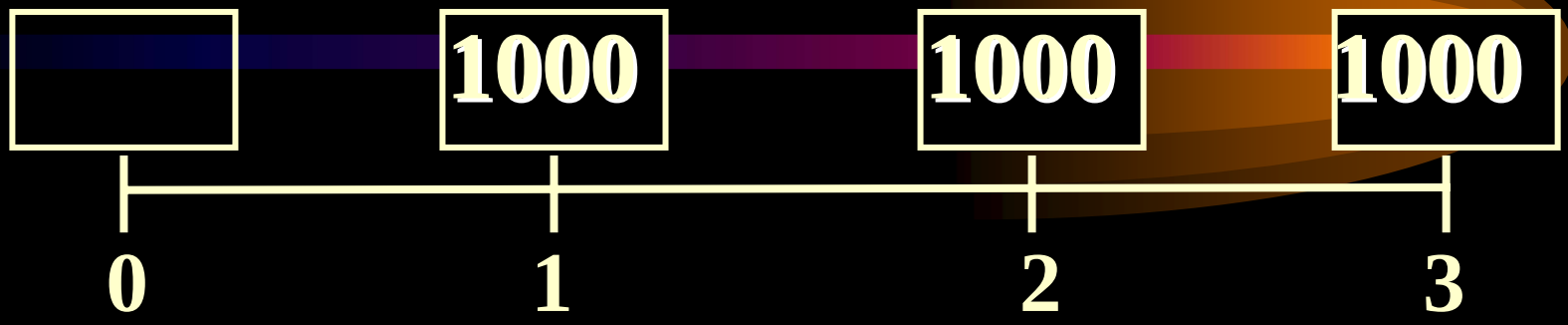
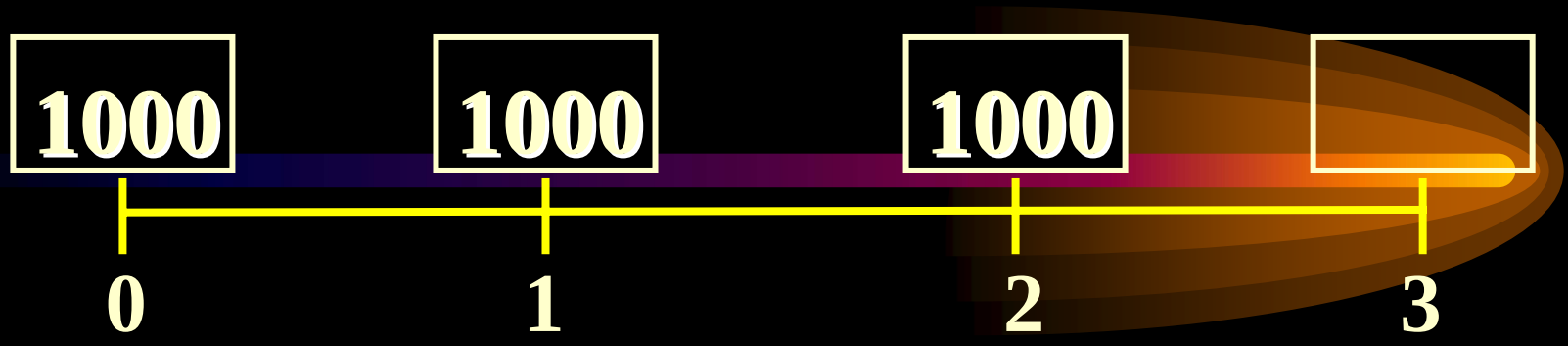


Using an interest rate of 8%, we find that:

- **The Future Value (at 3) is $3,246.40.**
- **The Present Value (at 0) is $2,577.10.**

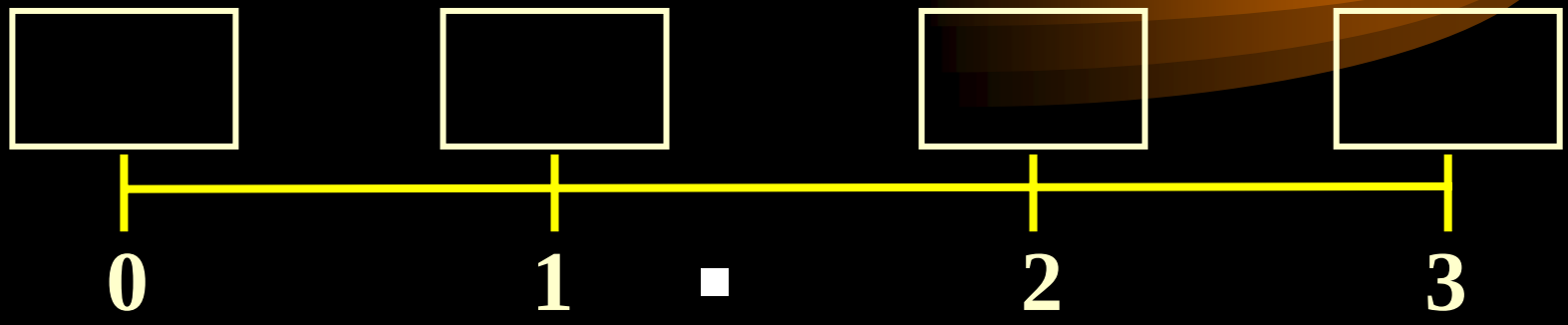
What about this annuity?



- Same 3-year time line,
- Same 3 $1000 cash flows, but
- The cash flows occur at the beginning of each year, rather than at the end of each year.
- This is an *"annuity due."*

Future Value - annuity due

If you invest $1,000 at the beginning of each of the next 3 years at 8%, how much would you have at the end of year 3?



Future Value - annuity due

If you invest $1,000 at the beginning of each of the next 3 years at 8%, how much would you have at the end of year 3?

Mathematical Solution: **Simply compound the FV of the ordinary annuity one more period:**

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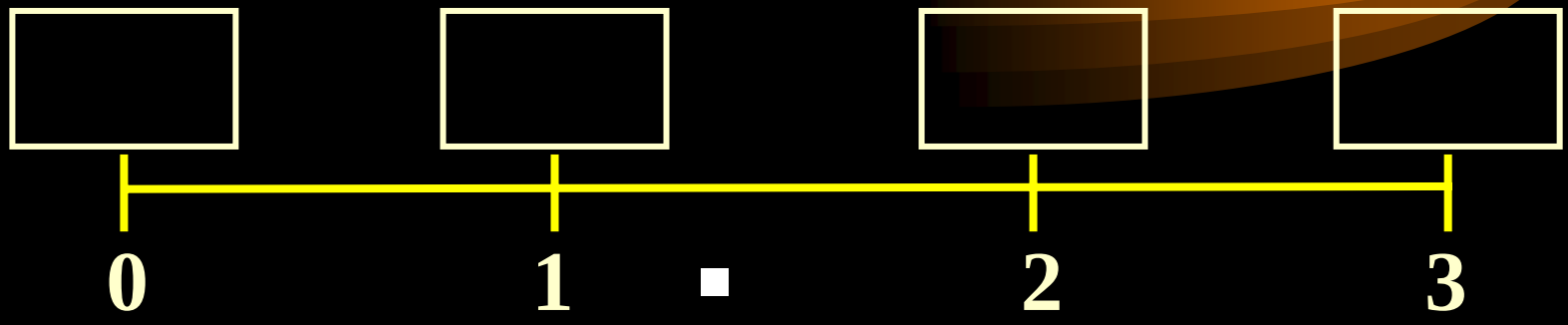
$$FV = 1{,}000\left[\frac{(1.08)^3 - 1}{.08}\right](1.08) = \$3{,}506.11$$

Present Value - annuity due

What is the PV of $1,000 at the beginning of each of the next 3 years, if your opportunity cost is 8%?



Present Value - annuity due

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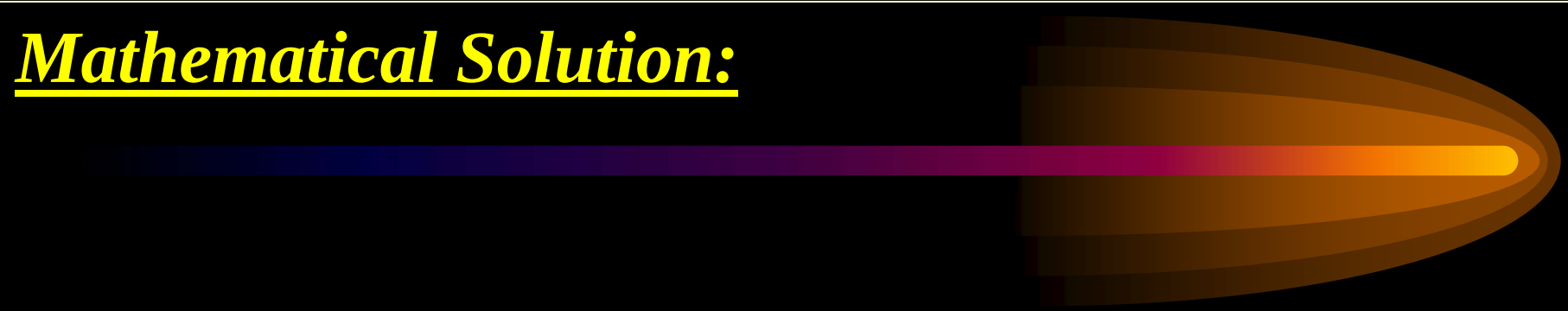

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$$PV = 1000 \left[\frac{1 - \frac{1}{(1.08)^3}}{.08} \right] (1.08) = \$2{,}783.26$$

Uneven Cash Flows

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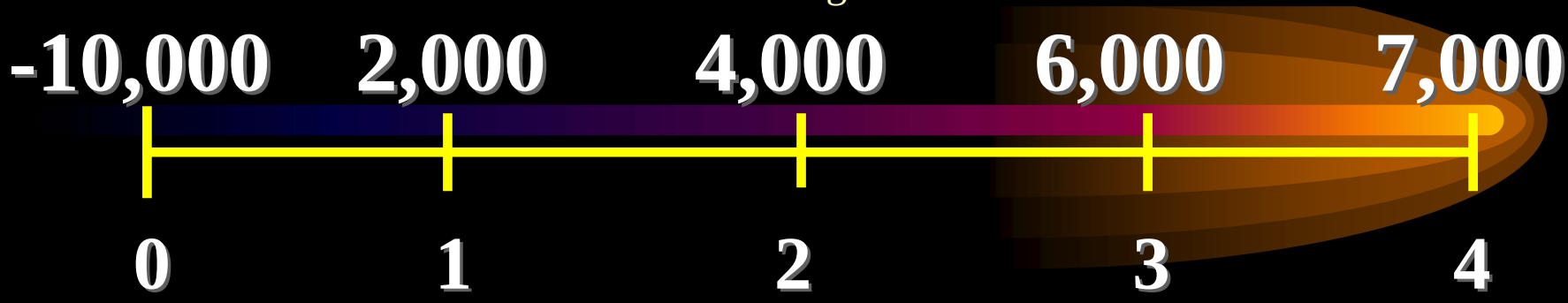
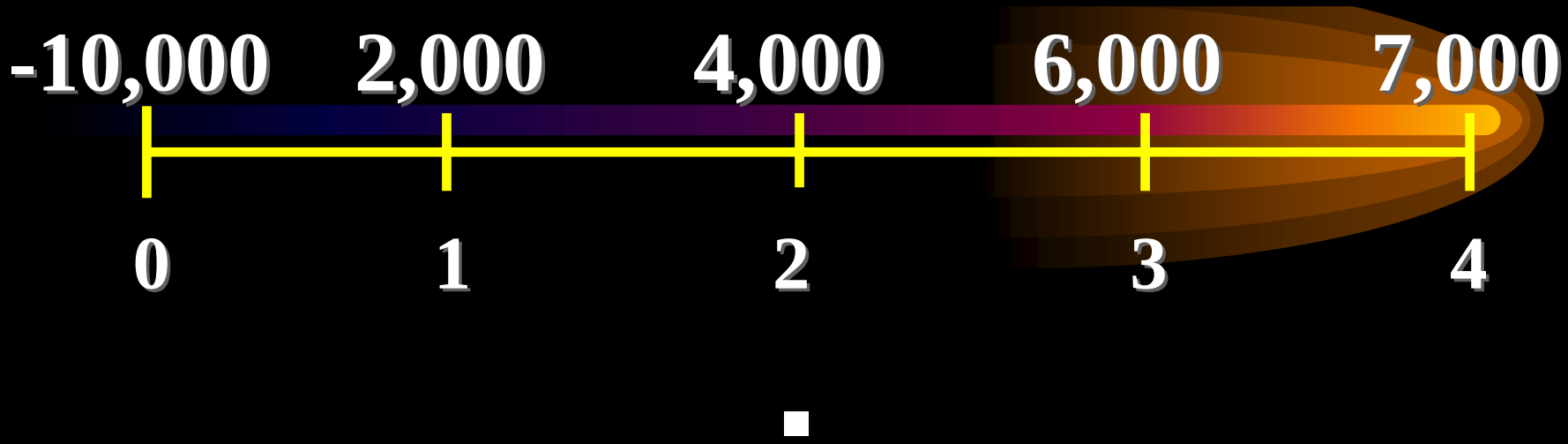


- **Is this an annuity?**
- **How do we find the PV of a cash flow stream when all of the cash flows are different? (Use a 10% discount rate).**

Uneven Cash Flows



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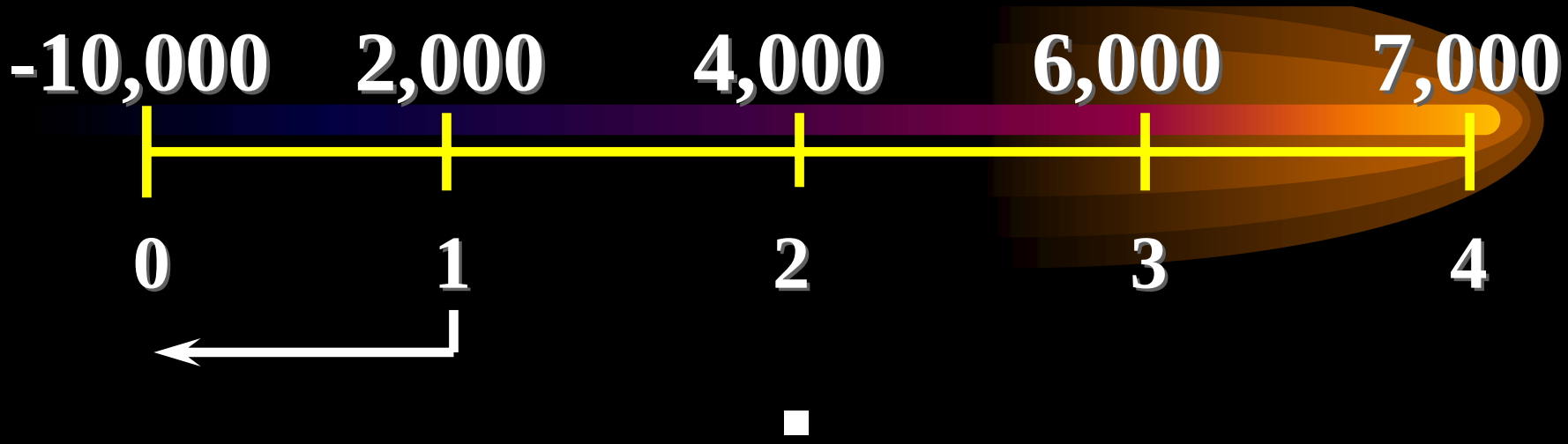

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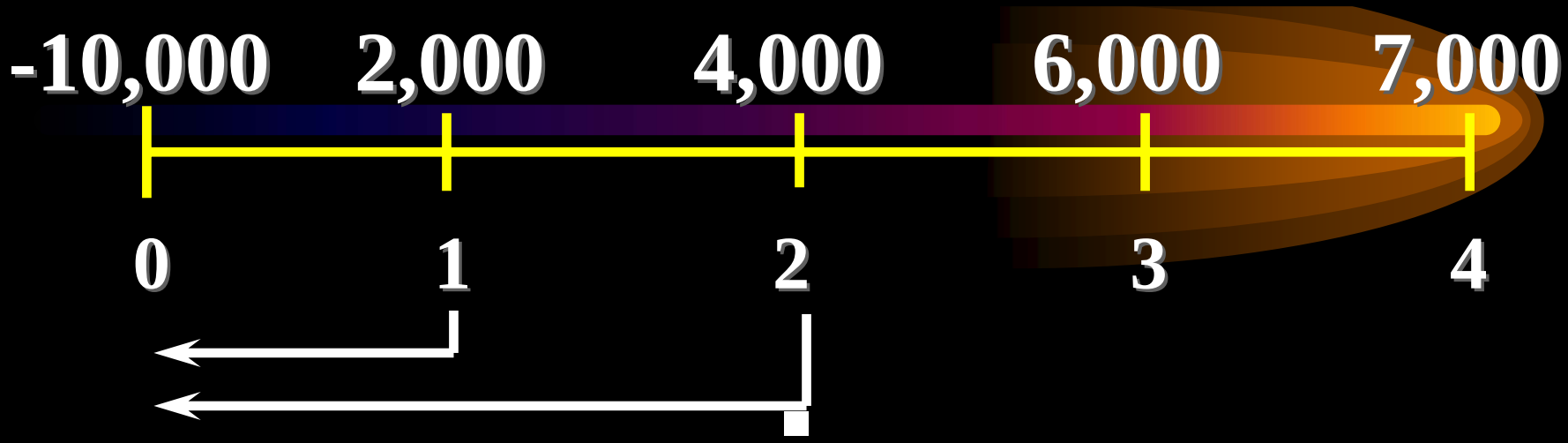

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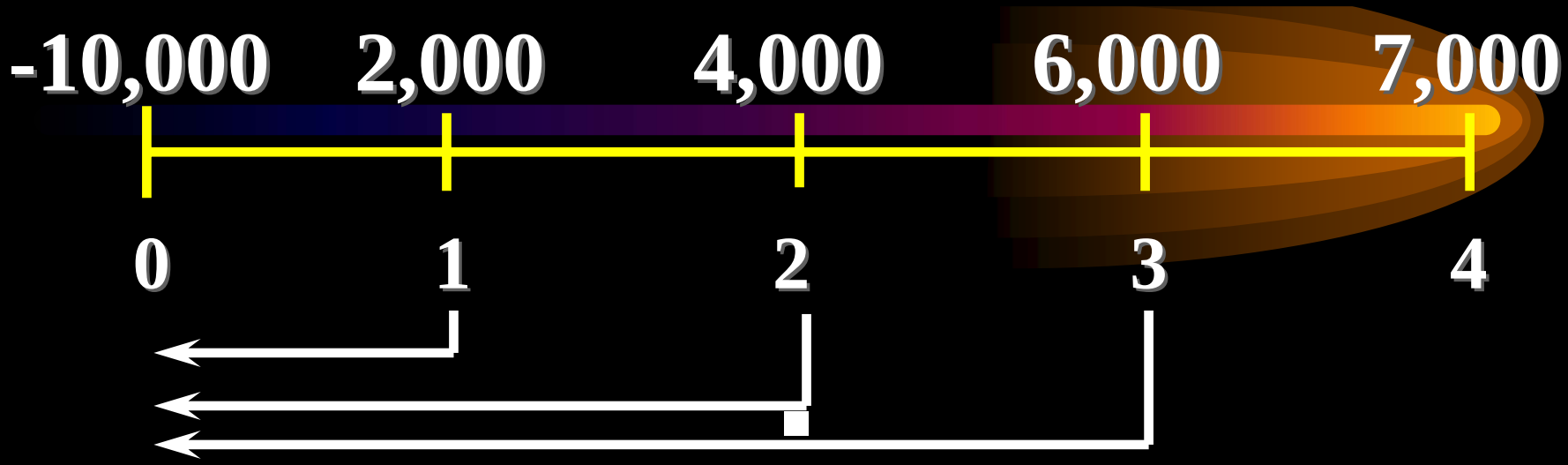

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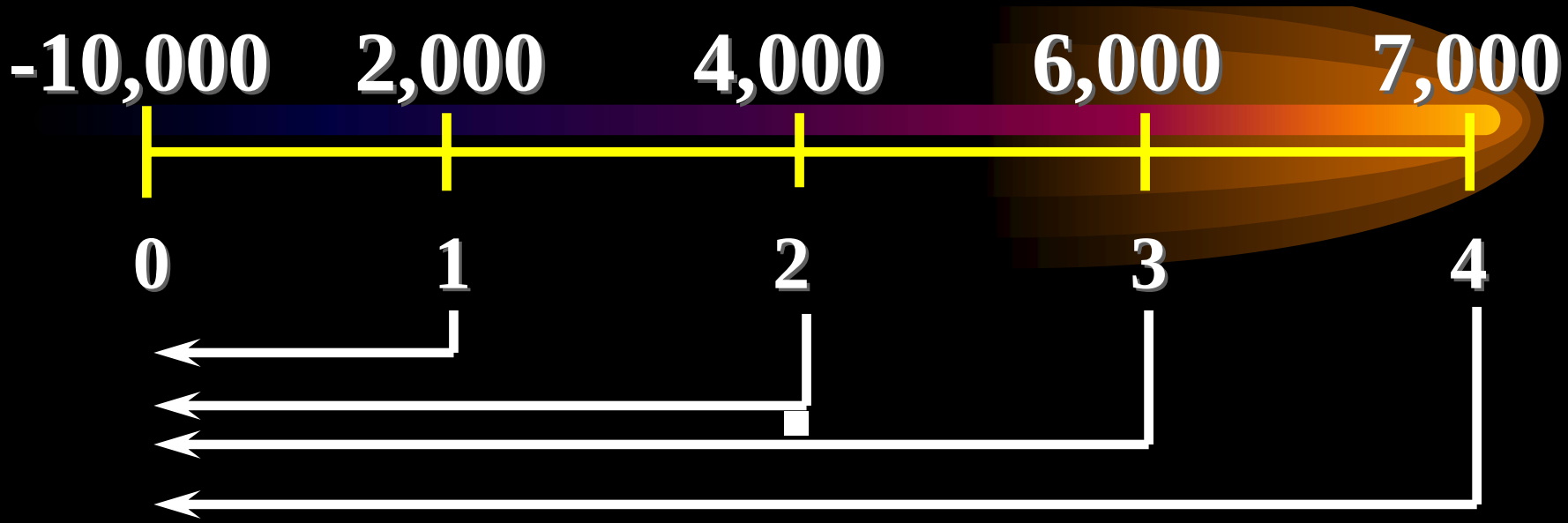

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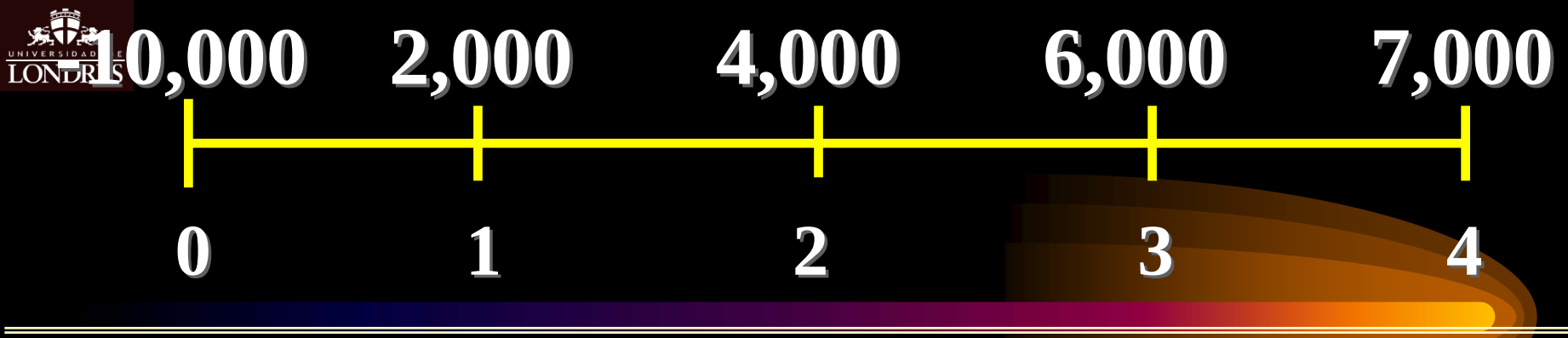

period	CF	PV (CF)
0	-10,000	-10,000.00
1	2,000	1,818.18
2	4,000	3,305.79
3	6,000	4,507.89
4	7,000	4,781.09
PV of Cash Flow Stream:		**$ 4,412.95**

Annual Percentage Yield (APY)

Which is the better loan:

- **8%** compounded annually, or
- **7.85%** compounded quarterly?
- We can't compare these nominal (quoted) interest rates, because they don't include the same number of compounding periods per year!

cotizadas

We need to calculate the APY.

Annual Percentage Yield (APY)

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$$APY = \left(1 + \frac{\text{quoted rate}}{m}\right)^{m} - 1$$

-

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- Find the APY for the quarterly loan:
  - ■

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$$APY = \left(1 + \frac{\text{quoted rate}}{m}\right)^{m} - 1$$

- Find the APY for the quarterly loan:

$$APY = \left(1 + \frac{.0785}{4}\right)^{4} - 1$$

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- Find the APY for the quarterly loan:

$$APY = \left(1 + \frac{.0785}{4}\right)^{4} - 1$$

$$APY = .0808, \text{ or } 8.08\%$$

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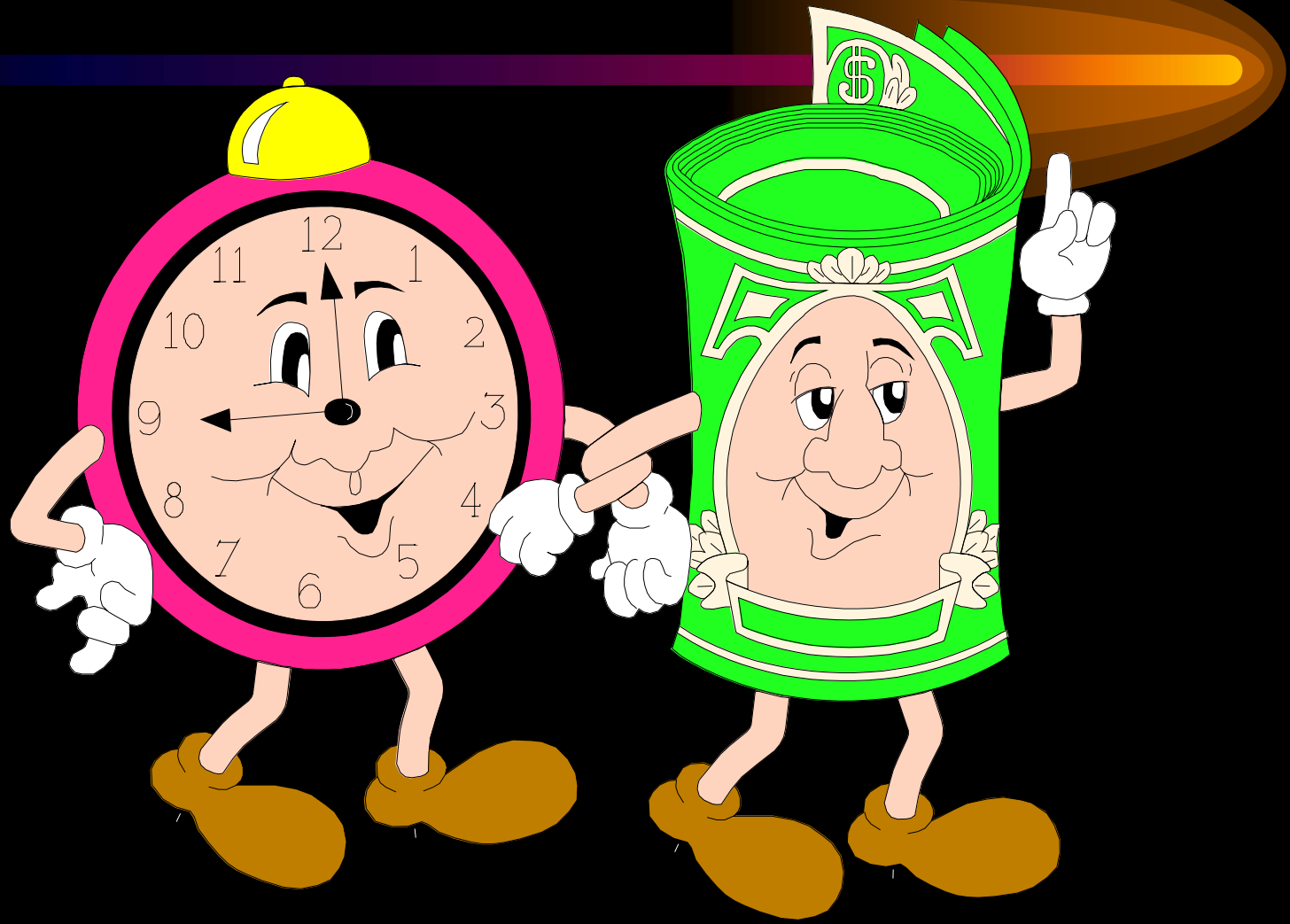
- Find the APY for the quarterly loan:

$$APY = \left(1 + \frac{.0785}{4}\right)^{4} - 1$$

$$APY = .0808, \text{ or } 8.08\%$$

- The quarterly loan is more expensive than the 8% loan with annual compounding!

Practice Problems



Example

- Cash flows from an investment are expected to be $40,000 per year at the end of years 4, 5, 6, 7, and 8. If you require a 20% rate of return, what is the PV of these cash flows?

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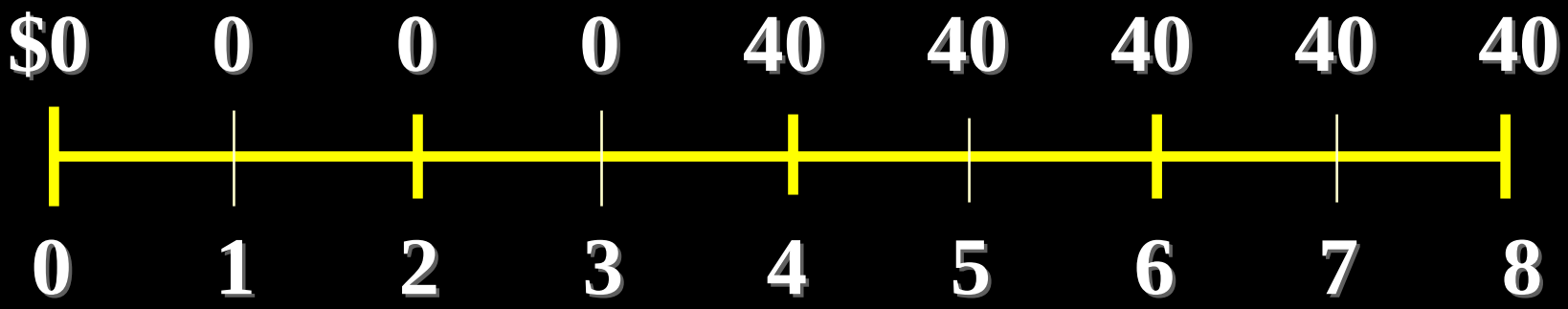
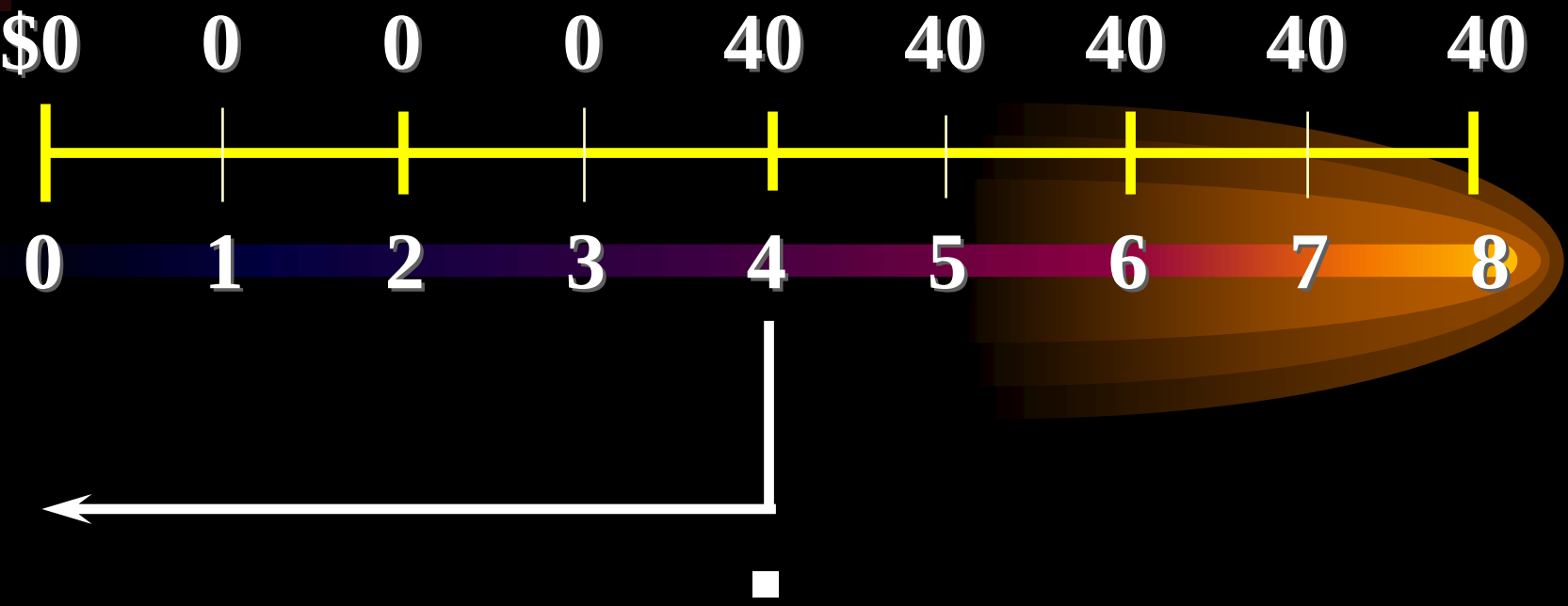


- This type of cash flow sequence is often called a "deferred annuity."

How to solve:

1) **Discount each cash flow back to time 0 separately.**



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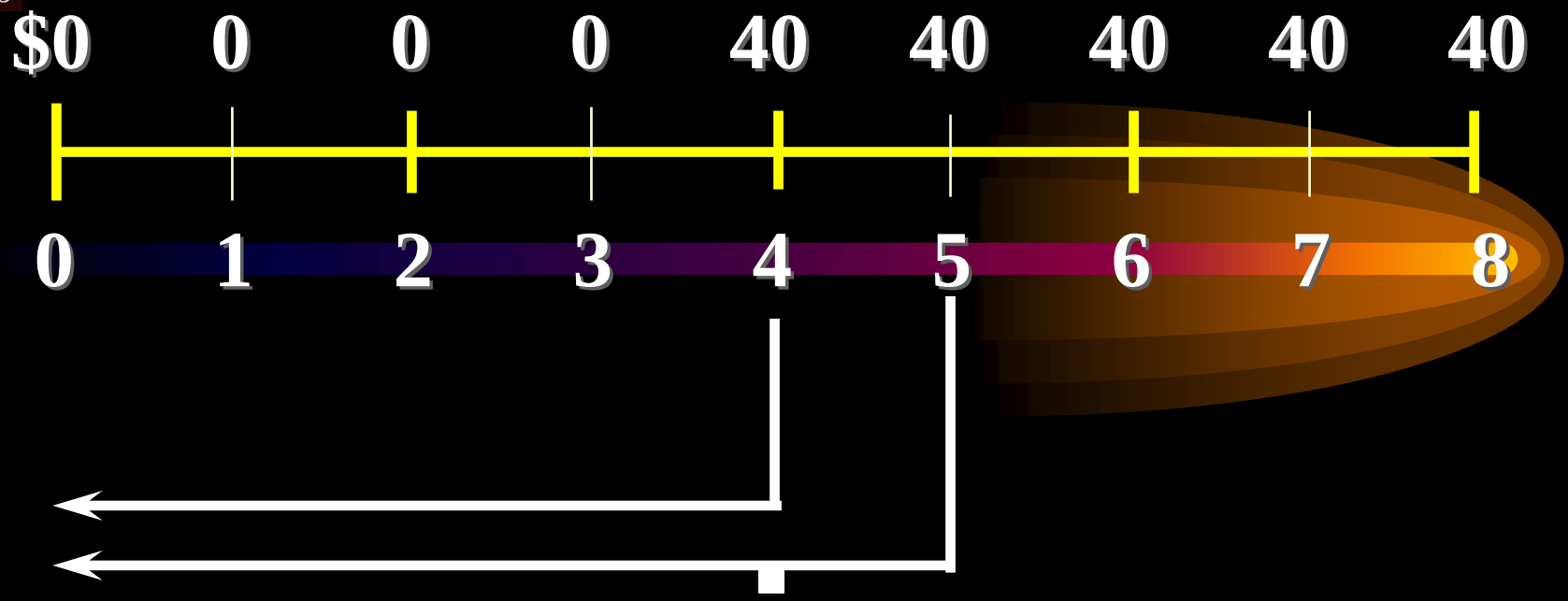

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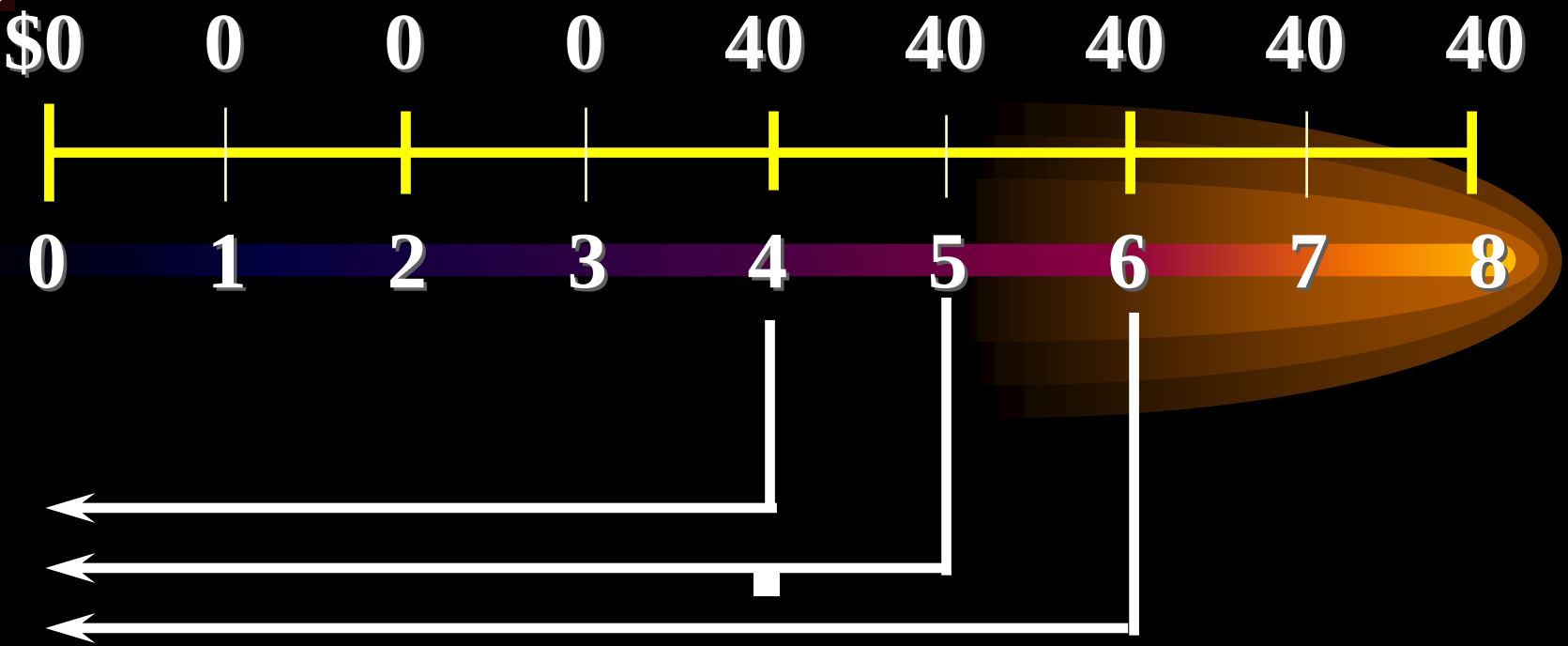

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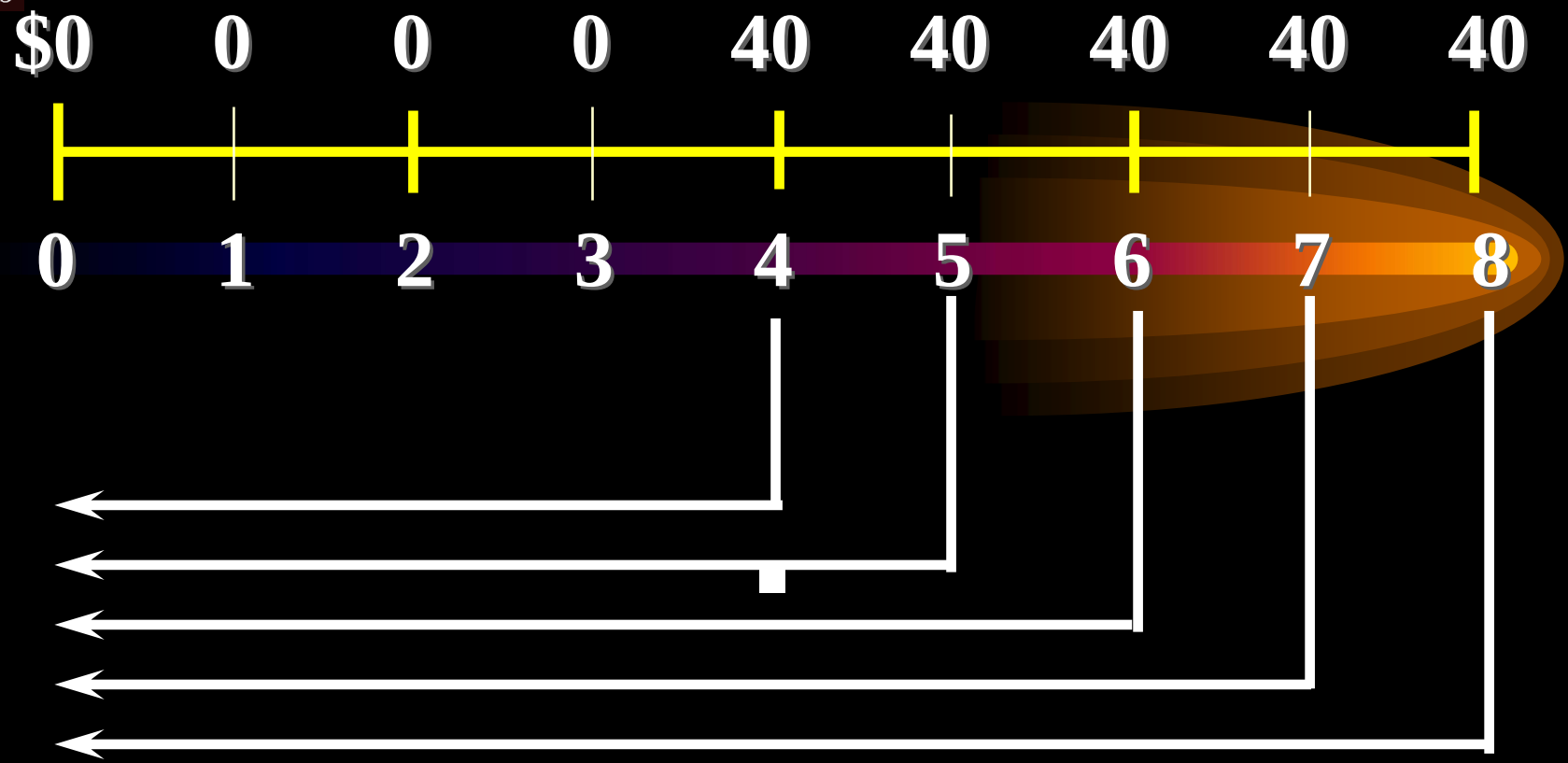

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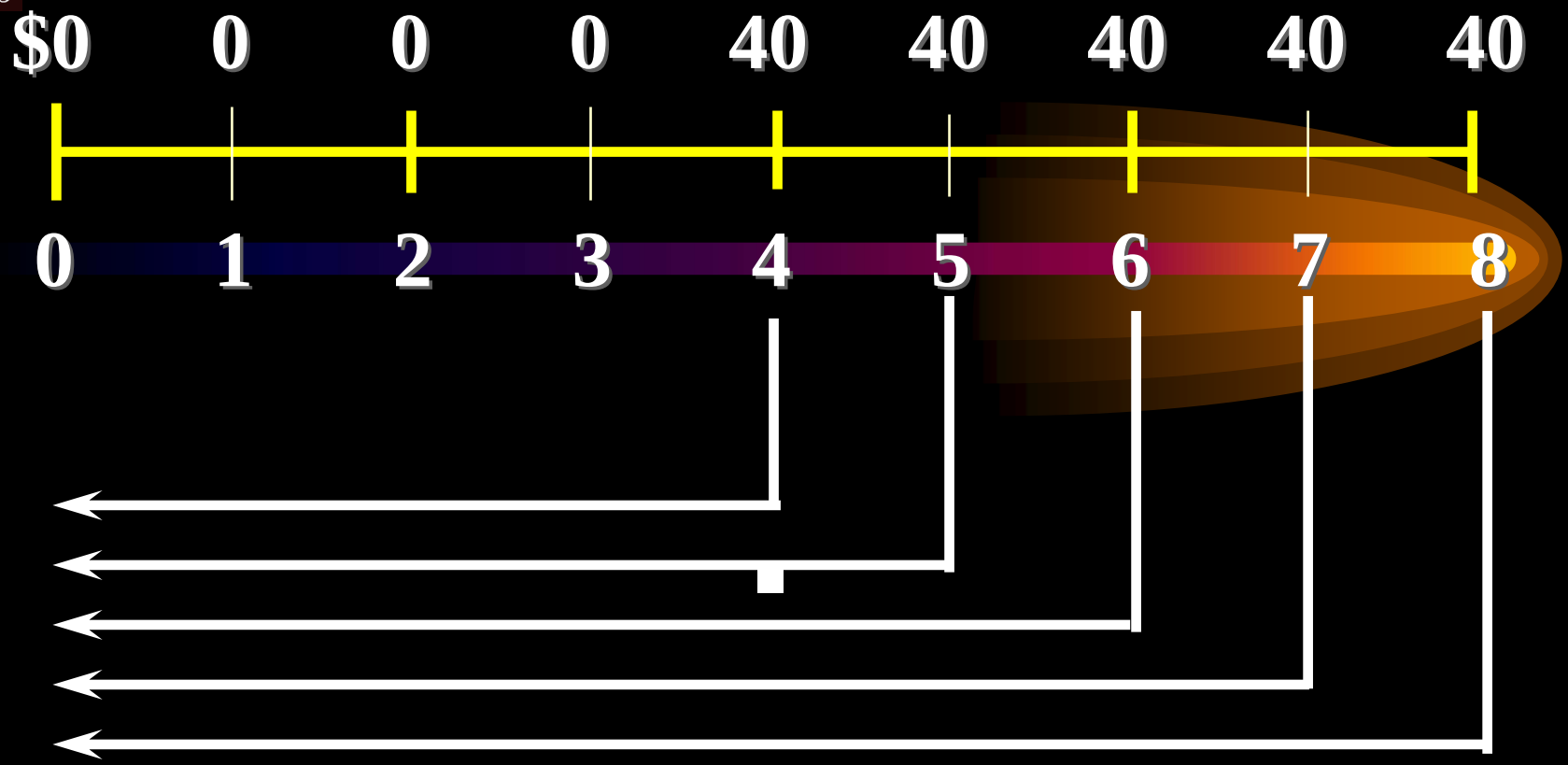


How to solve:

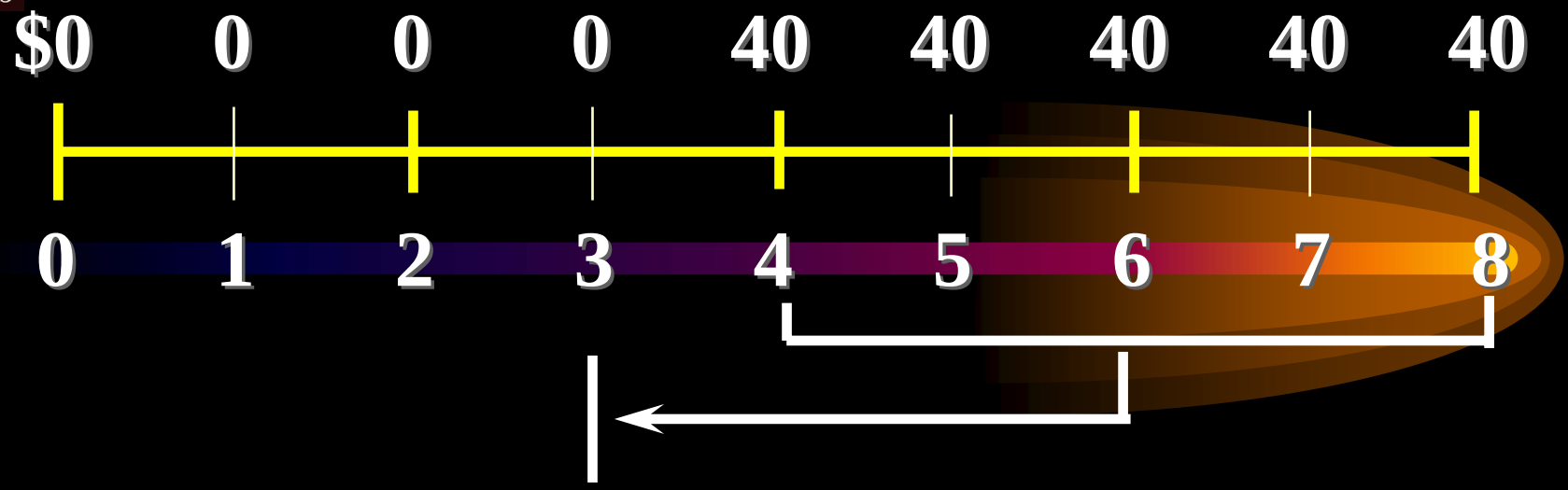
1) **Discount each cash flow back to time 0 separately.**

Or,

2) **Find the PV of the annuity:**

PV: End mode; P/YR = 1; I = 20;
PMT = 40,000; N = 5

PV = $119,624



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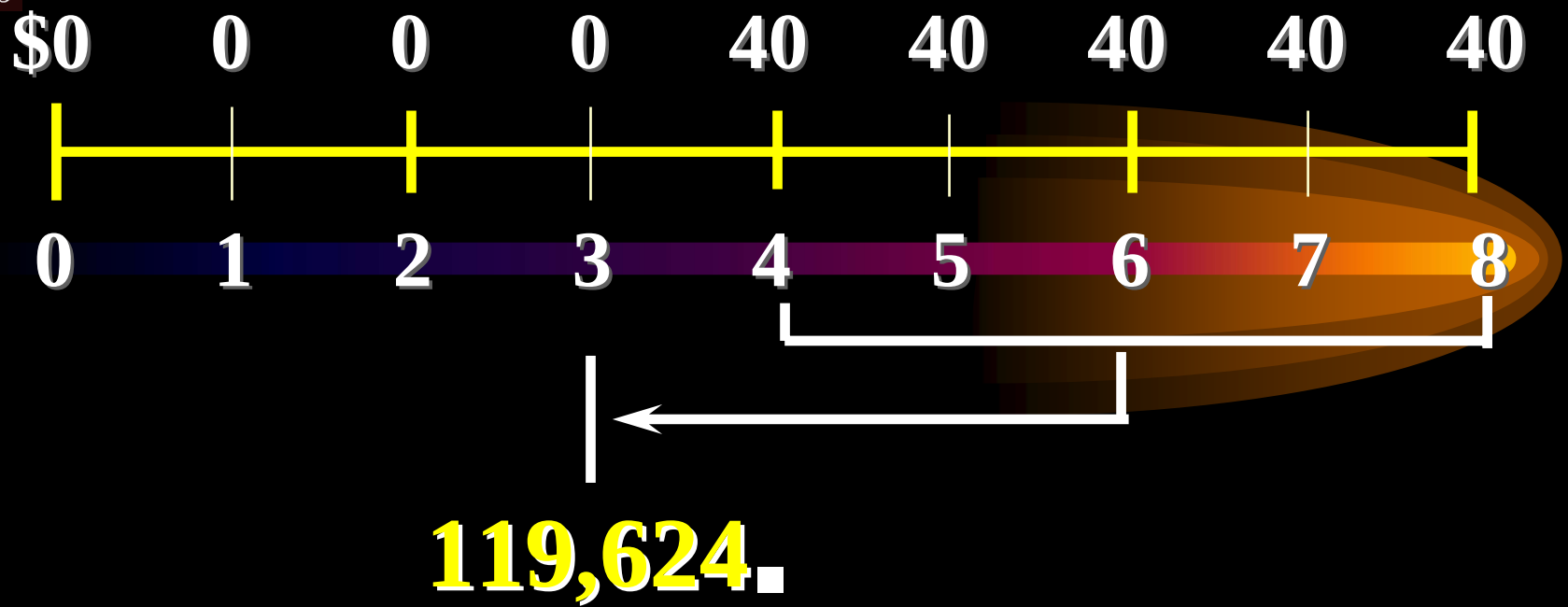
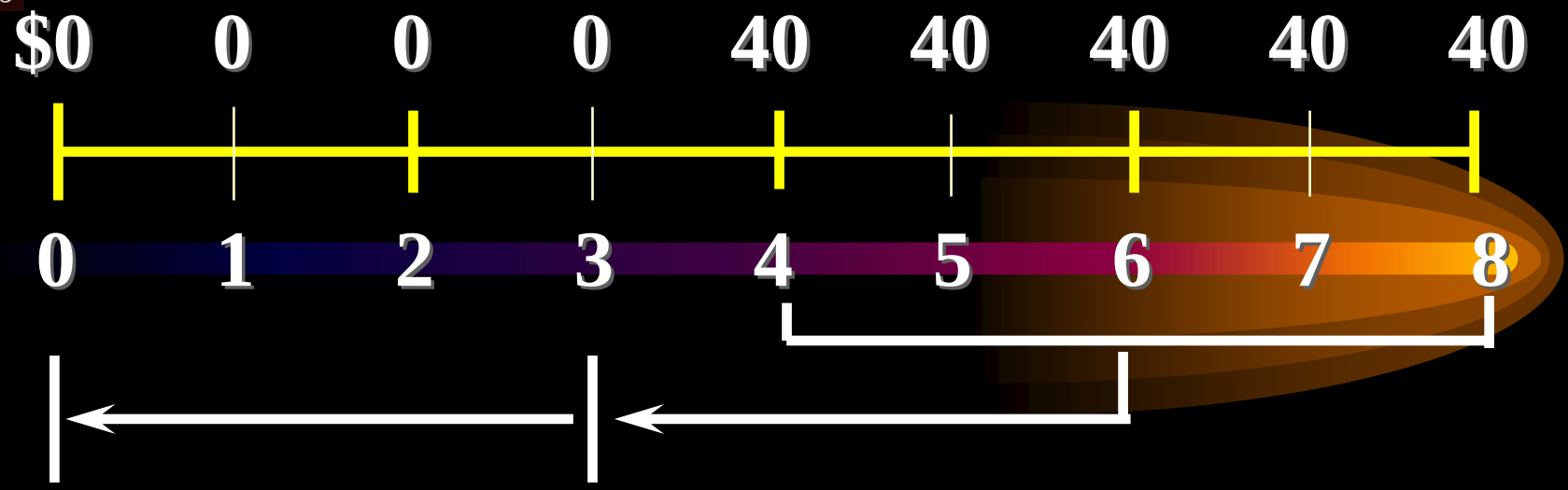
PMT = 40,000; N = 5

PV3= $119,624



119,624.



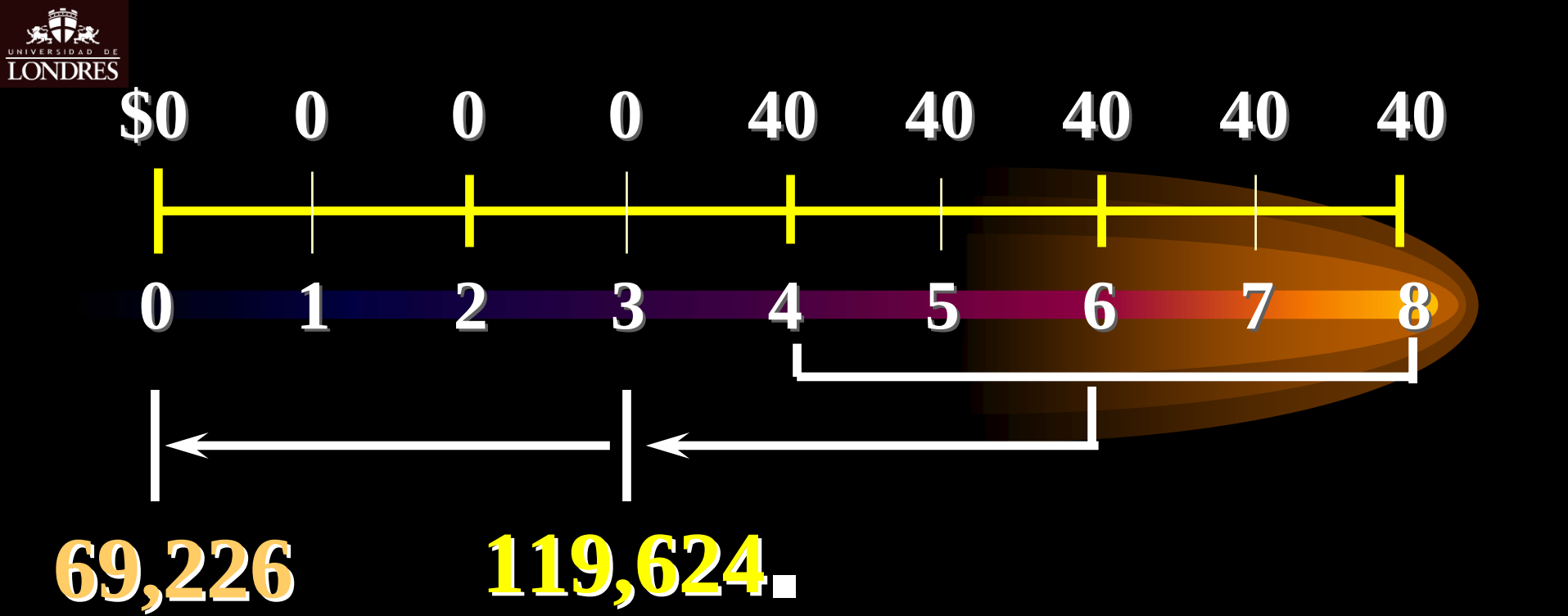
119,624.

Then discount this single sum back to time 0.

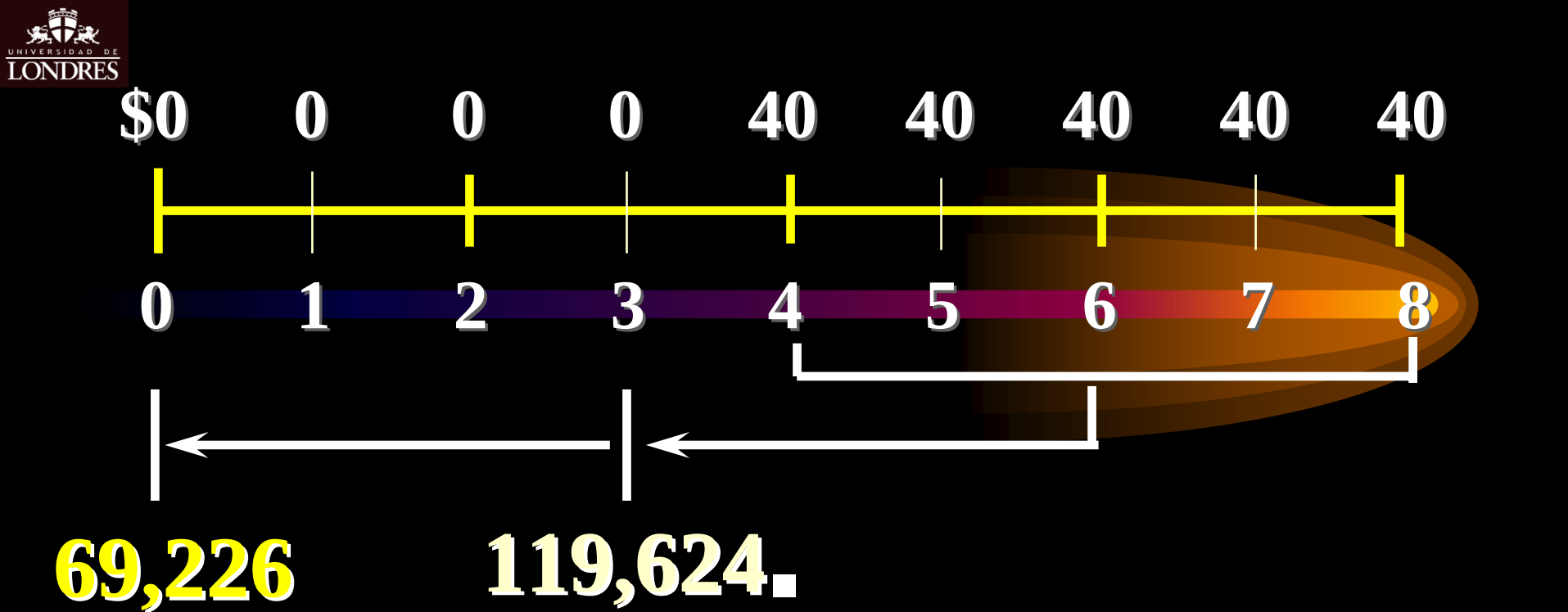
PV: End mode; P/YR = 1; I = 20;

N = 3; FV = 119,624;

Solve: PV = $69,226



69,226 119,624.



- The PV of the cash flow stream is $69,226.

Retirement Example

- **After graduation, you plan to invest $400 per month in the stock market. If you earn 12% per year on your stocks, how much will you have accumulated when you retire in 30 years?**

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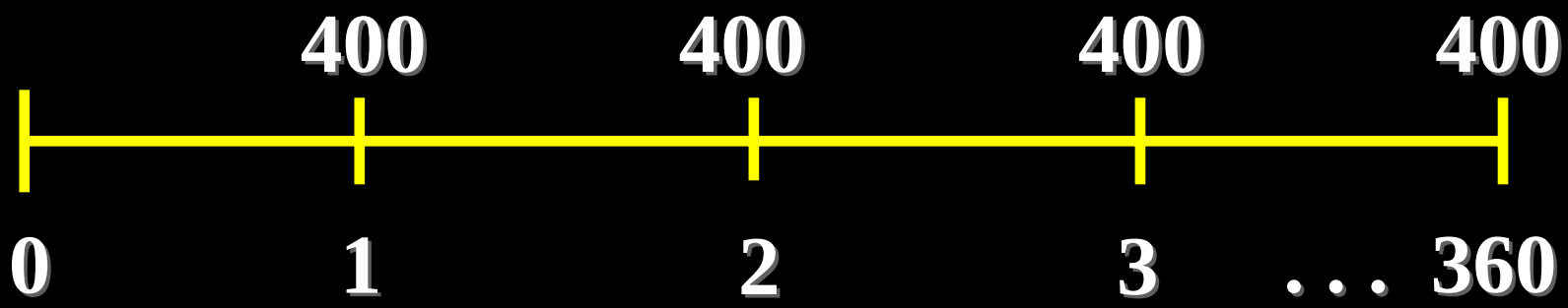

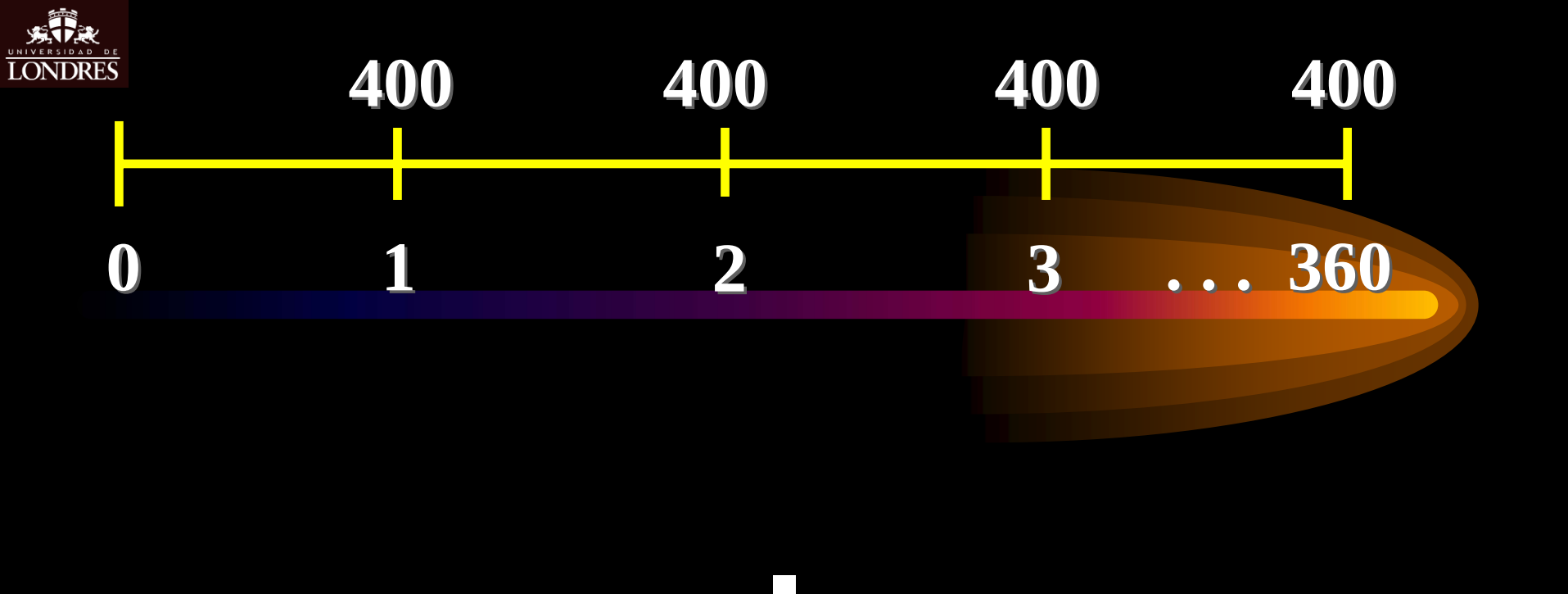
400
400
400
400
0
1
2
3
. . .
360

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If you invest $400 at the end of each month for the next 30 years at 12%, how much would you have at the end of year 30?

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$$FV = 400\,(FVIFA_{.01,\,360})$$ (can't use FVIFA table)

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$$FV = 400\left[\frac{(1.01)^{360} - 1}{.01}\right] = \$1{,}397{,}985.65$$

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If you borrow $100,000 at 7% fixed interest for 30 years in order to buy a house, what will be your monthly house payment?

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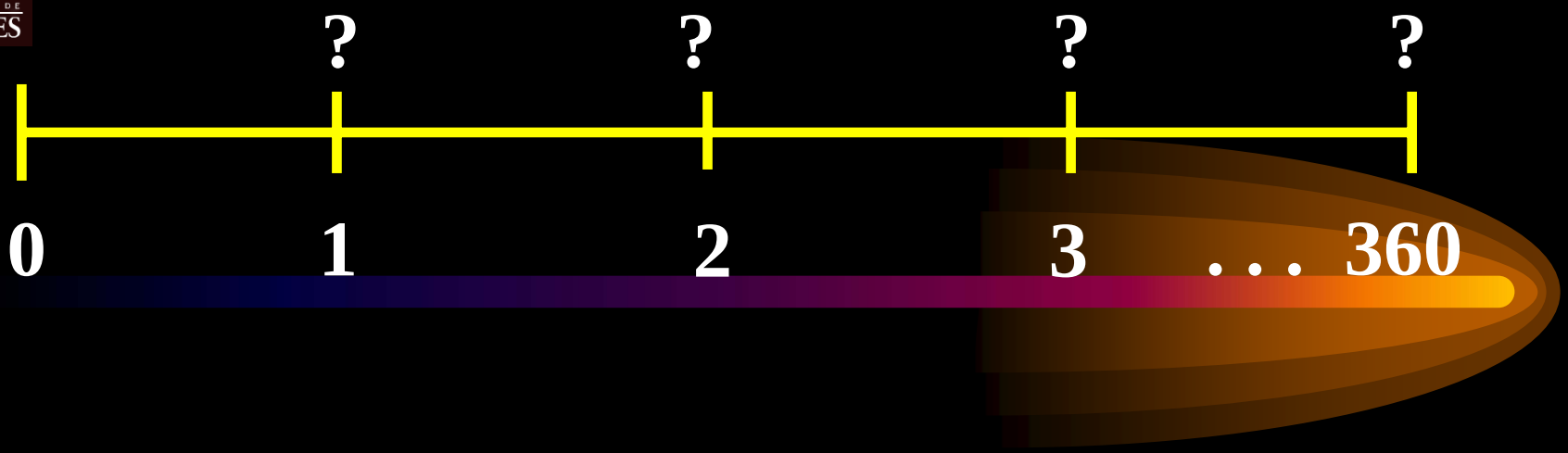
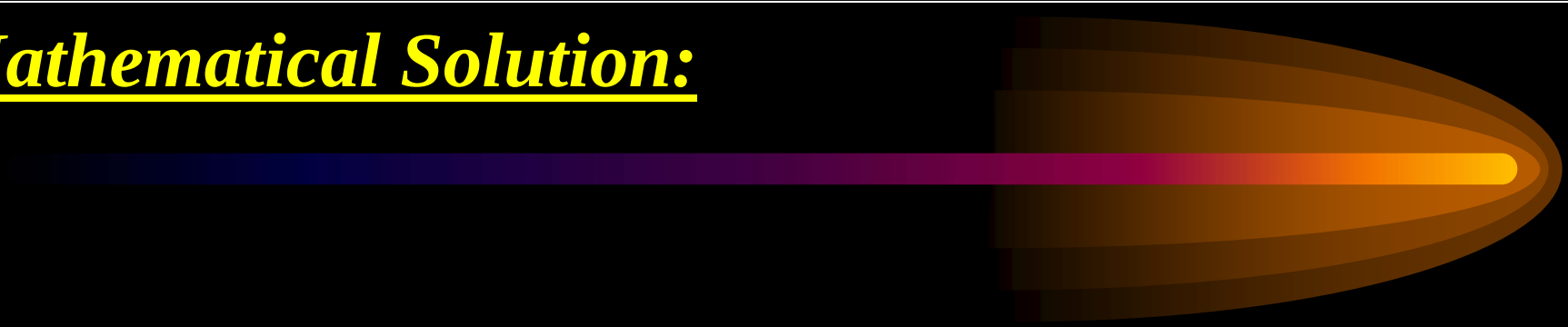

? ? ? ?
0 1 2 3 . . . 360

House Payment Example

Mathematical Solution:



House Payment Example

Mathematical Solution:

$$PV = PMT\ (PVIFA_{i,\,n})$$

$$100{,}000 = PMT\ (PVIFA_{.07,\,360})$$

(can't use PVIFA table)

$$PV = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

House Payment Example

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$$100{,}000 = PMT\ (PVIFA_{.07,\,360})$$

(can't use PVIFA table)

$$PV = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

$$100{,}000 = PMT \left[\frac{1 - \frac{1}{(1.005833)^{360}}}{.005833} \right]$$

PMT=$665.30